Three-Body Problem:

Symbolic dynamics as a tool

I. PM and SD

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Abstract

I stress the usefulness of numerical symbolic dynamics.

There are methods or tools standing between equations of motion and the target N-body systems.

The Poincare maps (PM) are between integration methods and systems with 2 degrees of freedom.

Symbol dynamics (SD) is in a similar position.

In the first talk, I compare the utilities of PM and SD.

Two methods are sometimes complementary.

In the second talk, I explain some techniques developed in SD in the three-body problem. Trying to extend to higher dimensions.

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0. A brief history of celestial mechanics in Japan (1868 - 1945)
1. Introduction
2. Poincare Maps and TBP
3. Symbolic dynamics and TBP
4. Comparison of PM and SD
5. Concluding remarks

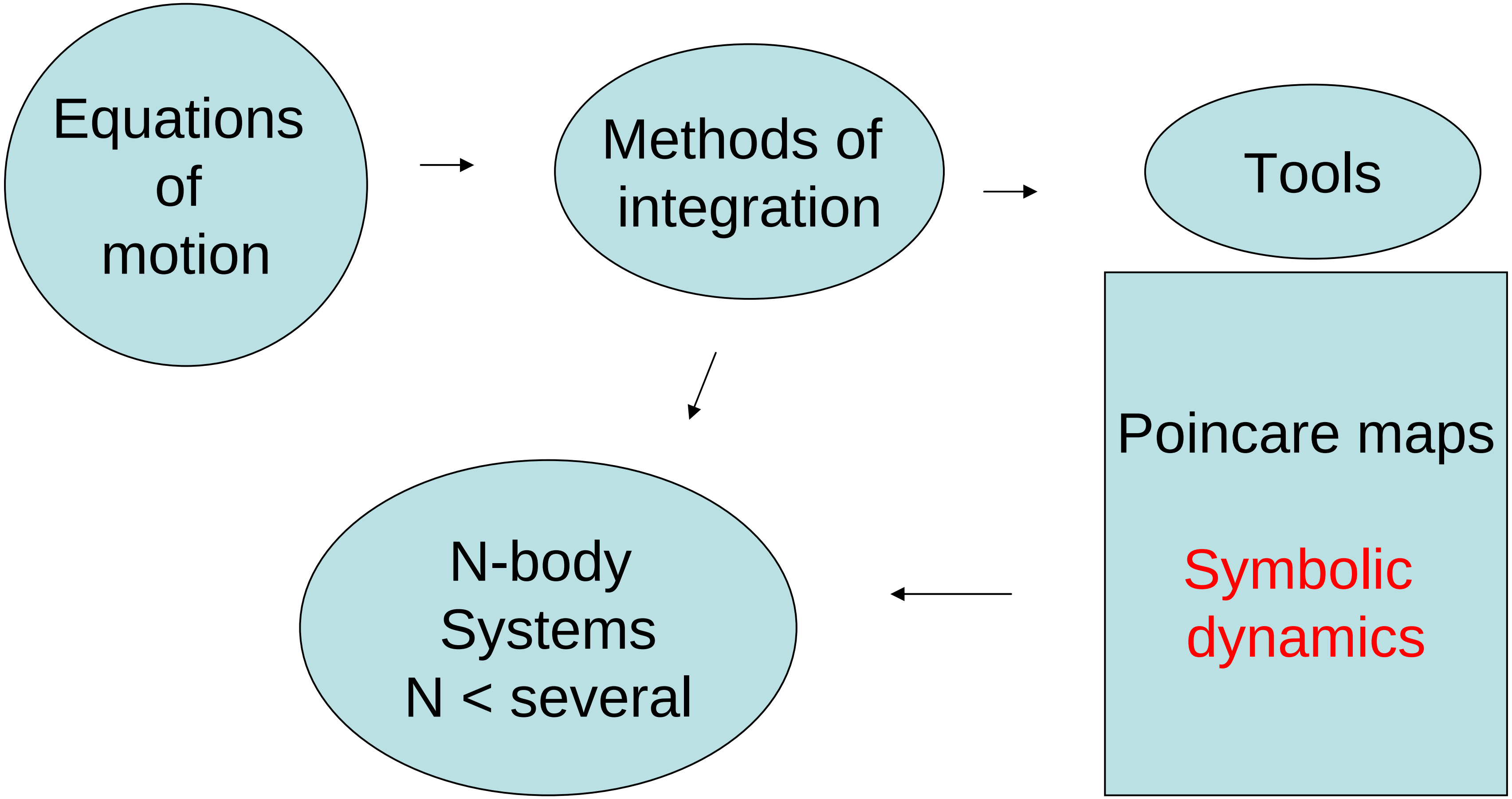
0. A brief history of celestial mechanics in Japan

