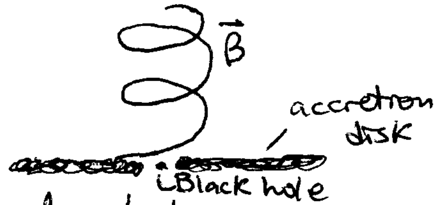
ENIGMA Winter School - Lesson 1 2004

Marscher



I. Launching of Jet by Magnetic Forces

If $\vec{B}$ is connected to accretion disk, differential rotation will wind field into mostly toroidal shape.

MHD eq.: $\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}\right) = \frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B} - \vec{\nabla}p + \vec{F}_{ext}$ $\underbrace{= \frac{1}{4\pi}\vec{B}\cdot\nabla\vec{B} - \vec{\nabla}\frac{B^2}{8\pi} - \vec{\nabla}p + \vec{F}_{ext}}_{\text{not used}}$

Choose cylindrical coordinates r, ϕ, z

not used $\left[\nabla\vec{B} = \hat{r}\left(\frac{\partial B_r}{\partial r} - \frac{B_\phi}{r}\right) + \hat{\phi}\left(\frac{1}{r}\frac{\partial B_\phi}{\partial \phi} + \frac{B_r}{r}\right) + \hat{z}\frac{\partial B_z}{\partial z}\right]$

$$\vec{\nabla}\times\vec{B} = \left[\frac{1}{r}\frac{\partial B_z}{\partial \phi} - \frac{\partial B_\phi}{\partial z}\right]\hat{r} + \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left[\frac{\partial(rB_\phi)}{\partial r} - \frac{\partial B_r}{\partial \phi}\right]\hat{z}$$

$\approx$ Toroidal $\vec{B}$: $\vec{B} \approx B(r,z)\,\hat{\phi} \Rightarrow \vec{\nabla}\times\vec{B} \approx -\frac{\partial B}{\partial z}\hat{r} + \frac{1}{r}\frac{\partial(rB)}{\partial r}\hat{z}$

$$(\vec{\nabla}\times\vec{B})\times\vec{B} = \hat{r}\left[-\frac{B^2}{r} - B\frac{\partial B}{\partial r}\right] + \hat{z}\left(-B\frac{\partial B}{\partial z}\right)$$

MHD eq. becomes $\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right) = -\frac{B}{4\pi}\left[\hat{r}\left(\frac{B}{r} + \frac{\partial B}{\partial r}\right) + \hat{z}\frac{\partial B}{\partial z}\right] - \vec{\nabla}p + \vec{F}_{ext}$

$-\frac{B^2}{4\pi r}\hat{r}$ is called the "hoop stress" or "magnetic pinch"

It is a force directed toward the axis
[Magnetic field lines tend to straighten themselves]

$\Rightarrow$ collimation as long as $-\frac{B}{4\pi}\frac{\partial B}{\partial r}\hat{r}$ term isn't stronger

generally negative [If $B \propto r^{-b}$, need $b < 1$]

Since $\frac{\partial B}{\partial z} < 0$ for physically reasonable situation,

$-\frac{B}{4\pi}\frac{\partial B}{\partial z}\hat{z}$ term is positive $\Rightarrow$ acceleration of flow along polar axis

$\hookrightarrow = -\vec{\nabla}\frac{B^2}{8\pi}$ gradient in magnetic pressure

Even if $B \propto r^{-b}$, $b > 1$, opening angle will narrow if $\left|\frac{\partial B}{\partial z}\right| > \left|\frac{\partial B}{\partial r}\right|$