# ENIGMA Winter School

# Contents

# 1 Launching of Jet by Magnetic forces

If B is connected to accretion disk, differential rotation will wind field into mostly toroidal shape.

MHD eg.:

$$\rho(\frac{\partial\vec{V}}{\partial t}+\vec{V}*\nabla\vec{V})=\frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B}-\vec{\nabla}p+\vec{f}_{ext}$$

Choose cylindrical coordinates r, $\phi$, z

$$\vec{\nabla}\times\ \vec{B}=[\frac{1}{r}\frac{\partial B_z}{\partial\phi}-\frac{\partial B_\phi}{\partial z}]\hat{r}+(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r})\hat{\phi}+\frac{1}{r}[\frac{\partial(rB_\phi)}{\partial r}-\frac{\partial B_r}{\partial\phi}]\hat{z}$$

$$\approx Toroidal\vec{B}:\vec{B}\approx B(r,z)\hat{\phi}\rightarrow\vec{\nabla}\times\vec{B}\approx-\frac{\partial B}{\partial z}\hat{r}+\frac{1}{r}\frac{\partial(rB)}{\partial r}\hat{z}$$

$$(\vec{\nabla}\times\vec{B})\times\vec{B}=\hat{r}[-\frac{B^2}{r}-B\frac{\partial B}{\partial r}]+\hat{z}(-B\frac{\partial B}{\partial z})$$

MHD eq. becomes

$$\rho(\frac{\partial\vec{V}}{\partial t}+\vec{V}*\nabla\vec{V})=-\frac{B}{4\pi}[\hat{r}(\frac{B}{r}-\frac{\partial B}{\partial r})+\hat{z}\frac{\partial B}{\partial z}]-\vec{\nabla}p+\vec{F}_{ext}$$

- $\frac{B^2}{4\pi r}\hat{r}$ is called the "hoop stress" or "magnetic pinch". It is a force directed toward the axis (magnetic field lines tend to straighten themselves) $\Rightarrow$ collimation as long as -$\frac{B}{4\pi}\frac{\partial B}{\partial r}\hat{r}$ isn't stronger.

Since $\frac{\partial B}{\partial z}<0$ for physically reasonable situation (if $B\propto r^{-b}$ need $b<1$, term is positive $\Rightarrow$ acceleration of flow along polar axis.

$-\frac{B}{4\pi}\frac{\partial B}{\partial z}\hat{z}=-\vec{\nabla}\frac{B^2}{8\pi}$ Gradient in magnetic pressure.

Even if $B\propto r^{-b}$, b > 1, opening angle will narrow if $|\frac{\partial B}{\partial z}|>|\frac{\partial B}{\partial r}|$

## 1.1 Considerations

1. Asumption of MHD = $\vec{B}$ field is frozen into plasma $\rightarrow$ fluid.

2. $B_\phi$ requires a current, $I(r) = \frac{cr}{2}B$. There must also be a return current, which will cause dissipation of energy where it crosses field line near base of flow.

Dissipation region could be near black hole(BH) $\rightarrow$ black hole "battery" (Lovelace 1916; Blandford  Znajek 1918)

In any case, ideal MHD does not include dissipation.

3. Current-carrying jets are unstable; Nakamura  Meier (2004, in Plasmas in the laboratory + in the Universe; available on astro-ph) propose that curren-driven kink instability might account for bends observed in pc-scale jets.

### 1.2 MHD Simulations

Meier, Koide  Uchida (2001, Science, 291, 84): Partly successful, but did not produce high-T jets.

Vlahakis  Königl (2004, ApJ, in press): Poynting-flux dominated jet$\sim \frac{1}{2}$ of MHD energy is converted to flow energy if $\frac{B}{\sqrt{4\pi\rho c^2}} \gg 1 \Rightarrow$ acceleration + collimation can continue out to pc scales.

Much more work needs to be done at higher dynamic range (largest to smallest scales of mesh).

## 2 Alternative: Hydrodynamical Acceleration + Collimation

(Brandford + Rees 1974, MNRAS, 169, 395)

Assume negative presure gradient in ambient thermal medium.

$$p \propto z^{-a}$$

Relativistic Bernoulli eq.: $\frac{\Gamma w}{n}$ = constant along each streamline, w = enthalpy density = p + e, e = energy density.