- 1868, The Meiji Restoration.
- 1877, The university of Tokyo was established.
- 1878 – 1883, American astronomers were employed.
- H. Terao studied celestial mechanics in France (1879 – 1883).
- 1902, H. Kimura; The z-term in the latitude variations:

  $$\Delta\varphi = x \cos \varphi + y \sin \varphi + z.$$

- 1908, The Astronomical Society of Japan.
- 1918, K. Hirayama; Discovery of families of asteroids.
- 1920s, T. Matukuma; Periodic orbits in the Hill's case.
- 1931, Y. Hagihara; The relativistic one-body problem.

1. Introduction



The purpose of the talk:

to advertise the usefulness of numerical symbolic dynamics.

How?

Compare the utilities of

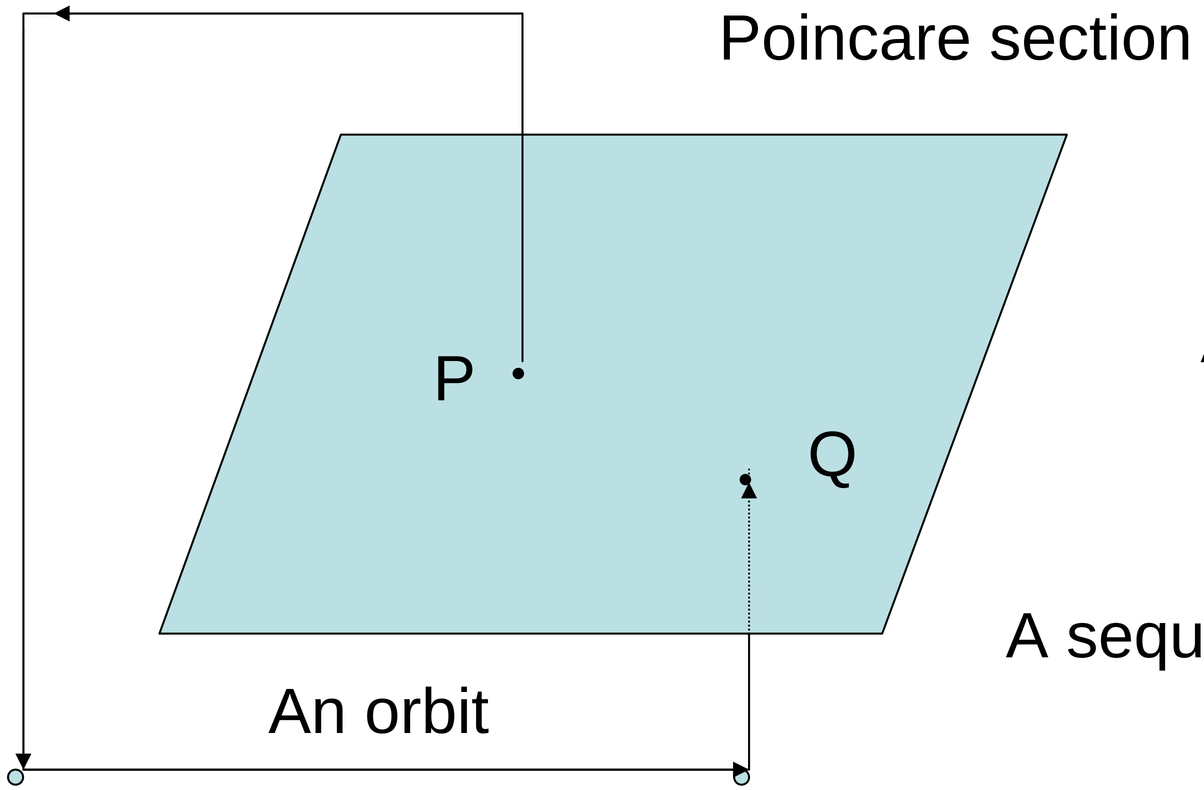
Poincare Maps and Symbolic Dynamics

See

- Similarity and difference
- Numerical examples

2. Poincare maps and TBP

2.1. Definition



An orbit

↓

A sequence of points

Analyze the structure of the set of point sequences

2.2. Some of the works

Poincare maps are essentially for systems with 2 degrees of freedom.

