UNIVERSITAS TARTUENSIS

# Post-Newtonian parameter $\gamma$ for multiscalar-tensor gravity with a general potential

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Once we have been attracted by such theories, we must determine what constraints must be validated in order to pass the Solar system tests. In order to check soundness the Parametrized Post-Newtonian (PPN) scheme has been put forth.

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1 Action functional
- Action
- Degrees of freedom

2 Motivation

3 Equations of motion

4 Parametrized Post-Newtonian approximation
- Sources
- Asymptotics
- Perturbations
- Equations for perturbed variables
- Solutions
- Geometric interpretation

5 Conclusions

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$$\mathcal{E}^\gamma = \mathcal{F}^{\gamma\alpha}\left(\frac{\kappa^2}{2\mathcal{F}}\frac{\partial\mathcal{U}}{\partial\Phi^\alpha} - \frac{1}{\mathcal{F}^2}\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\kappa^2\mathcal{U}\right),$$

- The PPN formalism assumes that asymptotically the spacetime is flat, i.e. that the background metric is Minkowskian $\overset{(0)}{g}_{\mu\nu} = \eta_{\mu\nu}$ and the scalar field is at some constant value $\overset{(0)}{\Phi} = \mathrm{const.}$
- In order to check the consistency conditions, let us study the equations of motion.

$$0 = \frac{1}{\mathcal{F}}\kappa^2 g_{\mu\nu}\mathcal{U} \qquad \Rightarrow \qquad \mathcal{U}|_{\overset{(0)}{\Phi}} \equiv \mathcal{U}_0 = 0\,,$$

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- Hence we obtain that the potential $\mathcal{U}$ and also its first derivative must be asymtotically vanishing (Minkowski background does not allow cosmological constant $\Lambda$).

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are relevant for the calculation of the PPN parameter $\gamma$.

- The PPN formalism has been developed to extract standardized information – the PPN parameters – characteristic of the slow motion weak field regime of metric gravity theories.
- The PPN calculations are done order by order. In other words orders of magnitude are ascribed to all quantities relative to the velocity $v^i = u^i / u^0$ of the source matter, which is taken to be a first order small quantity.
- In analogy with GR, gravity is sourced by matter, which is modeled by a perfect fluid.
- In the PPN approach it is argued that up to second order, that is necessary for calculating the Eddington parameter $\gamma$, the only surviving component of the stress-energy tensor $T^{(\chi)}_{\mu\nu}$ is

$$T^{(\chi)}_{00} = -T^{(\chi)} = \rho \propto \mathcal{O}(2) \,,$$

  where $\rho$ is the energy density.
- In the calculation we specify the matter source to be a point mass $M_0$ residing at the origin of spatial coordinates, $\rho = M_0 \delta(r)$.

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- The equation for the scalar field

$$\nabla^2 \overset{(2)}{\Phi}{}^\gamma = \mathcal{M}^\gamma{}_\alpha \overset{(2)}{\Phi}{}^\alpha + k^\gamma \rho \,,$$

where $k^\gamma = \mathcal{K}^\gamma|_0$ and the components of the "mass matrix" are

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$$\frac{\delta S}{\delta g^{\mu\nu}} = 0 \quad\Rightarrow\quad \mathcal{F}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + g_{\mu\nu}\Box\mathcal{F} - \nabla_\mu\nabla_\nu\mathcal{F} + \frac{1}{2}g_{\mu\nu}\mathcal{Z}_{\alpha\beta}\nabla_\rho\Phi^\alpha\nabla^\rho\Phi^\beta - \mathcal{Z}_{\alpha\beta}\nabla_\mu\Phi^\alpha\nabla_\nu\Phi^\beta + \kappa^2 g_{\mu\nu}\mathcal{U} = \kappa^2 T^{(\chi)}_{\mu\nu}\,,$$

$$\frac{\delta S}{\delta \Phi^\alpha} = 0 \quad\Rightarrow\quad \mathcal{F}_{\alpha\beta}\Box\Phi^\beta = \mathcal{E}_\alpha - \mathcal{K}_\alpha T^{(\chi)}\,, \qquad \mathcal{F}_{\alpha\beta} \equiv \frac{1}{4\mathcal{F}^2}\left(2\mathcal{F}\mathcal{Z}_{\alpha\beta} + 3\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\frac{\partial\mathcal{F}}{\partial\Phi^\beta}\right),$$

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Let us recall the form of metric perturbations

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The radius dependent part in both $G_{\text{eff}}$ and $\gamma$, namely

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$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{F}R - \mathcal{Z}_{\alpha\beta}g^{\mu\nu}\partial_\mu\Phi^\alpha\partial_\nu\Phi^\beta - 2\kappa^2\mathcal{U}\right) + S_m[g_{\mu\nu},\chi_m]\,,$$

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0 \quad\Rightarrow\quad \mathcal{F}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + g_{\mu\nu}\Box\mathcal{F} - \nabla_\mu\nabla_\nu\mathcal{F} + \frac{1}{2}g_{\mu\nu}\mathcal{Z}_{\alpha\beta}\nabla_\rho\Phi^\alpha\nabla^\rho\Phi^\beta - \mathcal{Z}_{\alpha\beta}\nabla_\mu\Phi^\alpha\nabla_\nu\Phi^\beta + \kappa^2 g_{\mu\nu}\mathcal{U} = \kappa^2 T^{(\chi)}_{\mu\nu}\,,$$

$$\frac{\delta S}{\delta \Phi^\alpha} = 0 \quad\Rightarrow\quad \mathcal{F}_{\alpha\beta}\Box\Phi^\beta = \mathcal{E}_\alpha - \mathcal{K}_\alpha T^{(\chi)}\,, \qquad \mathcal{F}_{\alpha\beta} \equiv \frac{1}{4\mathcal{F}^2}\left(2\mathcal{F}\mathcal{Z}_{\alpha\beta} + 3\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\frac{\partial\mathcal{F}}{\partial\Phi^\beta}\right)\,,$$

$$\mathcal{K}_\alpha = -\kappa^2\frac{1}{4\mathcal{F}^2}\frac{\partial\mathcal{F}}{\partial\Phi^\alpha}\,, \qquad \mathcal{K}^2 \equiv \mathcal{K}_\alpha\mathcal{F}^{\alpha\beta}\mathcal{K}_\beta$$

The radius dependent part in both $G_{\text{eff}}$ and $\gamma$, namely

$$\Gamma(r) = -\frac{4\mathcal{F}_0^2}{\kappa^4}\left[k_\alpha P^\alpha{}_{(\beta)} E^{(\beta)}_{(\delta)} (P^{-1})^{(\delta)}{}_\gamma k^\gamma\right] ,$$

seems rather complicated, but if the "mass matrix" is diagonalizable, then it has a nice geometric interpretation.

$$P^\alpha{}_{(\delta)} = v^\alpha{}_{(\delta)} , \qquad \mathcal{M}^\gamma{}_\alpha v^\alpha{}_{(\delta)} = m^2_{[\delta]} v^\gamma_{(\delta)} , \qquad E^{(\beta)}_{(\delta)} = \left(e^{-\sqrt{J}r}\right)^{(\beta)}_{(\delta)} = e^{-m_{[\delta]}r}\delta^{(\beta)}_{(\delta)} .$$

The deviation term can now be unrevealed as

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