Compact Object Binaries in Star Clusters

A Relativistic Treatment

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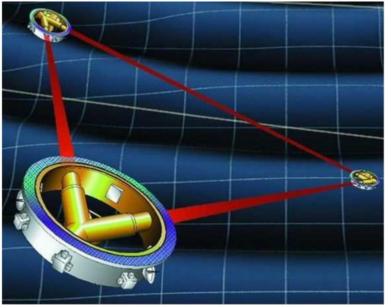
Motivation: Gravitational Wave Detectors

Stellar-mass compact object binaries are promising sources of gravitational waves for ground-based detectors (pictured: VIRGO).

Space-based detectors (LISA) will be able to detect gravitational waves from the inspiral of supermassive black holes.



Binary Parameters

Relativistic binary inspirals are sensitive to orbital parameters:

- Dependence can be significant.
- Star clusters can have large compact-object binary populations.
- Dense stellar environments affect binary parameters.

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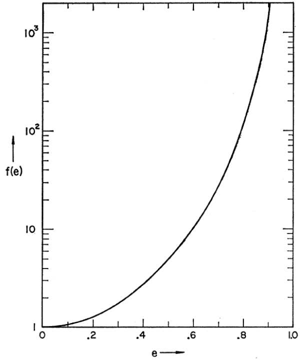
Investigate the Parameter Space

We will simulate relativistic binaries in star clusters in order to investigate the parameter space of relativistic inspirals.

Motivation: Dynamical Effects

Compact object binaries in star clusters interesting for a number of dynamical reasons.

- Relativistic dynamics
- Scattering interactions with field stars.
- "Stable" few-body systems.
- Highly eccentric binaries.



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Star Cluster Evolution:

- Isolated binary evolution from star cluster initial conditions:
  - Monte Carlo star cluster simulations.
- Binary evolution coupled with live star cluster dynamics:
  - Direct N-body simulations.

The Orbit-Averaged Approach

Models orbital and eccentricity evolution due to gravitational radiation.

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64 G^3 m_1 m_2 (m_1 + m_2)}{5 c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \quad (1)$$

$$\left\langle \frac{de}{dt} \right\rangle = -e \frac{304 G^3 m_1 m_2 (m_1 + m_2)}{15 c^5 a^4 (1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right) \quad (2)$$

- Multipole expansion by Peters and Mathews 1963, Peters 1964.
- Fast and easy to integrate.
- Good for isolated binaries on low eccentricty Kepler orbits.
- Only takes into account radiation and does not include direct orbit integration.
- Of limited use for perturbed binaries in strongly interacting systems.

Post-Newtonian Methods

The post-Newtonian method expands the linearised field equations in powers of v/c (e.g. Blanchet 2006).

$$a_{PN} = -\frac{GM}{r^2} + \frac{v}{c} a_{PN0.5} + \frac{v^2}{c^2} a_{PN1} + \frac{v^3}{c^3} a_{PN1.5} + \frac{v^4}{c^4} a_{PN2} + \frac{v^5}{c^5} a_{PN2.5} + \ldots$$

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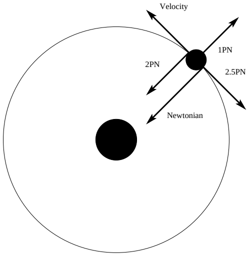
- Terms labelled by *relative* order.
- $1PN$, $2PN$, $3PN$ etc. conservative.
- $2.5PN$, $3.5PN$ etc. radiative.



