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# Structure of the near-periodic motion manifold in stellar triple systems

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**Abstract.** We consider the structure of the finite motion sets surrounding stable periodic orbits in the general three-body problem. We study the vicinities of three periodic orbits (von Schubart's orbit in the rectilinear problem, Broucke's orbit in the isosceles one, and the "Eight" orbit) in non-hierarchical triple systems. These three orbits have the special feature that sometimes one body passes through the center of mass of the triple system. Corresponding triple systems have zero angular momentum. The "Eight" orbit has an intermediate position between the two other orbits, which are the limiting cases. We have found a "bridge" of long-term metastable systems connecting these three orbits. The metastable trajectories can "stick" to the vicinity of one of the periodic orbits and sometimes shift from one vicinity to another one. The hierarchical Hill-type periodic orbits are also studied and their vicinities delineated. The structure of the near-periodic motion manifold is described.

## 1. Introduction

In stellar dynamics and celestial mechanics, periodic orbits play an important role. In particular, stable periodic orbits may be surrounded by sets of similar orbits with finite motions. These orbits form tube-like manifolds which can "attract" orbits in dynamical systems due to the complicated structure of the boundary layer. The unstable periodic orbits can generate sets of trajectories with chaotic motions.

The general three-body problem may reflect several main dynamical features inherent to a wider class of dynamical systems. So an investigation of periodic and near-periodic solutions in the three-body systems is of interest for a wider class of problems.

The first periodic solutions in the three-body problem were found by Euler and Lagrange. Subsequently, a huge number of periodic orbits have been discovered. One new method for constructing periodic solutions is based on minimization of the action functional (Moore 1993, Vanderbei 2004). Using such an approach, Moore (1993) has proposed a classification of simple periodic orbits in plane three-body systems for different potentials $\varphi(r) \propto r^{\alpha}$ (in the Newtonian case $\alpha = -1$).

Amongst periodic solutions in the three-body problem, one can distinguish so called "choreographies", when all three bodies move one after other along the same closed curve. Two famous examples of such orbits are the Lagrangian solution and the "Eight" orbit (Moore 1993, Chenciner and Montgomery 2000) in the equal-mass systems. One can also note "partial choreographies", when two bodies move one after other along the same closed curve, while the third body moves along a different closed curve.

Besides minimization of the action functional, one can search for periodic and near-periodic solutions by other means. One possible method is scanning an initial data region and use of some iteration procedure.

For the equal-mass free-fall three-body problem, three simple stable periodic orbits are well-known. These orbits are shown in Fig. 1. A number of periodic

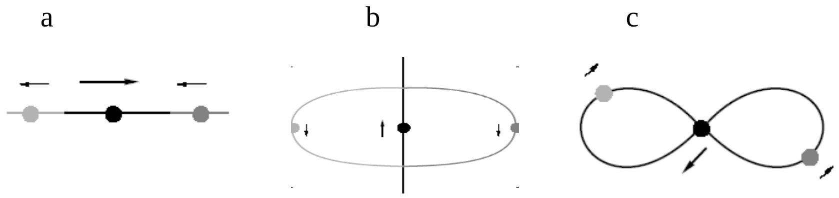


Figure 1. Three stable periodic orbits in the free-fall three-body problem: a) von Schubart (1956) orbit; b) Broucke (1979) orbit; c) "Eight" orbit.

orbits in rotating systems were found by Vanderbei (2004). We also refer the reader to the paper by V. Titov in this volume.

## 2. Properties of near-periodic orbits

A wide class of periodic and near-periodic orbits consists of so called Hill-type orbits (see, e.g., Vanderbei 2004, Martynova et al. 2005). In these orbits, two bodies form a dynamically isolated binary, and the third body moves around its center of mass along a quasi-circular curve. Near-periodic Hill-type orbits form loop-like (prograde motions) or petal-like (retrograde motions) structure (Fig. 2). When the number of loops or petals is odd, the components of the binary move along the same curve. When the number of loops or petals is even, the trajectories of the binary components are shifted by a definite angle.

The stable periodic orbits by von Schubart (1956), Broucke (1979) and the "Eight" orbit mentioned above are all surrounded by near-periodic orbits with finite motions. Three examples of such trajectories are shown in Fig. 3.