Start with a relativistically "hot" plasma. In this medium $p = \frac{e}{3}$, w = 4p $\propto z^{-a}$. Adiabatic index $\hat{\gamma} = \frac{4}{3}$

$$p \propto \rho^{\hat{\gamma}} \rightarrow n \propto \rho \propto p^{\frac{1}{\hat{\gamma}}} \Rightarrow \Gamma \propto \frac{n}{w} \propto p^{-(1-\frac{1}{\hat{\gamma}})} \propto p^{-\frac{1}{4}} \propto z^{\frac{a}{4}}$$

$$\Gamma \propto n^{-\frac{1}{3}} \propto r$$

Acceleration continues until $\Gamma \approx \frac{e_0}{\rho_0 c^2} = <\gamma_0>$ ( E= $\gamma mc^2$, conversion of internal energy to bulk of flow energy) $\Rightarrow$ need to start with ultrarelativistic plasma to get $\Gamma \gtrsim 10$.

If $e^-/e^+$ plasma, $< E_0 > \sim (6 * 10^9 K)k\Gamma$
If $e^-/p^+$ plasma, $< E_0 > \sim (10^{13} K)k\Gamma$

Opening angle $\psi \sim \frac{r}{z}$,
$r \propto n^{-\frac{1}{3}} \propto p^{-\frac{1}{4}} \propto z^{\frac{a}{4}}$
$\psi \propto z^{-(1-\frac{a}{4})} \propto z^{-\frac{4-a}{4}} \propto p^{-\frac{4-a}{4a}}$

Probably the most favorable case is if the ambient thermal medium is an isothermal, constant-velocity wind, in which case $p \propto n \propto z^{-2} \rightarrow a = 2, \psi \propto z^{-\frac{1}{2}} \propto p^{-\frac{1}{4}}$

Main constant is not to exceed observed X-ray luminosity. Collimation by factor $\sim 20$ (e.g., from 40° to 2°) by z $\sim$ 1 pc with p(1 pc) $\sim 0.1 dyne\ cm^{-2}$ reguires $p_0 \sim 10^4 dyne\ cm^{-2}$ at $z_0 \sim 0.01pc$

$L_x(x_0) \sim 3 * 10^{49} x_{0,pc}^{-3} p_0^2 T_6^{-3/2} erg\ s^{-1} \sim 3 * 10^{46} T_7^{-3/2} erg\ s^{-1}$

Needs to be $\lesssim 10^{45} erg\ s^{-1}$, so $T \sim 10^8$ K is needed $\rightarrow$ Might be possible if strong heating of wind by radiation.

# 3 Unconfined Ballistic Jets

If jet plasma's internal energy becomes rest-mass dominated beyond region of main collimation + acceleration, it will maintain $\sim$ constant $\Gamma$ + constant $\psi$ (cone)

If not ress-mass dominated ($e^- - e^+$ plasma with $< \gamma > > 1$), jet will accelerate hydrodynamically.

Observed: Jets may accelerate out to pc scales but not beyond.

# 4 Jets Confined by External Gas Pressure (on Parsec Scales?): 2-D + 3-D

Blandford-Rees treatment is 1-D. 2-D (or 2+D cylindrically symmetric) case is more complex

Consider first idealized scennario: cylindrical, supersonic jet enters a region where confining pressure drops suddenly. The pressure change is transmitted by sound waves that fan out from the point of change:

The jet expands + accelerates, but overexpands because of its inertia $\rightarrow$ pressure drops; jet pressure falls below that of medium, + compression sound waves pinch jet to $\sim$ original radius r

If pressure drop is high enough ($\gtrsim$ 30 $°/_{\circ}$?), compression waves steeper to form standing conical shocks

-get similar results in a non-steady jet with no sharp pressure drop

External pressure gradient: similar, but each recollimation section is wider then preceding one.

Divergin flow at inlet: Conival shocks form in first section usually "Mach disk" transverse shock near axis.

3-D: Jet does not tend to remain symmetric, so standing shocks are not exactly conical + Mach disk is weak or absent

# 5 Instabilities of Confined Jets

Already mentioned: Kink instability of current-carrying jet

Oscillations of cross-section discussed above is a "body mode" often called a pinch instability

Other body modes include helical instability. Jet can be concentrated in twisting ribbons of higher $\rho$

Kelvin-Helmholtz instability occurs when two parallel flows have different velocities, as at boundary of jet $\rightarrow$ causes turbulence + entrainment of external medium.