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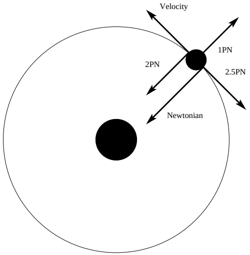
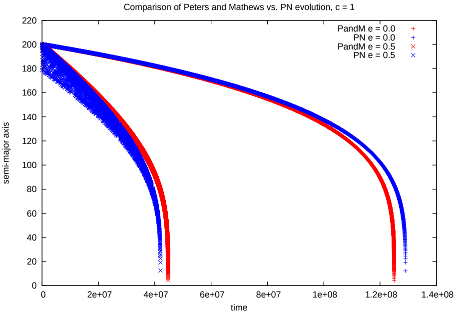
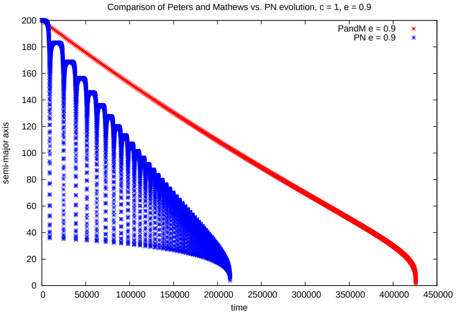


A natural choice to include in direct N-body codes.

Comparison for Isolated Binaries

The orbit-averaged approximation and the Post-Newtonian expansion agree well for isolated systems at low to moderate eccentricity (relativistic two-body code by Berentzen).





Low to moderate eccentricity.

High eccentricity.

Monte-Carlo Cluster Simulations

Star clusters can be simulated using Monte-Carlo methods.

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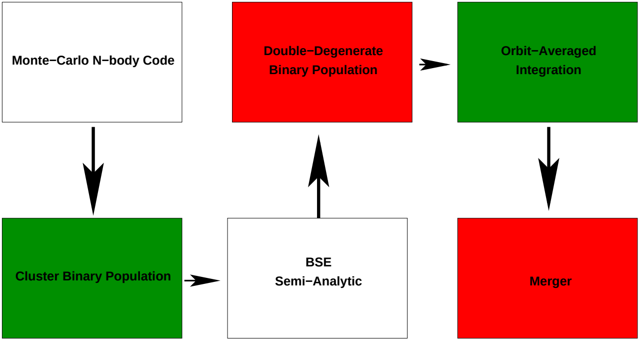
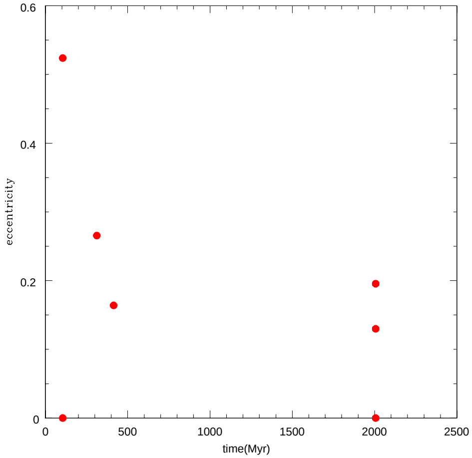


Monte Carlo cluster simulations kindly provided by Mirek Giersz.

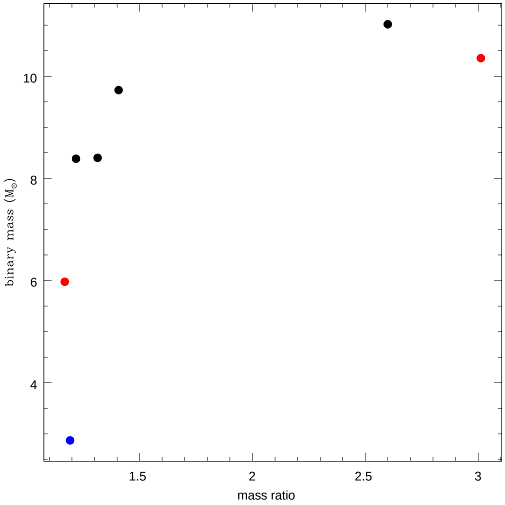
Some Results: M4

Initially binaries have only moderate eccentricity and are isolated so we use the orbit-averaged approximation.



- Of 159 merged binaries only 7 become double-degenerate before the merger.
- These binaries will merge due to relativistic effects.
- All mergers are circular.