RTBP: Poincare (1899), Henon(1960s), Jefferys(1971),
Markellos(1974), Benest(1974)
Wisdom(1982), …

GTBP:
Hietarinta&Mikkola(1993); collinear problem.

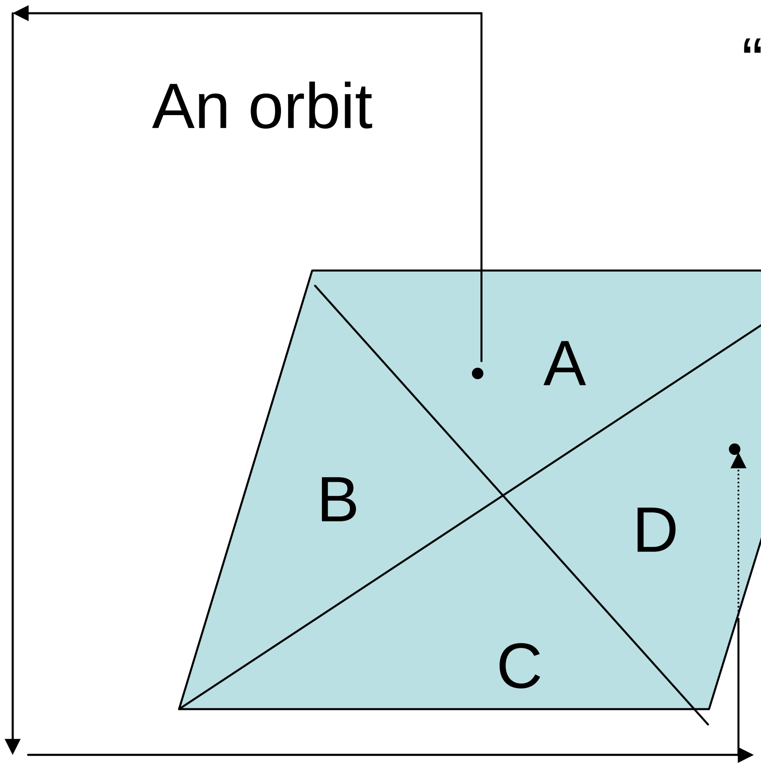
Galactic potential:Henon&Heiles(1964)

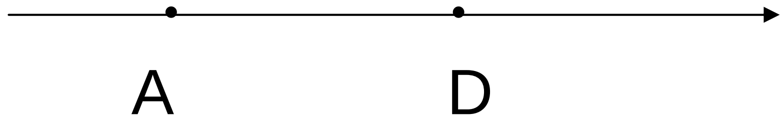
2-D Maps such as the Henon map, the standard map: many