Assigment:

Divide into 3 groups. Explain with 2 models each observation. For each model, make 2 further predictions.

1. 3C279 has trongly correlated optical + X-ray flux variations, with x-ray lagging optical by $\sim$ 10 days.

2. PKS 1510-089 has strongly correlated 15 GHz radio + X-ray flux variations, with X-ray lagging radio by $\sim$ 15 days. [Light curves + correlation plots were shown; see www.bu.edu/blazars]

# 6 Variability of Nonthermal Emission from Blazar Jets

## 6.1 Single-zone models

("blobs") vander Laan/Shklovskij models

Confict with radio data: predict strong depence of amplitude of variability on frequency: $F_m \propto \nu_m^\zeta, \zeta = 1-2$ for typical set of time dependences($B \propto r^{-1}$ $or$ $r^{-2}, K \propto r^{-(2\alpha+3)}$ where electron energy distribution $N(\gamma) = K\,\gamma^{-(2\alpha+1)}$

Also time lags at lower radio $\nu$'s are longer than observed. Light-travel effects were ignored $\rightarrow t_{evolve} > R/c$ ease only.

## 6.2 Gradients in maximum electron energy

(Marscher + Gear 1985, ApJ298, 114)

Accelerate particles at a front (probably a shock), which cool from radiative + expansion losses as they advect behind front (in frame of front). Critical frequency of electron with energy $\gamma mc^2 : \nu(\gamma) = 2.8 * 10^6 B\gamma^2$

Approximate B$\simeq$ constant + constant velocity of advection $B_{ad}C$ in cooling zone; then $\frac{d\,\gamma}{d\chi'} \propto \frac{d\,\gamma}{dt} = -bB^2\gamma^2, b = 1.3 * 10^{-9}$

Integrate, replace $\gamma$ with $[\frac{\nu}{2.8*10^6 B}]^{\frac{1}{2}}$, solve for $\chi' \approx 0.4B^{-\frac{3}{2}}\nu_{GHz}^{-\frac{1}{2}}(\frac{\delta}{1+z})^{\frac{1}{2}}B_{ad}$ $pc$, $\chi \approx \Gamma\chi'$

Assume that behind the shock B=const. $B_{jet}(z)$, $K = const.\ K_{jet}(z)$, $z$ = distant from jet.

Optical depth at peak in $F_\nu$ spectrum. $\tau_m \approx 0.5 \propto B^{\frac{2\alpha+3}{2}}Ks\nu^{\frac{-(2\alpha+5)}{2}}$ where s = line-of-sight thickness of region that emits at frequency $\nu$

Flux density $F_m \equiv F_\nu(\nu_m) \propto B^{\alpha+1}Kr^2\chi\nu^{-\alpha}$

See Section 6.3
s=$\chi$ for viewing angle $\theta = 0$
s=2r for viewing angle $\theta \approx sin^{-1}(\frac{1}{\Gamma})$
s$\propto$ $\chi$ for other viewing angles

Assume power-law dependences $B_{jet}(Z) \propto Z^{-a}, K(Z) \propto Z^{-\frac{2}{3}(2\alpha+3)}$

This expression for $K(Z)$ is based on adiabatic cooling in a constant-$\Gamma$ conicaloutflow. The $\frac{2}{3}$ factor in the exponent reflects that the expansion is only $\perp$ axis since flow speed is constant $\rightarrow$ dependence on Z same as on time.