The Merger Population



Binary masses between $2.87 M_\odot$ and $11.01 M_\odot$. Mass ratios between 1.17 and 3.01.

Black = BH-BH, Red = BH-NS, Blue = NS-NS

PN-Dynamics in Clusters on the Fly

Ideally we want to calculate binary dynamics as part of the cluster simulation.

- Modify NBODY6++ code to treat Post-Newtonian binaries.
  - Aarseth (1999), Spurzem (1999), Kupi, Amaro-Seoane & Spurzem (2006)
- NBODY6++ regularises binaries and external forces can be applied as a perturbation.

$$H = \frac{p^2}{2\mu} - \frac{GmM}{r} = E_0 \quad \Rightarrow \quad \Gamma = \frac{P^2}{8\mu} - GmM - E_0 u^2$$

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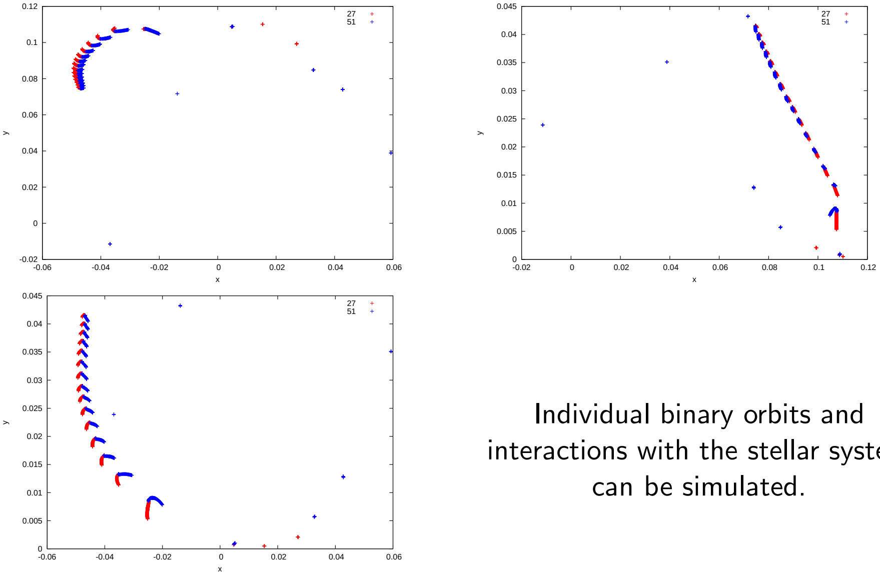
Semi-working Version

We have a version of NBODY6++ with up to 2.5PN corrections.

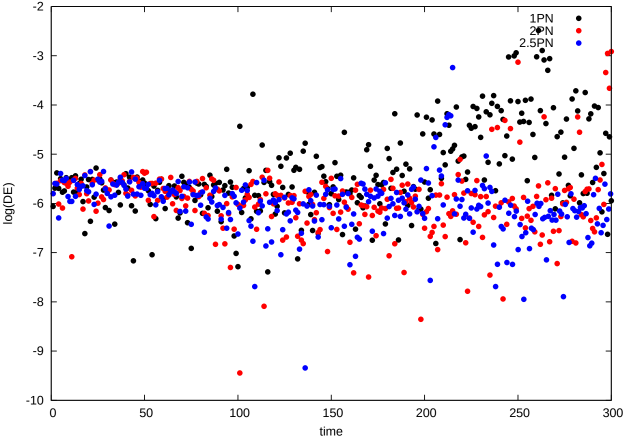
Individual Binary Orbits



Individual binary orbits and interactions with the stellar system can be simulated.

Still many Problems

So far simulations with no initial binaries.



Problems with global energy conservation in regions with many relativisic interactions.

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Direct N-body

- Have a version of NBODY6++ with PN terms in the testing phase.
- Still needs development to deal with repeated regularization of relativisic binaries.
- Need a more advanced version of the NBODY6++ code to deal with large numbers of initial binaries.