*Few-Body Problem: Theory and Computer Simulations*
*Annales Universitatis Turkuensis, Series 1A, Vol. 358, 2006*
*C. Flynn, ed.*

# Tidal interactions in N-body simulations

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**Abstract.**
The weak-friction model of tidal interactions between stars in the regularized few-body problem is applied to a study of the dynamics of non-hierachical multiple systems. The final state distributions and the properties of binaries, stable triples and single escapers are discussed. Tidal interactions and mergers increase the average and median values of binary and triple semi-major axes. These effects decrease the fraction of elongated binary systems compared to the gravitational few-body problem. They increase the average and median masses of binary and triple systems and decrease their mass ratios. Tidal interactions and mergers do not influence the distributions of angles between spin and orbital momentum vectors in binaries and stable triples. Tidal interactions lead to higher single escaper velocities compared to the gravitational few-body problem.

## 1. Numerical Approach

We use the weak friction model proposed by Hut (1981) to study the dynamical evolution of small non-hierarchical groups of stars. We assume constant tidal lag $\tau$ and restrict ourselves to the expansion of all time-dependent quantities to linear terms only. All quantities depending on distance are expanded to fourth order in $\frac{R}{r}$, where $R$ is the radius of distorted star and $r$ is the distance between perturber and distorted star. The equations of motion, including additional terms due to the tidal interaction, are regularized using the CHAIN regularization algorithm (Mikkola and Aarseth, 1993). Numerical integration of the regularized equations of motion is performed. The effect of star merging is taken into account according to the SPH models of star collisions (Benz and Hills, 1987, 1992). Escapes of single and binary stars are allowed.

## 2. Initial Conditions

We consider systems consisting of $N = 6$ stars. Initially all stars are randomly distributed within a sphere with a radius of 3 AU The stellar density in such systems is set unrealistically high to promote the effect of tidal interaction. The initial velocity vectors of the stars are isotropically distributed, and further, at the start of the integration the system is set at virial equilibrium. Three different initial mass spectra are considered: equal masses (EM), a Salpeter mass spectrum (SM), and a clump mass spectrum (CM) (see e.g. Sterzik and Durisen 1998 for details). The initial rotational velocities of the stars are chosen to match typical values for the Main Sequence. For each mass spectrum, 500 realizations of the initial conditions are integrated. The dynamical evolution is traced during 300 initial crossing times $T_{cr}$ of the system.

## 3. Final State Distribution

When the simulation is completed (after $300T_{cr}$), an analysis of the distribution of states is made. We distinguish the following states: binaries with negative total energy, binaries with positive total energy (two single stars), stable triples (according to the Golubev's (1967) stability criterion), unstable triples and systems of multiplicity higher than three.

Table 1 shows that the inclusion of tidal interaction and merging decreases the dynamical decay rate of the system. The fraction of final binaries is higher in the pure gravitational problem compared to simulations including merging and tidal interaction. The fraction of systems with multiplicity higher than three and the fraction of unstable triples is higher when tidal interactions and merging are taken into account. The likely explanation is that when merging and tidal interactions are taken into account, some close encounters lead to merging of objects which would otherwise have lead to escape. If we continue our integration to the time $1000T_{cr}$, we see that there is no difference in state distributions between the three factors under consideration.

## 4. Properties of Binary Stars

The analysis of the final binary semi-major axis distributions shows that the pure gravitational problem produces closer binaries than simulations including merging and tidal interactions. The median semi-major axis is $3-4$ times smaller (about 0.1 of the initial median system size) than in the case of merging. The difference between merging and merging + tidal interactions cases is not statistically significant. The reason for the growth of semi-major axes in the case of merging is the fact that the total energy of the system usually increases when the merging events take place. Tidal interactions do not increase the number of merging events significantly because the initial systems are dense enough to promote many merging events, lessening the effect of tidal interactions. The same tendency takes place if we suppose that a significant fraction of wide binaries is formed via merging in hierarchical triple systems. When we trace the dynamical evolution up to $1000T_{cr}$, we see that the median value of the semi-major axis is slightly higher than that what it was at $300T_{cr}$ for simulations in which merging and tidal interactions are taken into account.

The eccentricity distribution of final binaries in the pure gravitational problem satisfies the Ambartsumian–Heggie law, $f(e) = 2e$. Merging and tidal interactions slightly decrease the fraction of elongated binaries. This is due to the high probability of merging at the pericenter of orbits in close elongated binaries. Merging and tidal interactions increase the mean and median values of the final binary mass. Furthermore, the mass ratio $q = \frac{m_1}{m_2}$ is higher in the simulations with pure gravitational interaction, where $m_1$ is the mass of the light component in the binary and $m_2$ is the mass of the massive component. The angles between rotational and orbital momenta vectors are found to be randomly distributed.