3. Symbolic dynamics and TBP

3.1. Mark the external events along the orbit

External events:
"Visits" to particular parts of the phase space





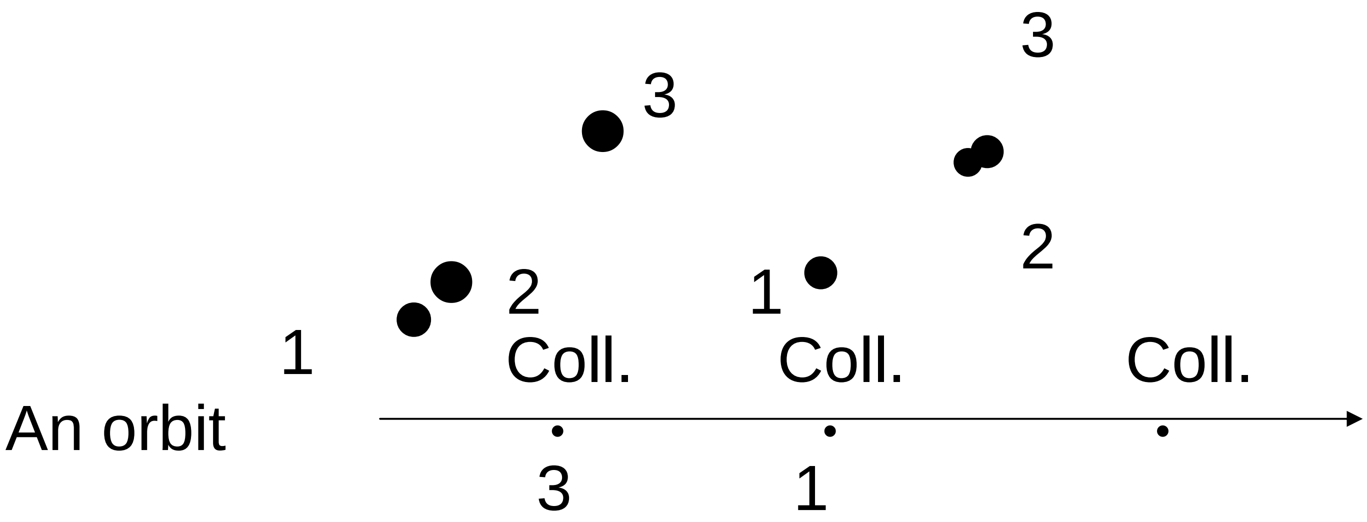
An orbit → A sequence of symbols

Analyze the structure of the set of symbol sequences

3.2. Mark the internal events along the orbit

Internal events:

Collision (zero length), zero area, or zero volume



An orbit → A sequence of symbols