Also, conical flow assumption gives $r \propto Z$

Algebra gives as $F_\nu(\nu > \nu_m) \propto Z^{-\frac{8\alpha+3a(2\alpha-1)}{6}}\nu^{-(\alpha+\frac{1}{2})}$ (OK for any angle) $\propto Z^{-1.13}\nu^{-1.2}$ for $\alpha = 0.7, a = 1$

Dependence of flux at turnover in spectrum on frequency of turnover $\nu_m$; $F_m \propto \nu_m^{\frac{(4\alpha-3)(2+3a)}{4(2\alpha+3)+6a\alpha}} \propto \nu_m^{-0.05}$ for $\alpha = 0.1, a = 1, \chi < \chi_{chock} + \theta \not\approx sin^{-1}(\frac{1}{\Gamma})$

For a viewing angle that maximizes superluminal apparent motion, $\theta \approx sin^{-1}(\frac{1}{\Gamma})$
$\nu_m \propto Z^{-\frac{(3a+4)(2\alpha+3)-6}{3(2\alpha+5)}} \propto Z^{-1.45}$
$F_m \propto \nu_m^{\frac{10\alpha-12a-3}{(3a+4)(2\alpha+3)-6}} \propto \nu_m^{-0.36}, \alpha = 0.1, a = 1$
The depence of $F_m$ on $\nu_m$ is much weaker then - and even inverted relative to - that of the expanding blob model. It is much closer to the observed behavior.

Once $\nu_m$ reaches frequency at which $\chi' = \chi_{chock}$ (assume $\propto Z$) the flare reaches the "adiabatic" stage at which $F_m \propto \nu^{\frac{5[4\alpha+3(a-1)]}{6a\alpha+9a+9\alpha+6}} \propto \nu_m^{0.56}, \alpha = 0.7, a = 1, all\ \theta$

What causes the flare to rice? Orginal guess: reduction in SSC losses - criticized by Björnssen, probably not valid model.
Formation of shock? Acceleration/bending that increases $\delta$?

## 6.3 Light-travel effects + aberation

If $\Delta t_v ar \lesssim R/C$, light-travel delays are important $\rightarrow$ need to consider "retarded" time:

Aberation acts in a similar sense: Radiation travels toward observer from each point in feature when the observer-frame view is adopted.

See Gomez et al.(1994,A+A,292,33). For equations + discussion on how to compute aberation + time delays in bent jets.

## 6.4 Synchrotron Self-Compton Emission + Energy losses: A few notes

Already discussed by Dr.Kirk in $\delta$-function approximation
- But $\delta$-function approximation is only good if seed photon spectrum is strongly peaked, as for a blackbody.

A. In SSC case (McHardy et al.1999 MNRAS,310,571) all seed photons that can scatter up to $\nu$(X-ray, $\gamma$-ray) contribute equally

$\rightarrow$ lower-$\nu$ photons (abudant) are scattered by higher-E electrons (rare)

$\rightarrow$ mid-$\nu$ $\gamma$'s (abudant) are scattered by mid-E electrons

$\rightarrow$ higher-$\nu$ $\gamma$'s (abudant) are scattered by lower-E electrons (abudant)

B. During a flare, electrons need to wait for flare's seed photons to arrive $\rightarrow$ causes delay in SSC emission if $\theta \approx 0°$ (Sokolov, Marscher, + McHardy 2004, ApJ, submitted).

C. SSC Losses are very difficult to calculate: they depend on retarded radiation field seen by electrons at each location of source. But radiation field depends on history of electron energy distribution which depends on history of losses. Nonlinear problem!

Easy way out (but how accurate is it?): Determine $(L_\gamma/L_{IT}+1)$ observationally + multiply this by $B^2$ in synchrotron loss formula

- Should work well for external Compton (EC) model in which seed photons are from source outside of jet.

D. Electrons may see a different synchrotron photon field than observer because of geometry + time delays.

E. SSS flux density can be predicted from observations of features in the jet with VLBI at several $\nu$'s - only done thus far for a few blazars. For formula, see Marscher (1983 ApJ); if I don't forget, I will add it to these notes on the ENIGMA website.

# 7 Values of Γ

A. From superluminal motion

$$beta_{app} = \frac{\beta sin\theta}{1-\beta cos\theta} \geq \Gamma_\beta$$

Observed in compact radio sources (flux-limited sample): $\beta_{app}$ as high as $\approx 40$ ($\gamma$-ray blazars: Jorstad et al. 2001 ApJS, 134,181; Kellermann et al. 2004, ApJ, submitted) [Statistical studies indicate that high-Γ jets are rare in volume but easy to detect because of beaming $\rightarrow$ vastly over-represented in flux-limited surveys.]