## 5. Stable Triples

Here we use the representation of a hierarchical stable triple as a superposition of inner and outer binaries. Semi-major axes of the inner binaries are $10 - 100$ times shorter than the initial median system size. The semi-major axes of the outer binaries are comparable with the initial median system size. The behaviour of the median values of inner and outer semi-major axes does not differ from that of the final binaries (see section 4). The hierarchy of stable triple systems is rather high: the mean inner and outer binary semi-major axis ratio is 1:20.

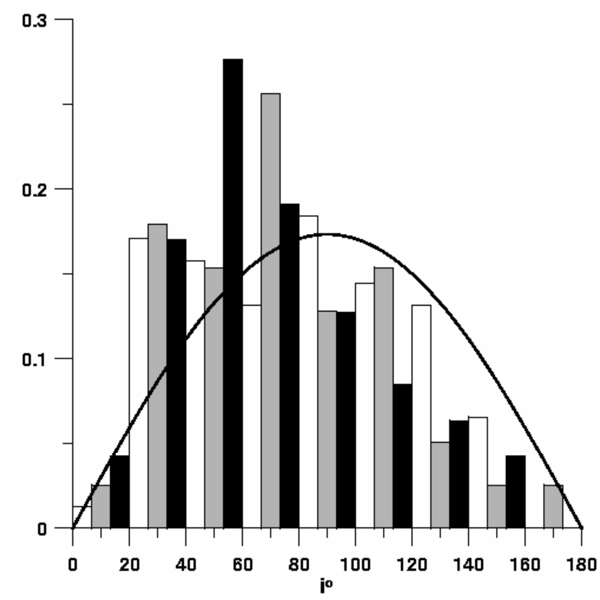


Figure 1. Distributions of angle $i$ between inner and outer binary orbital momentum vectors for the initial clump mass spectrum. White columns – no tides, no merging; grey columns – merging; black columns – merging + tidal interaction. The solid line corresponds to a random distribution.

The eccentricities of the outer binaries are smaller than the eccentricities of the inner binaries (the median values are $e_{out} \approx 0.5$ and $e_{in} \approx 0.7$). Merging and tidal interactions lead to a decrement in the fraction of stable triple systems with elongated inner binaries. These factors slightly decrease the fraction of the outer binaries with high eccentricities.

The inclusion of tidal interactions and merging decreases the mass ratios in the outer and inner binaries, whereas their inclusion increases the mean masses of stable triple systems.

Fig. 1 shows the distribution of angle $i$ between orbital momentum vectors of the inner and outer binaries. The solid line corresponds to a random orientation of these vectors $f(i) = \frac{1}{2} sin(i)$. The distribution is asymmetric for all the considered effects in the simulations. Prograde motions are found to be prefered: this is because system stability is greater in prograde compared to retrograde systems (Golubev's stability criterion).

When the masses of the stars are unequal, in the case of merging and tidal interactions, the fraction of systems with retrograde motions is smaller than in the case of the pure gravitational problem. When the masses of all stars are equal, the distribution of angle $i$ shows no dependence on the extra factors acting on the system.

The distribution of angles between orbital and rotational momentum vectors is consistent with being in a random distribution.

## 6. Single Escapers

We find that mean single escaper velocities are typically 10 – 40 km/s and that mean escaper velocity depends on the mass spectrum adopted. In the case of the clump mass spectrum the stars possess the highest mean velocity. The slowest mean escaper velocity takes place in the case of the Salpeter initial mass spectrum. Tidal interactions lead to higher mean and median velocities compared to the merging case. Velocities of single escapers from the systems where only gravitational interactions are taken into account are higher than in the case of merging but slightly lower than in the case of tidal interactions.

The mean and median masses of single escapers do not depend on the extra factors acting on the system.

## References

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Table 1. Final states distribution

| Factor | Mass spectrum | Binary system | Two singles | Stable triple | Unstable triple | Multiple system |
|---|---|---|---|---|---|---|
| tides+merging | EM | 0.47 | 0.02 | 0.09 | 0.19 | 0.23 |
| tides+merging | CM | 0.44 | 0.01 | 0.10 | 0.26 | 0.19 |
| tides+merging | SM | 0.49 | 0.02 | 0.12 | 0.20 | 0.17 |
| merging | EM | 0.45 | 0.02 | 0.08 | 0.19 | 0.26 |
| merging | CM | 0.47 | 0.02 | 0.08 | 0.27 | 0.16 |
| merging | SM | 0.51 | 0.01 | 0.11 | 0.21 | 0.16 |
| no tides, no merging | EM | 0.56 | 0.06 | 0.15 | 0.08 | 0.15 |
| no tides, no merging | CM | 0.56 | 0.01 | 0.15 | 0.18 | 0.10 |
| no tides, no merging | SM | 0.55 | 0.02 | 0.18 | 0.15 | 0.10 |