3.3. Some of the works

Isosceles GTBP:

Alekseev(1968,1969), Simo&Martinez(1988),
Zare&Chesley(1998)

Rectilinear GTBP:

Tanikawa&Mikkola(2000a,b), Saito&Tanikawa(2007,2008),
Orlov et al.(2008)

Planar GTBP:

Chernin et al.(2006), Tanikawa&Mikkola(2008),
Sano&Tanikawa(2008)

4. Comparion of PM and SD

4.1 Poincare maps

Object systems: Two degrees of freedom systems

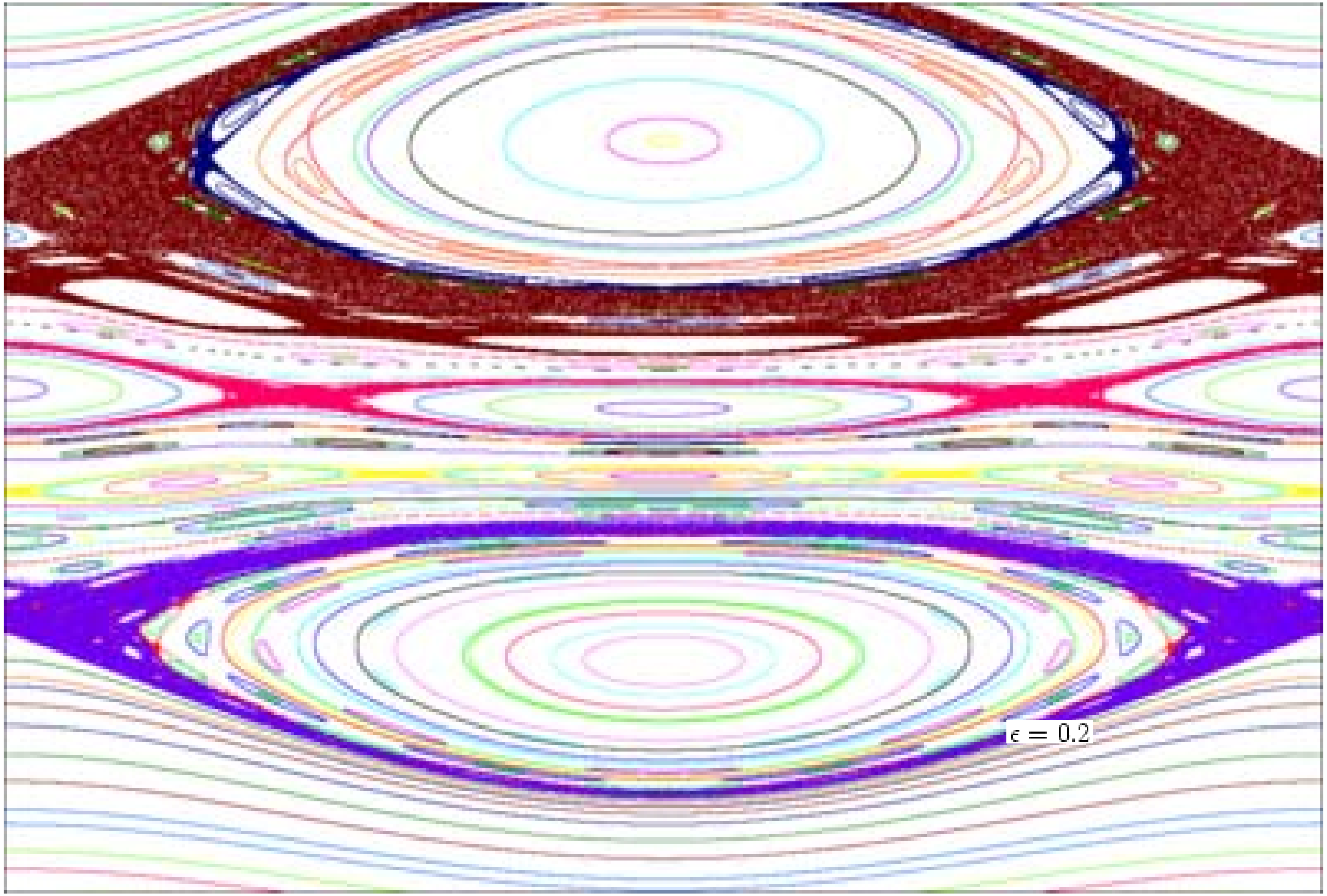
The 3-dimensional equi-energy surface.