B. Theoretically:
$\Gamma \approx \frac{1}{2}(\frac{B^2/8\pi}{\rho c^2})$ at base of jet (Conversion of Poynting flux to kinetic energy in magnetic launching models) But: If $\Gamma \gtrsim 10$ close to central engine, high-dencity photon field will interact with electrons $\rightarrow$ momentum loss ("compton drag"- Phinney 1987, in Superluminal Radio Sources, ed. Zeusus + Pearson, 301) $\Rightarrow$ argues in favor of acceleration of jet out to pc-scales.

# 8 Opening Angles

FRII Radio galaxies viewed at $\theta \approx 90^\circ : \psi$ typically $1^\circ - 3^\circ$
-But most of these pobably have low $\Gamma$'s

Blazars: ex. 3C279: $\psi_{obs} \approx 5^\circ$ but $\psi_{obs} = \psi/sin\theta$ ($\theta << 1$)
Derivation: from figure, in l.o.s coordinates, sin $\psi_{obs} = \frac{r\omega\theta}{Zsin\theta}$ and $r = Ztan\ \psi \rightarrow sin\psi_{obs} = \frac{Ztan\psi cos\theta}{Zsin\theta} = \frac{\psi}{sin\theta}$ for $\psi << 1, \theta << 1$

in 3C279 , $\Gamma \gtrsim 25$; $\beta_{app}$ as high as 25 $\Rightarrow \theta \lesssim 2^\circ$, so $\psi \approx 5^\circ sin 2^\circ \approx 0.2^\circ$!

## 8.1 Polarization

A. Linear: Sychrotron radiation $\Pi \approx \frac{3(\alpha+1)}{3\alpha+5} \frac{B^2_{unif}}{B^2_{unix}+B^2_{rand}}$ (Burn 1966, MNRAS, 133,67)
Observed: most jet features have $\Pi \lesssim 10^\circ/_\circ \rightarrow B_{rand} \sim 2.5 B_{unif}$
Direction = electric vector position angle $\chi$ (EVPA) $\perp$ projected $< \vec{B} >$
-But optically thick case: $\Pi \lesssim 0.05, \chi || \vec{B}_{proj}$

Scohk: Component of $\vec{B}$ that is —— shock front is amplified $\Rightarrow$ EUPA $\rightarrow$ $\perp$ shock front.

Observed: EVPA $\perp$ jet axis in most BL Lac jets but tend to be more often obligue in quasar jets

B. Faraday rotation $e^- - p^+$ plasmas only

$\Delta\chi = \frac{e^3}{2\pi m^2 c^2}\nu^{-2} \int_0^s n_e \vec{B} * d\vec{s} = 1.3 * 10^4 \nu^{-2}_{GHz} \int_0^{s_{pc}} n_e B_{l.o.s} ds$ Radians

$n_e = n_e(thermal) + \frac{n_e(relastivistic)}{<\gamma>^2}$

If $n_e B_{l.o.s}$ varies avross source + $\Delta\chi$ is large, effect causes depolarization from differential $\Delta\chi$'s.

C. Circular polarization

Tends to be at level $\Pi_{circ} \lesssim \gamma^{-1}(\nu)\frac{B^2_{unif}}{B^2_{unif}+B^2_{rand}}$, $(\gamma(\nu) = 19\nu^{\frac{1}{2}}_{GHz}B^{\frac{1}{2}}(\frac{1+Z}{\delta})^{\frac{1}{2}})$ (Legg +westfold 1968, ApJ,154,499) for optically thin source. Typically $\Pi_{circ} \lesssim 0.1°/_\circ$ expected.

Observed (Wardle, Homan, ed al.): Up to $\sim 1°/_\circ$ (e.g., 3C279) $\rightarrow$ conversion of linear to circular polarization in optically thick region (plasma effect)

D. SSC Polarization (Celotti + Matt 1994, MNRAS,268,451)

Linear $\Pi \approx 50^{circ}/_{circ}$ of synchrotron $\Pi$, same $\chi$.

## 8.2 Matter Content of Jets: $e^- - p^+ ore^- - e^+$ or both $\sim$ egual?

Galactic cosmic-rays: $e^- - p^+, u(p^) \sim 100u(e^-)$

$e^- - p^+$ advantages: More momentum, inertia $\rightarrow$ harder to distrupt, $\Gamma$ = constant after initial acceleration, as observed in many objects ($\beta \approx$ constant)

$e^- - e^+$ advantages: High-E processes make $e^- - e^+$ pairs, not much rest mass so easier to accelerate.

