Few-Body Problem: Theory and Computer Simulations
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How to change the relative inclination in a hierarchical triple-star system by tidal dissipation

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Abstract. A simple conceptual theory for the changes in the relative inclination of a hierarchical triple-star system during tidal evolution is formulated and tested by numerical integrations. Low inclinations are generally stable, while highly inclined 'Kozai-cycle' systems generally show some decrease of the relative inclination during the evolution. If such a system has an unusually low-mass secondary, in a short-period outer orbit, the relative inclination may change more drastically. For Algol, the present perpendicular orbit is hard to explain by these mechanisms, and is probably the outcome of chaotic encounters at the birth of the system.

1. Introduction

An important characteristic for a hierarchical triple-star system is the relative inclination between the two orbital planes, hereafter denoted j. An original goal for this paper was to shed some light on the origin of the peculiar $j \approx 90°$ in the Algol system (Lestrade et al., 1993). The reasoning by Söderhjelm (1975) can be used to show that the offset from perpendicular is at most some 3°, but the bottom line of the present note is that this is probably accidental. In other cases, however, tidal dissipation in a close orbit may sensibly change the relative inclination.

Starting from a simplistic model including only the orbital angular momenta, it is easy to derive an equation for j as a function of a variable angular momentum in the close orbit. The well-known 'Kozai-cycle' eccentricity modulations in an inclined close orbit (Kozai, 1962) thus naturally gives a reflex periodic change in the relative inclination. An irreversible change of the angular momentum can be had from orbit shrinkage due to tidal dissipation, and in such a system, the relative inclination may thus also change irreversibly. By skipping over many of the complex details in the tidal evolution, both as regards the strength of the interaction, and in particular, by neglecting the rotational motions of the stars, the present study gives a qualitative explanation of the inclination variations as observed in numerical simulations.

More refined studies of the tidal evolution of particular systems have been made by e.g. Beust et al. (1997) and Borkovits et al. (2004), but for a general picture, the present investigation should hopefully be of some value.

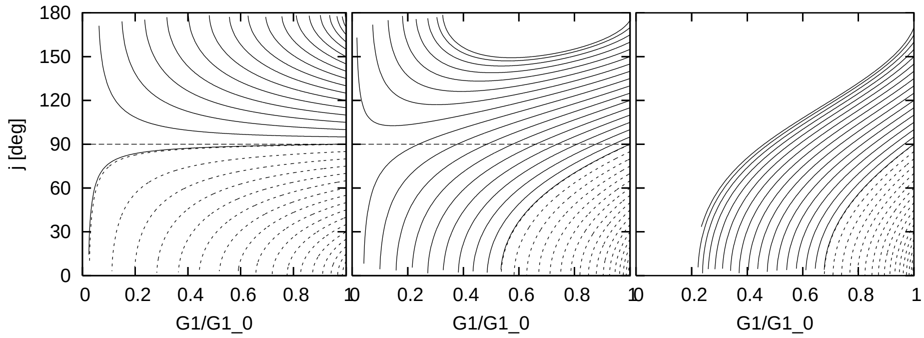

Figure 1. Theoretical variation of the relative inclination with decreasing G_1, for original $G_1/G_2 = 0.05$(left panel), 1.5 (middle) or 2.5 (right). Full lines are for the originally retrograde orbits, dashed lines for the direct ones.

2. The simple theory

With the magnitudes of the orbital angular momenta termed G_1(close) and G_2(wide), and neglecting the rotational contribution, the total angular momentum is their vector sum, of magnitude

$$C = \sqrt{G_1^2 + G_2^2 + 2G_1G_2 \cos j}.$$

With G_2 and C constant, this directly gives a relation $j(G_1)$, with a shape dependent on the original ratio of the angular momenta. This ratio is given in terms of the orbit sizes (a_i) and eccentricities (e_i) as

$$G_1/G_2 = \frac{\mu(1-\mu)}{\nu}\sqrt{(1+\nu)[a_1(1-e_1^2)]/[a_2(1-e_2^2)]}$$

where the mass-ratios are $\mu = m_2/(m_1 + m_2)$ and $\nu = m_3/(m_1 + m_2)$. In typical hierarchical triples with $a_1 \ll a_2$, we will thus have $G_1 < G_2$. When the third mass is small ($\nu \ll 1$), G_1 may become dominant, however, which has some interesting consequences. Fig 1 shows one 'typical' ($G_1/G_2 = 0.05$) case, where the relative inclination diminishes rather symmetrically on both sides of the perpendicular configuration, and two extreme ($G_1 > G_2$) cases, where retrograde high-inclination systems may become more inclined, even past 90°.