A 2-D surface divides the 3-D space.

Results: stable structures like

Stable periodic orbits, quasi-periodic orbits,

separatrix structure, sea of chaos.



Taken from

http://www.scholarpedia.org/article/Hamiltonian_systems

4.2 Symbol dynamics

4.2.1. The case external events are used.

Object systems: similar to the PM cases

Results

- the boundaries of the structure;
- unstable periodic orbits;
- The sea of chaos is resolved.
- The structure inside the stable structure may be smeared out.

4.2 Symbol dynamics

4.2.2. The case internal events are used.

Object systems: systems with higher dimension

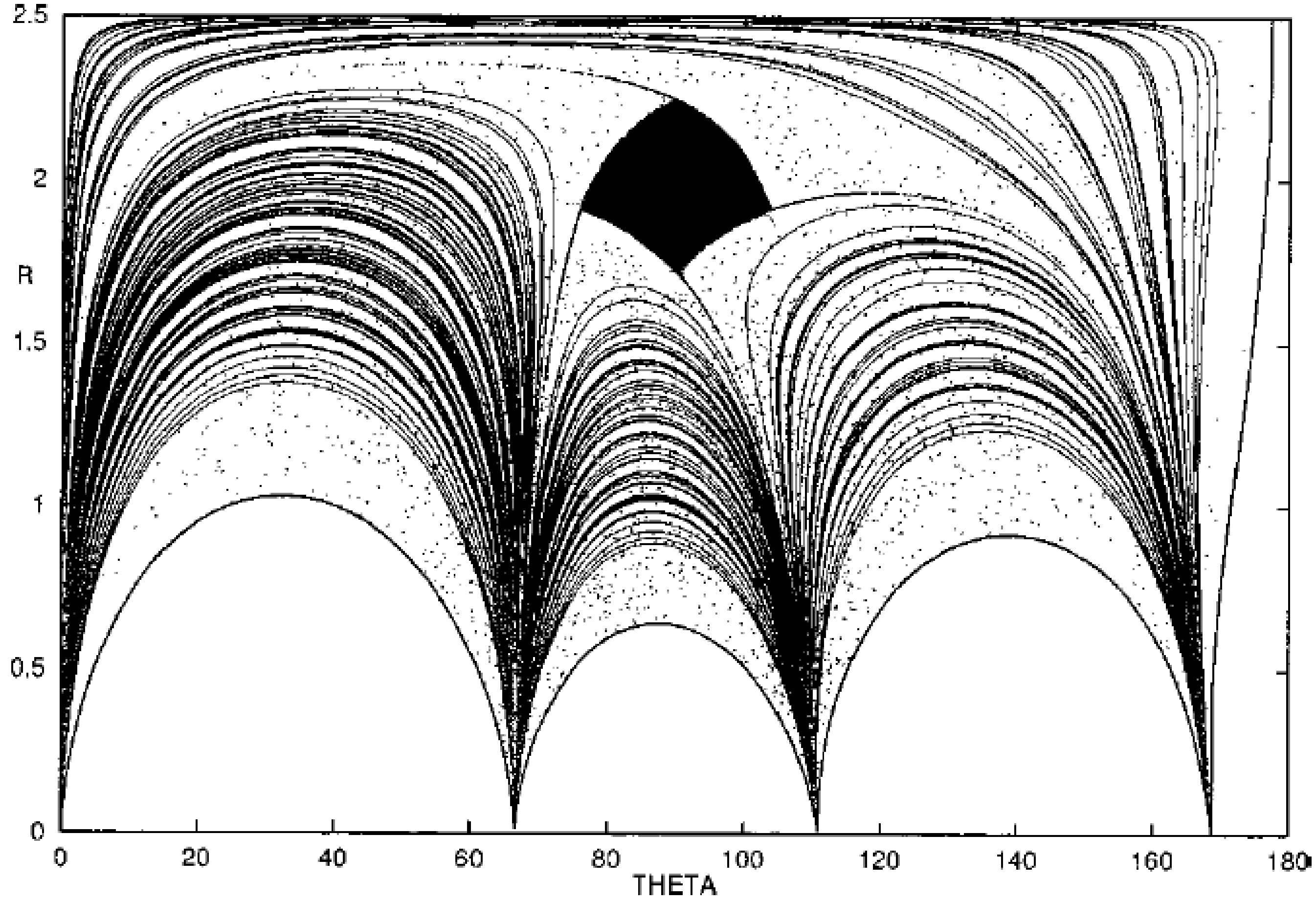
not restricted to a 3D space.

