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Behaviour of a weakly perturbed two-planetary system on very long time-scales

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Abstract. We study the orbital evolution of planetary systems similar to the Solar System. The averaged equations of motion are integrated numerically, and the orbital evolution of Sun-Jupiter-Saturn type systems is investigated on time-scales of order 10 Gyr. This allows us to analyse resonance conditions for exoplanetary systems.

1. Introduction

The stability of the spatial planetary three-body problem has been investigated by Robutel (1993a, 1993b), Laskar and Robutel (1995) and Robutel (1995) using Kolmogorov-Arnold-Moser (KAM) theory. A source of chaotic behaviour for the Jovian planets (both for two and four planets), namely, the 2 : 5 resonance between the mean motions of Jupiter and Saturn has been explored by Varadi, Ghil and Kaula (1999). The motion of the Jupiter–Saturn planetary system near the 2 : 5 resonance has been modeled analytically by Michtchenko and Ferraz-Mello (2001) in the frame of the planar three-body problem.

In the present paper we continue our study of the spatial planetary three-body problem (Kholshevnikov, Greb and Kuznetsov (2001, 2002), Kholshevnikov and Kuznetsov (2004, 2005), Kuznetsov and Kholshevnikov (2004)). The aim of the present work is the investigation of the dynamical evolution of a weakly perturbed spatial two-planetary system on time-scales of order 10 Gyr. We start by presenting the Hamiltonian as a Poisson series with respect to all elements, using the Lie transforms method to construct the averaged Hamiltonian up to squares of small parameters. Numerical integration of the averaged equations of motion allows us to study the orbital evolution of Jupiter–Saturn system on a very long time scales. Analysis of the variable change functions shows boundaries of validity of this approach with respect to eccentricities, inclinations and semi-major axes of the planets. In particular this shows that the resonant properties of extrasolar planetary systems can be validly investigated.

2. The Sun-Jupiter-Saturn System

The averaged equations are integrated numerically by 15th order Everhart and 11th order Runge-Kutta methods (for slowly changing orbital elements) and by the spline interpolation method (for rapidly changing elements). The accuracy of the integration is monitored by computation of the integrals of energy and area.

The lower and upper limits for averaged eccentricities are 0.017, 0.051 (Jupiter), 0.019, 0.078 (Saturn), and for averaged inclinations are 1.3°, 2.0° (Jupiter), 0.73°, 2.5° (Saturn). The results obtained for the two integrators are in good agreement. The relative differences between the first and second approximations exceeded the small parameter $\mu = 1 \cdot 10^{-3}$, but are less than $\sqrt{\mu} = 3.2 \cdot 10^{-2}$.

In the first approximation the evolution of the longitudes of ascending nodes with respect to the ecliptic plane turns out to be a libration, with amplitudes of 12.9° and 32.8° for Jupiter and Saturn respectively, in agreement with the results of Smart (1953). The evolution of the logitudes of ascending nodes with respect to the Laplace plane turns out to be secular.

In the second approximation the evolution of the longitudes of ascending nodes with respect to the ecliptic plane turns out to be a combination of oscillations with a large amplitude and a slow secular motion.

On the Laplace plane the difference between the longitudes of ascending nodes of Jupiter's and Saturn's orbits, $\delta\Omega$, is identically equal to 180° (Charlier 1927). We find from our calculations $|\delta\Omega - 180°| < 0.0085°$ through time interval 10 Gyr, in a good agreement with the theory.

The evolution of the pericentre longitudes of Jupiter's and Saturn's orbits turns out to be secular for both approximations.

The short-period perturbations of Jupiter's and Saturn's semi-major axes do not exceed 0.0023 AU and 0.0122 AU, respectively. The short-period perturbations for the other elements are much less than the amplitudes of the long-period perturbations.

3. Resonances in the Two-Planetary Problem

The resonant condition $n\omega = n_1\omega_1 + n_2\omega_2 = 0$ defines the resonant value of the semi-major axis of the orbit of the second planet a_2^{res}. The solution based on the Poisson series expansions is not valid or restrictedly valid for $a_2 \in [a_2^{res} - \Delta a, a_2^{res} + \Delta a]$. Here Δa is the resonant zone width. We can determine Δa from various conditions. We here use estimates for the narrow and wide resonant zones widths introduced in Sokolov (1980), Sokolov and Kholshevnikov (1981).

Let us turn now to extrasolar planetary systems, having two planets with the more massive one situated closer to the parent star (Marcy, Butler, Fisher et al. 2005), Schneider (2005). Resonant semi-major axes a_2^{res} are presented in table 1 for five two-planetary systems. Here $\mu \sin i_0$ is an estimate of the small parameter μ from the observational data. The orbital inclination i_0 with respect to the plane of the sky is unknown and is thus a free parameter. The indices n_1 and n_2 correspond to the value a_2^{res} close to a_2. The interval data for $\mu \sin i_0$,

m_2/m_1 and a_2/a_1 correspond to different sources: Marcy, Butler, Fisher et al. (2005) and Schneider (2005). The mark "?" indicates uncertain data.

Table 2 presents estimates of the resonant zones for four two-planetary systems. All distances in table 2 are expressed in units of the semi-major axis of the first planet. We did not obtain results for HD 12661 as the expansions in the present work do not contain terms for which $n_1 = 2$ and $n_2 = -11$ simultaneously. The semi-major axes of the planets 47 UMa c and Saturn lie in the wide resonance region, while the semi-major axes of HD 202206 c and HD 169830 c are located near the boundary of the wide resonance region. So deep resonance takes place in none of the systems considered.

4. Conclusions

The evolution of the orbits of Jupiter and Saturn turns out to be almost periodical on a time-scale of 10 Gyr. The differences in the results for the first and second order approximations are explained by the influence of the small divisors. They appear in the right-hand sides of the averaged equations from the second order approximation on. The investigation of the dynamical evolution of extrasolar planetary systems demands the taking into account of resonance effects.

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Table 1. THE RESONANT SEMI-MAJOR AXES

Planet	$\mu \sin i_0$	m_2/m_1	a_2/a_1	n_1	n_2	a_2^{res}
47 UMa c	0.0024–0.0025	0.30–0.32	1.78–1.79	3	−7	1.758
Saturn	0.0010	0.299	1.827	2	−5	1.842
HD 202206 c	0.0174–0.0175	0.14	2.89	1	−5	2.921
HD 12661 c	0.0022–0.0023	0.65–0.68	3.08–3.17	2	−11	3.113
HD 169830 c	0.0021–0.0040	0.79–1.40	3.35 (?)	1	−6	3.299

Table 2. THE RESONANT ZONES

Planet	Narrow resonance			Wide resonance		
	Δa	a_{min}^{res}	a_{max}^{res}	Δa	a_{min}^{res}	a_{max}^{res}
47 UMa c	0.013	1.745	1.771	0.036	1.722	1.794
Saturn	0.012	1.830	1.854	0.027	1.815	1.869
HD 202206 c	0.004	2.917	2.925	0.009	2.912	2.930
HD 169830 c	0.001	3.298	3.300	0.003	3.296	3.302