Few-Body Problem: Theory and Computer Simulations
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Classification of orbits and recoverability of initial conditions in some three-body systems

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Abstract. We have studied some properties of metastable orbits in the equal-mass free-fall three-body system through computer simulations. Firstly we have determined the relative abundances of each of the three known metastable orbit types. Secondly we have looked at the number of each type of orbit as a function of time of escape. Finally we have computed these orbits backwards in time to examine the recoverability of the initial conditions.

1. Introduction

In this work we have studied some properties of what is probably the simplest case of the Newtonian general three-body problem (TBP): an equal-mass free-fall three-body system. In such systems all three bodies have equal mass, and lie initially at rest, i.e. the systems studied did not have any additional angular momentum.

The study consists of three parts. In the first part, a selection of these systems has been simulated and the resulting orbit type has been classified. In the second part, we have analysed differences in the escape times of the system in terms of orbit type. In the final part, we attempt to recover the initial conditions of the simulation from the time when the escape occured. All calculations were done with code using the Bulirsch-Stoer integrator with the chain regularization algorithm (Mikkola & Aarseth 1993) and optional slow-down treatment (Mikkola & Aarseth 1996).

2. Classification of interplay

Even though orbital classification in three-body systems is a complicated matter, the main idea remains clear. The basis of our classification was originally presented by Martynova *et al.* (2003) in which work they suggest dividing orbits into three classes in terms of the 'interplay stage'. These three classes are the Schubart orbit, the Broucke orbit and the eight-like orbit.

On the left in Fig. 1 we show the so-called D-region or the homology map, or the escape times of some equal-mass free-fall systems. (For notes on the construction of the D-region see e.g. Heinämäki *et al.*, 1999). In our study we simulated about 2100 systems from the Lagrangian and the aligned-triangles parts of the D-area. These areas respectively resemble the dark areas 1 and 2 presented on the left in Fig. 1. Some orbits were rejected because of instability

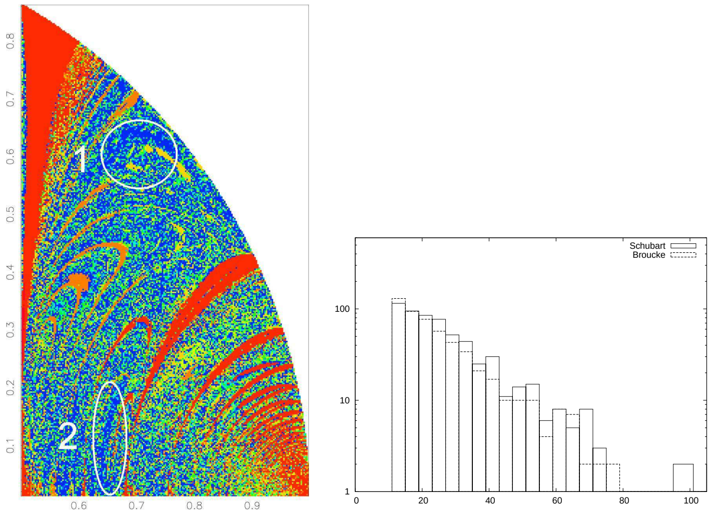

Figure 1. *Left:* D-region of equal-mass free-fall three body problem (TBP). The darkest pixels represent metastable systems with time of escape $\geq 13\,ICT$. Simulated orbits are selected from the dark areas 1 and 2. (Lehto *et al.*, in preparation) *Right:* Numbers of individual orbits as a function of the time of escape. Both Schubart and Broucke orbits are presented on the same semi-logarithmic scale.

or when most of the time the system was in an 'escape with return'-state. After the rejections about 1400 orbits remained.

In the aligned triangles area, the Schubart type forms 94% of orbits while in the Lagrangian area orbits are divided into 30% Schubart and 67% Broucke types. The eight-like orbit is nowhere common (total 3%). As assumed in Martynova *et al.* (2003) the Schubart type is most common in the aligned triangles area.

Orbital type is not necessarily constant: changes in orbital type during the interplay do appear. This is a prominent feature in the Lagrangian part of the D-region where all systems start with the Broucke orbit but change to Schubartian or eight-like motion. Such changes may take place multiple times during the interplay. In this work we decided to classify orbits by the most prominent type of orbit during the interplay. In the area of aligned triangles, the Schubart orbit is to be expected: in fact, all systems start their motion with this type, although it is also the case that type changes are then rare and Schubartian motion almost always persists throughout the whole interplay period. If a type change did appear, the resulting type was always Brouckean. Schubart orbits thus appear more stable than Broucke orbits.

There is another feature that reorganizes the system during the interplay: changes between the relative positions of the bodies during their motion on a certain type of orbit. This is typical in Broucke orbits, where the body moving in the middle of the system might change place with one of the binary companions. This does not change the type of the orbit.

3. Distribution of escape times

It is probably not very surprising that the relative number of systems which survive a given time, before an escape occurs, decreases exponentially with time. The same kind of exponential decrease is apparent both in the set of all simulated orbits and in sets of Broucke and Schubart types separately. The right panel of Fig. 1 shows the observed number of Schubart and Broucke orbits as a function of escape time if it is $\geq 13\,ICT$ (Initial Crossing Time). Because the numbers of Schubart and Broucke orbits were quite similar, comparison of these distributions was easily done with two well known non-parametric statistical tests: Tukey's quick and compact test (Sachs 1984 p. 289) and the Mann-Whitney U-test (Green & Margerison 1977 p. 145. Both tests indicate there is no significant difference in the distributions of escape times between these two types.