Yet to be looked for.

Results

Similar to the case of external events.

Not so much is obtained.



2.5
2
1.5
1
0.5
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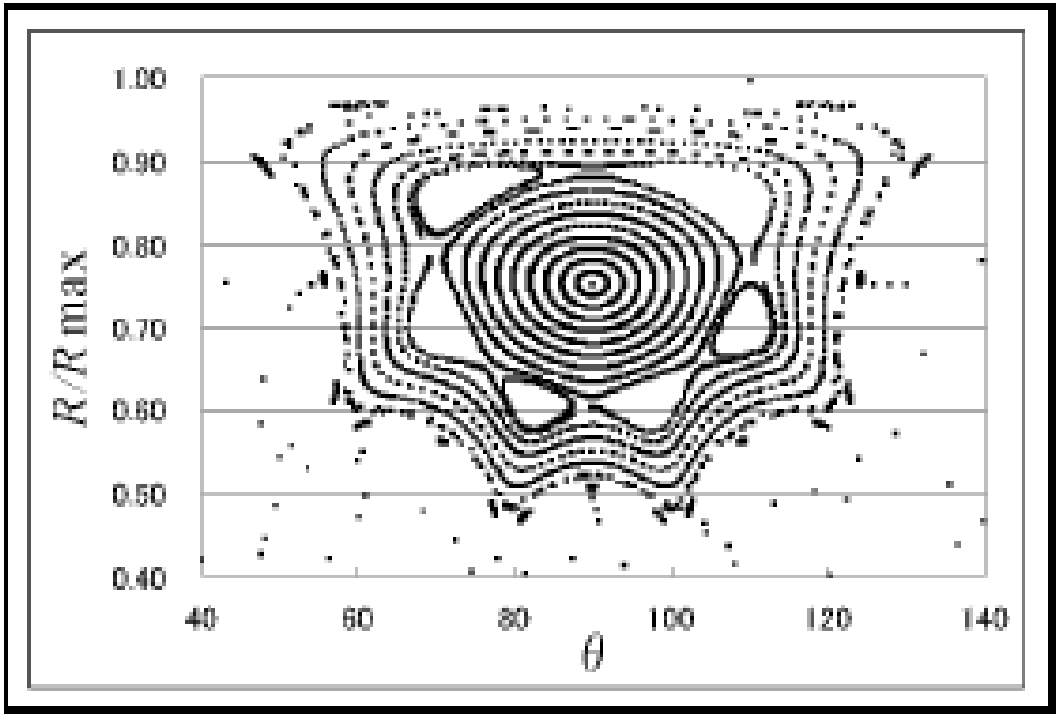
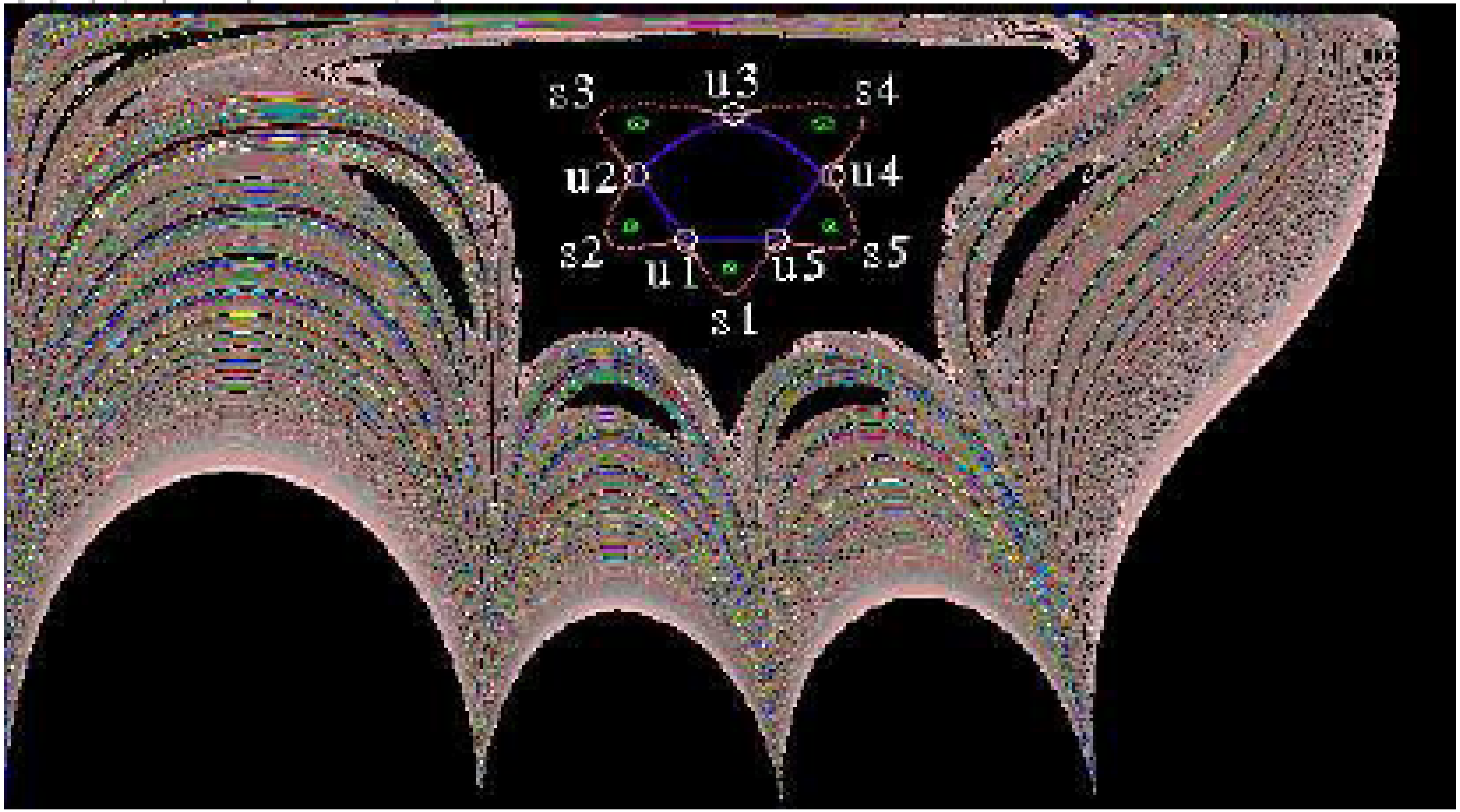
40
60
80
100
120
140
160
180
THETA



3: A Poincaré map in order to select candidates for periodic points. In this map, $a = -0.2$
pected periodic points are those with $\nu = 4/6$. The centres of elliptic motions under the Poi
re the candidates.

(a) $(a, b) = (-0.150, 0)$



5. Concluding remark

(1) Symbolic dynamics is a tool with the same level of utility as Poincare maps.

(2) The role of SD is sometimes complementary to that of PM.

(3) SD is not yet fully developed in the three-body problem.