3. The proto-Algol system

To have some rough idea about the evolution of Algol through large-scale mass-exchange, I used the rapid binary evolution code BSE by Hurley et al. (2002). An initial system with $M_1 = 3.26$ M$_\odot$, $M_2 = 1.14$ M$_\odot$and $P_1 = 1.95$ days evolves through rapid mass-transfer at 300 Myr to a state at age 570 Myr with Algol's observed (cf. e.g. Söderhjelm, 1980) $M_1 = 0.80$, $M_2 = 3.6$ M$_\odot$. In the sequel, various 'proto-Algol' systems are taken to have these close orbit masses, with the third star ($M_3 = 1.6$ M$_\odot$) in its present ($P_2 = 680$ d, $e_2 = 0.22$) orbit at a high relative inclination.

With a 2-day close period, the stars are tidally deformed, and reasonable $k^{(2)}$ stellar structure constants give a fast enough apsidal motion that the high-eccentricity Kozai cycles are completely damped out (cf. Söderhjelm, 1984). For a more interesting case, we may start with a larger close period, say $P_1 = 4.0$ days, and this 'pre-proto-Algol' is hereafter called 'System A'. In this case, three-body effects dominate for $k^{(2)}$ up to around 0.05, and typical main-sequence values (≈ 0.005 for the dominating large star, cf. Claret and Giménez, 1992) give peaks with a maximum e_1 around 0.7. In reality, these peaks will be rapidly damped by tidal dissipation, and successively, the close orbit may become almost circular, with a period not far from proto-Algol's 2-day one.

4. Tidal dissipation

As described e.g. in Eggleton et al. (1998), a fully realistic model of the tidal dissipation in a close binary is not yet available. In the present numerical simulations, I have used the simplified prescription by Kiseleva et al. (1998), containing a free parameter $\lambda(\ll 1)$ describing the strength of the dissipation. (The nondissipative tidal accelerations are in essence, although not in form, identical to the ones in Söderhjelm, 1984.) Interestingly, it turns out that the final outcome of the tidal evolution is usually very insensitive to the value of λ. A higher value gives a more rapid shrinkage of the orbit, but approximately the same final state is reached over several orders of magnitude for λ, and one need not know a 'correct' value for it.

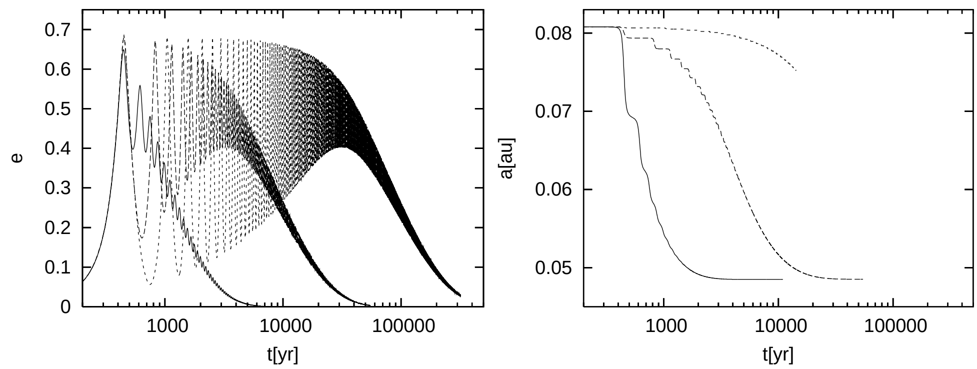

Figure 2. The tidal circularization in 'System A', for $\lambda = 10^{-5}, 10^{-6}$ and 10^{-7}. The eccentricity changes are shown at left, and at right are the corresponding changes in the close semi-major axis.