Attemps to decide issue are all flawed or not definitive

-Power needed to feed lobes - depends on value of $\gamma_{main}$ $e^- - e^+$

-Power needed to create high-E emissions - uses $\delta$-function approx. for SSC that is incorrect for X-ray emission.

- Circular polarization: Wardle + Homan claim low $\Delta\chi$ needed while high optical depth is present $\rightarrow$ mostly $(80^{circ}/_{circ})$ $e^- - e^+$ plasma $\rightarrow$ not true according to calculations by Ruszkowski + Begelman (2002, ApJ, 573,485)

## 8.3 Velocity Gradients Across Jet "Spine-sheath" structure

-Good evidense for this in extended jets of FRI radiogalaxies + milliarcsec-scale jet of Mkn501 (Givoletti et al. 2004 ApJ) [Could be caused by Kelvin-Helmholtz instability at boundary]

Often suggested fo compact jets when more free parameters are needed because observation data conflict with model- Dangerous! On the other hand, jet launching theorists say it's possible.

## 8.4 Jet Energy Density + Luminosity

Studies of FRII radio galaxies give $< L_{jet} > \gtrsim 3 * 10^{44\pm2} erg\ s^{-1}$. Lifetimes of jets $\approx 10^{7\pm1} yr$.

Compact Jets: Can solve for B,K from equations for $\tau_m + F_m$ if Z, $\Theta$, $\alpha$, $\nu_m, +F_m$ are measured with multi-$\nu$ VLBI

Or: Take easy way out + assume equipartition

First method gives B= $10^{-5} b_1 \Theta^4_{mas} \nu^5_{m,GHz} F^{-2}_{m,Jy} (\frac{\delta}{Hz})\ G\ (b_1 = 2to4 for \alpha = 0.3to1$

+ similar expression for K (see Marscher 1983 ApJ)

Equipartition gives $B = b_2(\alpha)\theta^{-1}(1+Z)^{\frac{5+\alpha}{3+\alpha}} [\frac{1+y}{f} D^{-1}_{Gpc} \Theta^{-3}_{mas} F_{\nu,Jy} \nu^{\alpha}_{GHz} \gamma^{-(2\alpha-1)}_{min}]^{\frac{1}{3+\alpha}}$ where $b_2(\alpha) = 0.3, 1.5, 4.4$ for $\alpha = 0.5, 0.75, 1$ respectively. Here $y \equiv \frac{u_{protons}}{u_{e^-,e^+}}$, $f \equiv$ volume filling facto of emission, $\gamma_{min} \equiv$ minimum energy of relativistic electrons. For $\alpha = 0.5$, replace $\gamma^{-(2\alpha-1)}_{mtn}$ by $ln(\gamma_{max}/\gamma_{min})$

For the most compact features($\Theta \sim 0.1 mas$) we can resolve, $B \sim 10^{0\pm1} G$, $u_{mag} = \frac{B^2}{8\pi} \sim 4 * 10^{-2\pm2} ergcm^{-3}, L_{mag} \approx \Gamma u_{mag} * \pi r^2 c \sim 10^{46\pm1} erg\ s^{-1}$ Warning: only measured in a few objects!

## 8.5 Model for Core

A. Low frequencies (radio) $\nu < \nu_m$: core=place where $\tau(\nu) \approx \tau_m$. To calculate $Z_{core}(\nu)$, need to specify B(Z), K(Z)

Frozen-in field: $B(Z) \propto Z^{-1} (\vec{B} \perp \hat{Z})$, $\quad B(Z) \propto Z^{-2} (\vec{B} || \hat{Z})$

$K(Z)$ is more involved:
$u \propto p \propto n^{4/3}$ and $u =< \gamma > n$
$< \gamma > \propto n^{1/3}$