End

3. Comparion of PM and SD

3.1 Poincare maps

Object systems

1. Two degrees of freedom – four dimensional space

   RTBP, 2D maps

Find stable objects like

stable periodic orbits,

quasi-periodic orbits, (stable structure)

Limitations

1. more than 2 degrees of freedom
2. chaotic orbits

4. The rectilinear case of the Three-body problem

4.1 The rectilinear (collinear) problem

1) Masses m_1, m_0, m_2 are aligned on a fixed line.

2) Three masses repeat binary collision.

3. Importance of COs and POs

3.1 PO (periodic orbits)

- Poincare's conjecture;
  Any solution can be approximated arbitrarily by POs of arbitrarily long periods (Methodes Nouvelles).
- Counter examples
- In which systems, are periodic solutions dense?
- In which part of systems, are periodic solutions dense?
- How in the three-body problem?

3. Importance of COs and POs

3.2 CO (collision orbits)

- COs are not the solutions to the problem.
- COs divide the phase space into cells.
- Escapes are frequently related to COs
- Other phenomena like exchange, capture may have intimate relations with COs.

4. Symbolic representation

- Give a symbol at a particular instant along an orbit if some special event takes place:

  (1) when the orbit visits a specified place of the phase space.

  (2) when collision takes place.
- Replace orbits as continuous curves in the phase space by a sequence of symbols (symbol sequences)
- Apply techniques of symbol dynamics developed in mathematics

0. A brief history of celestial mechanics in Japan

- 1868, The Meiji Restoration
- 1877, The university of Tokyo was established.
- T.C. Mendenhall (1878 - 1881), H.M. Hall 1880 – 1883)

φ

- H. Terao studied celestial mechanics in France (1879 – 1883).
- 1902, H. Kimura, The z-term in the latitude variations:

$$\Delta\varphi = x \cos \varphi + y \sin \varphi + z$$

- 1918, K. Hirayama, Discovery of Families of asteroids.
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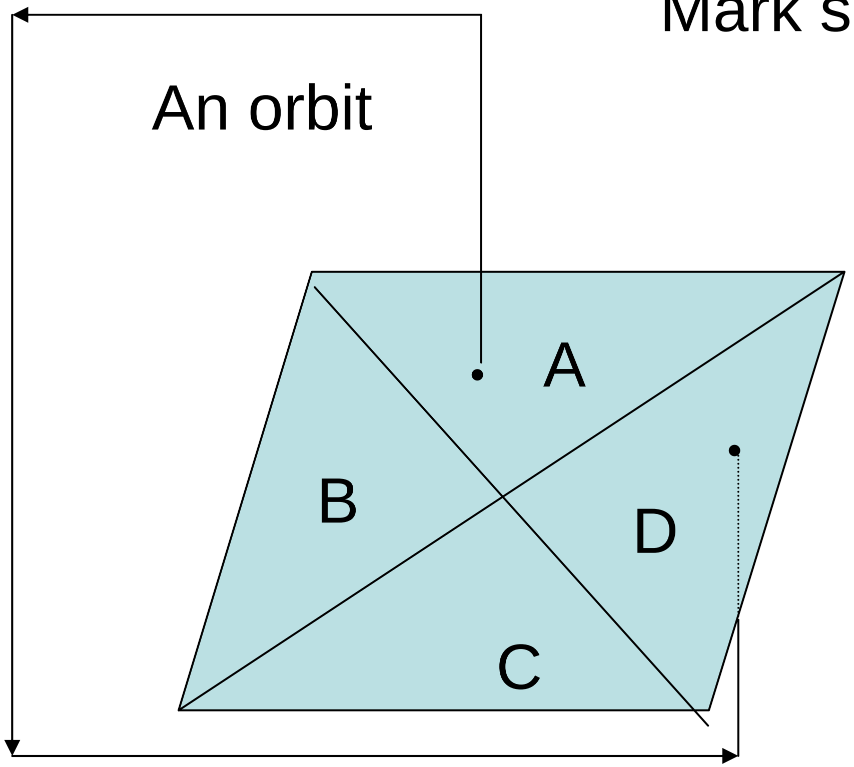
2.2. Symbolic dynamics



Mark special events along the orbit

“Visits” to particular parts of the phase space

External events

Collision (zero length), zero area, or zero volume

Internal events

An orbit

An orbit → A sequence of symbols