4.1. Results for proto-Algol (system A)

For 'system A', we have $G_1/G_2 = 0.11$, and from the theory above, it is clear that for j around 90°, even a large decrease of the close orbit angular momentum will change the inclination only marginally. As an example (Fig 2), runs with $j = 95°$ and $\lambda = 10^{-4}$ to 10^{-7} all give a 40% decrease of a_1 (with a timescale inversely proportional to λ), but this large decrease of G_1 changes the relative inclination only to $j = 94.8°$, both in the theory and in the actual numerical integrations.

If we start with even longer close periods, the Kozai-cycles have larger amplitudes (because the tidal deformation is relatively insignificant), and very small λ-values suffice for significant orbit-shrinkage (actually having a larger effect than smaller ones). For not too small λ (10^{-4} or 10^{-5}), we may start with anything between 4 and 40 days and still get a final close period around 2 days. In all cases, the changes in relative inclination are very moderate, and proto-Algol must have started in an almost perpendicular configuration. This is not unreasonable if we envision it as the stable remnant of a dynamically unstable small multiple system, where the relative inclination between the orbit planes seems to be more or less random (cf. Rubinov et al., 2004).

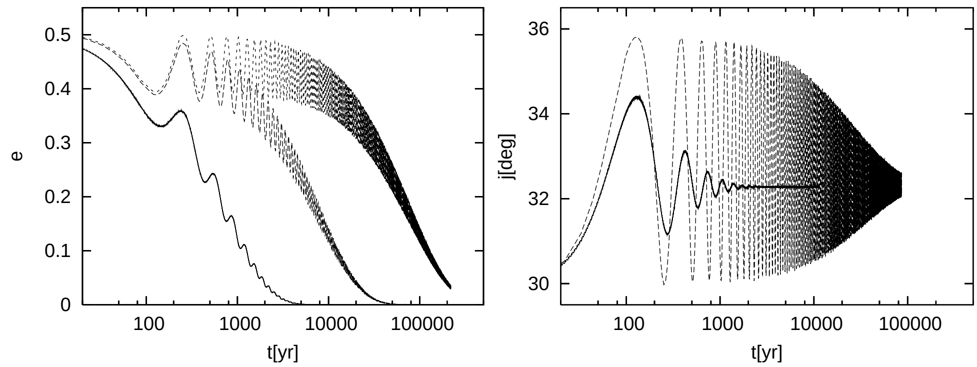

Figure 3. The tidal circularization in 'System B', for $\lambda = 10^{-4}, 10^{-5}$ and 10^{-6}. The eccentricity changes (starting from e_1=0.5) are shown at left, and the oscillatory but secularly constant relative inclinations at right, for $\lambda = 10^{-4}$ and 10^{-6}.

4.2. Results for low j (system B)

When the relative inclination is below some 40(140) degrees, there are no high-eccentricity Kozai cycles, but one may ask what happens if the close eccentricity is large to begin with. Interestingly, from a number of numerical trials, the circularization seems to proceed with secularly constant close orbit angular momentum, and consequently with no secular change of the relative inclination. As an example, the 4d proto-Algol system was started at relative inclination 30°, with varying start e_1 and varying λ. In this 'system B', the circularization proceeds as usual with a time-scale proportional to λ^{-1}, and with the final a_1 only dependent on the starting e_1 (see Table 1 and Fig 3). The small change of G_1 (often actually an increase) is readily apparent, giving an oscillating but secularly constant relative inclination.