The number of eight-like orbits was very small ($N = 26$) compared to two leading types and therefore meaningful statistical comparison was not possible.

4. Stability of simulation program and recoverability of initial conditions

The final part of the study consists of two subparts: first the simulations were done backward-in-time from the escape (i.e. running backwards to the initial conditions) and the simulations were left to run until an escape appears. This (second) escape represents the 'formation' of the system. The second subpart consists of simulations run forwards from 'formation' to the original free-fall situation until the original escape re-appears. We wanted to see if the (original) initial conditions had been recovered sufficiently well to see this escape take place again. The problem has two aspects: a) the system itself is sensitive to initial values and, perhaps, the accuracy is not high enough to lead the system to the correct initial positions and velocities again and b) does the lose of numerical accuracy in the simulation process prevent orbital recovery? In theory, with infinite numerical accuracy all orbits are recovered, but in practice finite decimal precision prevents this. When time reversal is done with fixed numerical accuracy we get information on how sensitive a particular system actually is.

Because the equations of motion are autonomous (i.e. there is no explicit time dependence) the time reversal was easily done by changing the sign of each velocity vector, i.e. by mapping $\vec{v} \mapsto -\vec{v}$ and then using these as new initial velocities. In this part of the study, the 'slow-down' treatment was turned off.

Initially, we made some check-out calculations with very low accuracy (single precision). No orbits were recovered. Moving to double precision accuracy, in some 200 backward-in-time simulations, about 90% of the orbits found their initial conditions again. The maximum difference allowed in spatial coordinates was $\Delta q < 0.05$ but actually all orbits fitted well inside this limit if the initial conditions were found. There appeared no obvious correlation between the escape

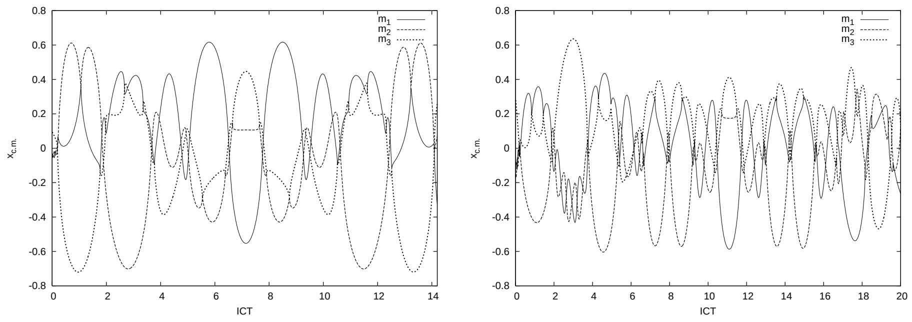

Figure 2. *Left:* An example of time symmetric orbit that found its initial conditions. *Right:* An example of orbit that found its initial conditions but has an asymmetric time evolution. Notice the time-symmetry near the initial conditions at $11ICT$.

time and recoverability of initial conditions. It was also seen that escape with return before the final escape did not make the initial conditions unrecoverable.

When the simulations were started from the formation time, one is technically dealing with a three-body scattering event of a binary and a single star. From the set of more than 100 simulations, approximately 70% of orbits found their initial free-fall conditions. Most of these systems had asymmetric time evolution from formation to escape, but motion was still time symmetric near the initial conditions. One such orbit is shown in the right panel of Fig. 2. Note that in some cases the total motion was time symmetric in the manner seen in the left panel in Fig. 2. In the cases where initial conditions weren't found the orbits usually had quite random-looking interplay from formation to escape.

5. Discussion

We have presented a relatively simple study of the equal-mass free-fall three body problem. The main practical issue of this work has been the classification process. Because we have no automatic procedure of orbital classification, all orbits had to be classified by eye, and the number of individual simulations remained small. This limitation is most severe for eight-like orbits, since they are quite rare.

An automatic classifier would have to cope with the intermittent changes of orbital type and changes between the relative positions of bodies.

The number of Schubart and Broucke type orbits is experimentally found to decrease exponentially towards longer escape times, with no significant statistical difference between the two exponential laws. This indicates that the escape time is determined by a stochastic process. The number of eight-like orbits found here is too small to study whether this property also holds for these orbits.

Our study of the recoverability of the initial conditions of the simulations involved running the orbits forward to escape (i.e. system breakup); then running them backwards through the initial conditions and onward to a new escape (termed the system formation), and finally forward again from formation to breakup.

For most of orbits, the initial conditions were recovered, running the simulations backwards from the breakup point, and more than 50% of the orbits achieved this condition when run forward from system formation. We conclude that these individual cases are particularly sensitive to their initial conditions compared to most of the orbits. Comparing to the fact that with single precision accuracy no orbits were recovered this conclusion seems natural and the existence of this behaviour is a typical property of chaotic systems. In future it would be interesting to see what kind of exact relation binds together the number of recovering orbits and the increasing numerical accuracy and also if this relation expresses differences between different forms of general three-body problem.

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