Constant $\Gamma$ jet:$n \propto Z^{-2} \rightarrow < \gamma > \propto Z^{-2/3}$
$n = \int_{\gamma_{min}}^{\gamma_{max}} K\gamma^{-(2\alpha+1)} d\gamma = \frac{K}{2\alpha}[\gamma_{min}^{-2\alpha} - \gamma_{max}^{-2\alpha}]$
$K \propto n < \gamma >^{2\alpha} \propto n^{1+\frac{2\alpha}{3}} \propto Z^{-\frac{2}{3}(3+2\alpha)}$
Prior equations for $\tau_\nu$ and $F_\nu$ can be used to solve for $Z_{core}(\nu)$:
$Z_{core}(\nu) \propto \nu^{\frac{-(2\alpha+5}{2b+a(2\alpha+3)-2}}$ when $B \propto Z^{-a}, K \propto Z^{-b}$
$F_\nu(\nu < \nu_m) \propto Z_{core}^{3-b-a(1+\alpha}(\nu)\nu^{-\alpha} \propto \nu^{\alpha_{thick}}$
$\alpha_{thick} = \frac{[b+a(1+\alpha)-3](2\alpha-5)}{2b+a(2\alpha+3)-2} - \alpha = \frac{4\alpha(a-1)+5(a+b-3)}{2b+a(2\alpha+3-2}$
e.x.: $\alpha = 0.7, b = frac23(3 + 2\alpha), a = 1; \alpha_{thick} = 0.56$
Testable: $Z_{core}$ dependence on $\nu$ is measurable with phase-reference VLBI (with mixed results!).
Why does $\alpha_{thick}$ not continue above $\nu_m$?
-Self-similarity of jet must stop at $Z_{core}(\nu_m)$
Two ideas: A. standin shock(s) accelerates electron at $Z_{core}(\nu_m)$, jet is "cool" at lower $Z$'s.
B. Jet accelerates out to parsec scales, $Z_{core}(\nu_m)$ is where $\sigma$ reaches maximum value.

# 9 Spine-Sheath Jet Model

On Kpc scales, there is evidence for velocity gradient between the jet axis + boundary (Laing is the primary proponent of this, with papers in MNRAS starting in the late 1990's)

In Mkn501, Giroletti et al.(2004 ApJ) interpret the observations on milliarcsec scales in the same way, but with $\Pi_{Spine}$ ~5-10 and $\Pi_{Sheath} \sim 2$

A number of researchers appeal to such structure in order to increase the number of free parameters when a model does not fit the variability data well. Beware!

Problem with extreme version: There are not enough "parent population" objets (galaxies with $L > L_*$) to allow $\Pi_{Spine} > 50$ except for the most extreme, rare jet, unless the spine is invisible (carries energy but not radiate).

# 10 "Scandals"-plenty of Data, but Weak Models

Blazar field is still not "mature". Some examples:

a. We don't have a working model of the core - the most prominent feature in a jet - as seen at $\nu > \nu_m$.

b. Two of the most extreme objects - the TeVblazars Mkn421+501- have slow apparent speeds + weak radio varibility, but high $\delta \gtrsim 10$ and extreme variability are seen in X-ray + TeV $\gamma$-ray.

c. The shock model explains the occurrence of flares, superluminal knots + decays of flares very well, simble models for the rise don't exist.

d. The flare shapes are typically $\sim$ symmetric with quasi-exponentialrises + decays. No model I'ha seen reproduces this. In general, flares with timescales limited by light-crossing time should have flat peaks.

e. Particle acceleration models remain only partially developed. Why do they lead to all particles being relativistic?How is the value od K related to the physics?

f At least one blazar (a southern source whose name escapes me, 0405-376?) has a minimum brightness temperature (measured by scintillation $\sim 2 * 10^{14} K$ This requires $\sigma \gtrsim 200$ to avoid SSC catastrophe.

g. When deprojected, the opening angles of some blazars $\gtrsim 0.2°$. It's not clear whether jet collimation theories can explain this.

h. Accretion disk models do not explain the optical $\rightarrow$ X-ray emission well in radio-quiet objects (Seyferts). The spectrum is reproduced well, but expected correlations of light curves at different frequency ranges are not observed.

i. The high $\gamma$-ray luminosities of some quasars can be explained by inverse Compton scatterin of seed photons from some non-jet source. But there is a reverse time delay of $\gamma$-ray variations which lag superluminal radio “ejections” (Jorstad et al. 2001, ApJ, 556,138). The radio core is thought to lie outside the broad emission-line region, which was the prime candidate dor making seedphotons.