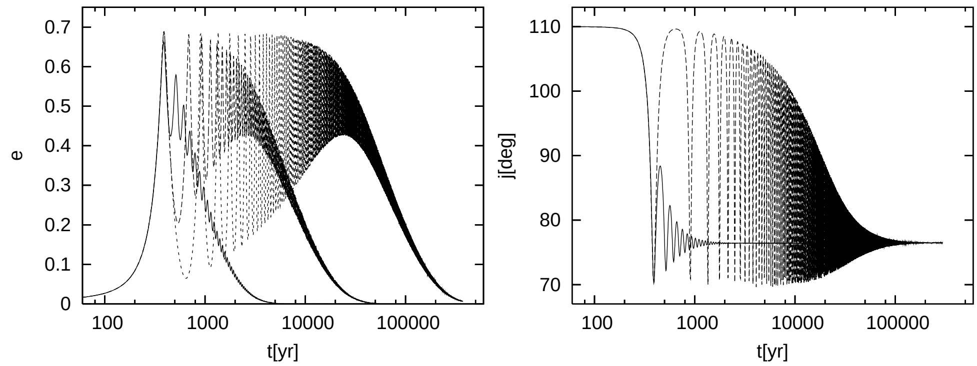

Figure 4. The tidal circularization in 'System C', for $\lambda = 10^{-5}, 10^{-6}$ and 10^{-7}. The eccentricity variations are at left, with the corresponding large changes in relative inclination at right, for $\lambda = 10^{-5}$ and 10^{-7}.

Table 1. The final a_1-values for 'System B', from different initial e_1. The approximate 'circularization time' for $\lambda = 10^{-6}$ is indicated, as well as the ratio of final to initial close orbit angular momentum.

e_1(start)	a_1(fin)[au]	t_{circ}[yr]	$G_1/(G_1)_0$
0.1	0.0802	2×10^6	1.00
0.3	0.0751	1×10^6	1.01
0.5	0.0632	5×10^5	1.02
0.7	0.0422	2×10^4	1.01

4.3. Results for large G_1/G_2 (system C)

In order to see larger inclination changes, one has to have a dominating close orbit angular momentum. As an illustration, we may take a system with the masses and 4d close period of 'System A', but with a third star mass of only 0.1 Msun in a 172d (1.0 a.u.) orbit. These parameters correspond to $G_1/G_2 = 2.49$, and from theory, we may expect large changes of inclination if the close orbit is shrunk appreciably. Some results of numerical integrations of this 'System C', started with j =110°, and are shown in Fig 4. The eccentricity and semi-major axis of the close system decrease indeed very similarly to 'System A' in Fig 2, but for the relative inclination, there is now a dramatic effect. For the observed

a-decrease by 42%, the theory predicts a 'decrease' of j to around 76°, which is exactly the result of the numerical integrations.

5. Conclusions

To secularly change the relative inclination in a hierarchical triple-star system, one has to change secularly the angular momentum of the close pair. For an almost constant eccentricity, circularization by tidal friction seems to proceed at constant angular momentum, and there are no changes of the relative inclination. Only when there are 'Kozai cycle' eccentricity variations, does the orbit shrinkage seem to be accompanied by a real decrease of the angular momentum. A necessary condition for inclination change is thus a start value between about 40 and 140 degrees, and a third-body orbit small enough to induce a faster apsidal motion than the tidal deformation of the close pair.

The effect of the angular momentum change is also very dependent on the ratio of angular momenta G_1/G_2, and in most cases, there is only a moderate decrease of the inclination (retrograde orbits remaining retrograde, with a decrease of $180 - j$). When the angular momentum in the inner orbit is larger than that in the outer, direct orbits still decrease their inclination, but now an initially retrograde orbit may become steeper, even passing 90° to turn into a direct one. Triples with a small inner/outer period-ratio and a low-mass third component would thus be expected to have a distribution of relative inclinations with more direct orbits than their 'birth' distribution. Since close triples with a well-determined relative inclination are still rare, no observational test of this prediction can be expected in the near future.

Finally, in the light of the present investigation, Algol's curiously exact perpendicular configuration must be assumed to be a coincidence. This result comes mainly from the high mass of the third star, giving a low G_1/G_2. Even if the large-scale mass-transfer was not fully conservative, any further changes of G_1 in this stage do not suffice to give a significant change of the relative inclination.

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