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Symbolic dynamics in the general three-body problem

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Abstract. Free-fall equal-mass three-body systems are numerically studied using the approach of symbolic dynamics. We scan the two-dimensional homology map of initial configurations in steps of 0.001 along both axes. States of binary and triple encounters as well as changes of configuration types are used to construct symbolic sequences. The symbolic sequences are characterized by Shannon and Markov entropies. The dependencies of these entropies on initial configurations are plotted and analyzed. An intermittence between chaotic and regular behaviour is observed. The subregions of different behaviour are outlined.

1. Introduction

Symbolic dynamics is widely used in studies of dynamical systems. The approach is based on the topological conjugacy between continuous evolution of the dynamical system and a shift map on the space of sequences of integer numbers reflecting the state of the evolution. Constructed symbolic sequences allow one to describe various orbit types (periodic orbits, their bifurcations, triple collisions etc.).

Alexeyev (1981) has applied symbolic dynamics to one special case of the three-body problem (Sitnikov's (1960) problem). Using the symbolic dynamics approach, Alexeyev (1968a,b; 1969; 1981) found an intermittence of motions of different types in the Sitnikov problem.

The symbolic dynamics approach was also applied to two other special cases of the three-body problem: the rectilinear problem (Tanikawa and Mikkola 2000a,b; Saito and Tanikawa 2004) and the isosceles problem (Zare and Chesley 1998, Chesley 1999).

We have also started a similar study for the free-fall equal-mass three-body problem (Mylläri et al. 2004). The present work continues this investigation.

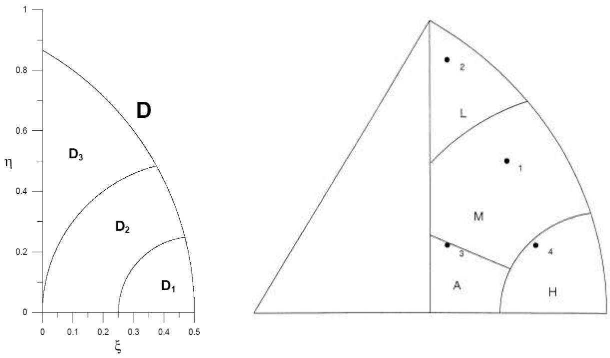

Figure 1. Two possible partitions of the homology region D.

2. Methods and Results

A symbolic dynamical system consists of three basic parts: an alphabet Ω, a space X of infinite sequences

$$\{\omega_i\},\ i \in Z,\ \omega_i \in \Omega,$$

and a shift transformation σ that moves (shifts) the sequence one position to the left. We construct symbolic sequences for the free-fall equal-mass three-body problem using several different means. One approach consists of introducing some partition of the phase space and fixing in what subregion the trajectory is at the current moment. Since not all transitions between subregions are possible, this approach corresponds to sub-shifts of finite type. A second approach is to fix some dynamical states (double encounters, triple encounters etc.) during the evolution of the triple system.

Symbolic sequences can be characterized by Shannon (H_1) and Markov (H_2) entropies:

$$H_1 = -\sum_i p_i \ln p_i,\ H_2 = -\sum_i p_i \sum_j q_{ij} \ln q_{ij}.$$

Here p_i is the frequency of symbol "i'' in the sequence, and q_{ij} is the frequency of transitions from "i'' to "j''. When we consider double and triple encounters, we can take ω_i as the number of the distant body.

Two possible partitions of the homology region D (Agekian and Anosova, 1967) are shown in Fig. 1. The first partition is suggested in this paper; the second is taken from Chernin et al. (1994).

We have estimated the entropies H_1 and H_2 along the whole trajectory and fixed their maximum values. The results are shown in Figs. 2 and 3. Different shades of grey correspond to different ranges of the entropies.

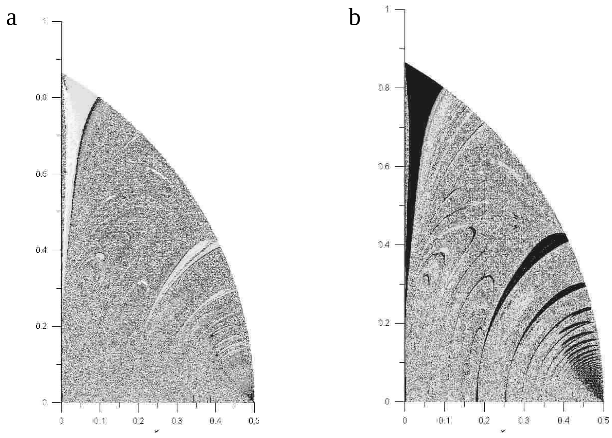

Figure 2. Entropies $H_1(\xi, \eta)$ for double (a) and triple (b) encounters.

3. Conclusions

An intermittence of subregions of high and low entropies has been observed. In particular, zones of fast escape after a few triple encounters are clear. One remarkable pattern is the system of self-similar arcs around the point (0.5, 0). Also a few concentrated structures emanating from the abscissa axis of the point (0.4, 0.4) are indicated. Note that the periodic orbit "figure-of-eight" (Moore 1993) approaches the boundary of region D near this point. The typical feature of distributions of points with different entropies is the ray $\xi + \eta = 0.5$ coming from the point (0.5, 0). This ray can separate the initial conditions for triple systems with different types of dynamics. Although we cannot strictly prove the existence of the topological conjugacy and thus the validity of the symbolic dynamics approach for the general three-body problem, there are some experimental results which argue in favour of adopting the symbolic dynamics approach. Some of these results are given in Mylläri et al. (2004). Here we can add that there are distinct peaks on the entropies' histograms for the sequences constructed using double encounters (see Fig. 4). Similar peaks are observed for triple encounters. Different peaks should correspond to different ergodic components.

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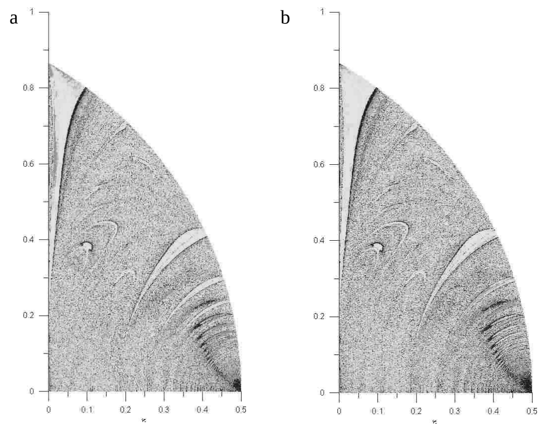

Figure 3. Entropies $H_2(\xi, \eta)$ for the partitions indicated in Fig. 1.

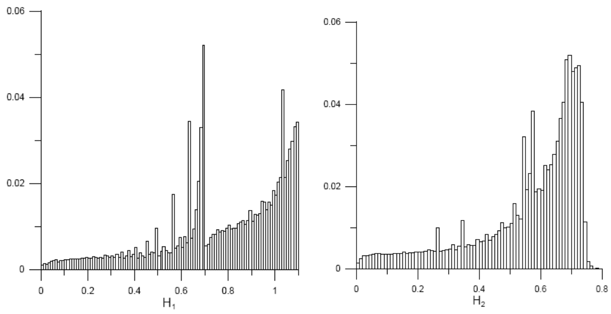

Figure 4. Histograms for the entropies $H_1(\xi, \eta)$ and $H_2(\xi, \eta)$ for double encounters.

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