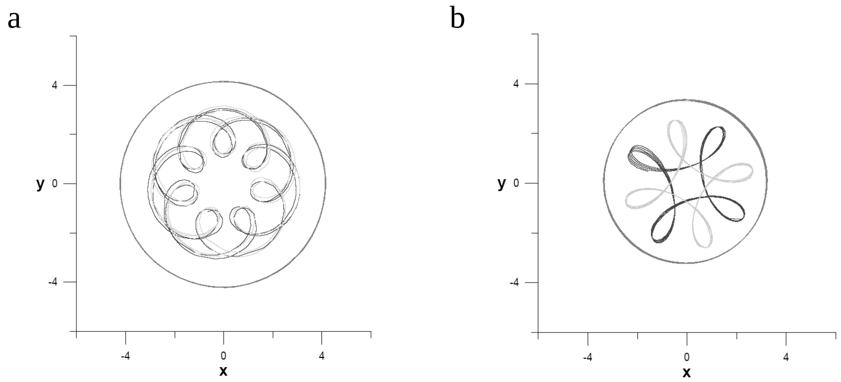


Figure 2. The examples of loop-like (a) and petal-like (b) Hill-type near-periodic orbits.

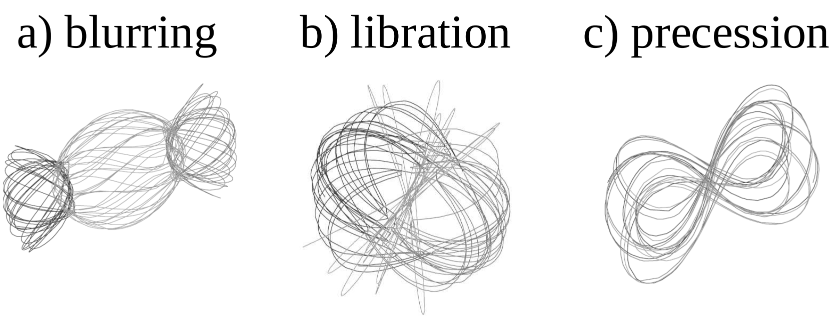


Figure 3. The examples of near-periodic motions in vicinities of von Schubart (a), Broucke (b), and “Eight” orbits (c).

We can see three possible types of motion in vicinities of these stable orbits: a) blurring of trajectory windings; b) libration with respect to the center of mass; c) circular precession.

Orlov et al. (2004) have investigated possible connections between these three stable periodic orbits for plane equal-mass systems. Initial positions of bodies were chosen in *syzygy* crossing, when one component is placed in the center of mass of the triple system. The initial conditions could then be determined by three parameters: the virial ratio $k$ and two angles $\varphi_1$ and $\varphi_2$ between the velocity vectors of the two outer bodies and the line connecting these bodies. The dependence of life-time $T(k,\ \varphi_1,\ \varphi_2)$ was studied, and the regions with $T > 1000\tau$ ($\tau$ is crossing time) were outlined. Three sections $T(\varphi_1,\ \varphi_2)$ at $k\ =\ 0.2,\ 0.4,\ 0.5$ are shown in Fig. 4. Three regions of near-periodic finite trajectories generated by these periodic orbits do not join each other, however they are connected by long-living unstable systems.

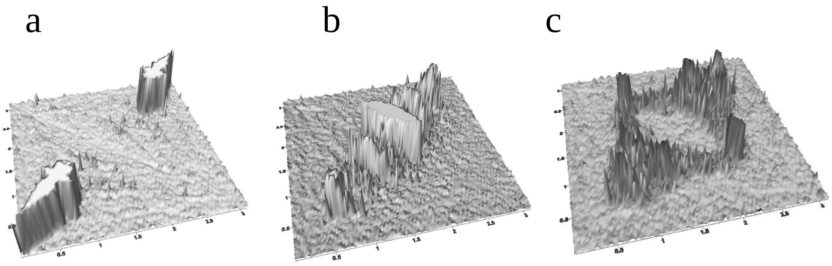


Figure 4. The dependencies $T(\varphi_1,\ \varphi_2)$ for $k = 0.2$ (a), 0.4 (b), and 0.5 (c). The stability regions are the upper plateaus.

We note that the two periodic orbits mentioned above are realized in two special limit cases of the three-body problem: i.e. the isosceles and the rectilinear. These orbits are surrounded by regions of finite motions.

In the free-fall problem, some trajectories can “stick” to the stable periodic orbit regions for a long time ($\sim 100\tau$). These trajectories were called metastable (see Martynova et al. 2003). Sometimes the same trajectory may visit the vicinities of stability regions around different periodic orbits during its evolution. The evolution of metastable systems is terminated by escape.

## 3. Conclusions

We summarise as follows:

1. Stable periodic orbits in the general equal-mass three-body problem are connected by long-living unstable systems.

2. Hill-type stable orbits correspond to the resonances between periods of the inner and outer binaries.

3. Petal-like and loop-like structures for near-periodic orbits correspond to retrograde and prograde motions.

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