Three-Body Problem

Symbolic dynamics as a tool

II. Some techniques

Kiyotaka TANIKAWA

National Astronomical Observatory of Japan

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0. A brief history of celestial mechanics in Japan

(after 1945)

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0. A brief history of celestial mechanics in Japan

- 1945, Japan was defeated by the USA.
- 1957, The launch of the sputnik by the USSR.
- Celestial mechanists were invited to the USA.
- 1961, Kozai, Perturbation theory for satellites ;1962, Kozai mechanism.
- 1966, G. Hori, general transformation theory.
- 1974, J. Yoshida, Escape criterion.
- 1977, H. Kinoshita, Nutation theory.
- 1983, H. Yoshida, Non-integrability; 1990, Higher order symplectic integrators.
- 1990s -, GRAPE(J. Makino, E. Kokubo), T. Ito, …

1. Introduction

1.1 Poincare

- Poincare(1890) proved the non-integrability of the RTBP in two senses.

  Transverse intersections of stable and unstable manifolds of periodic orbits.

  RTBP does not have analytic integrals.

Then, Poincare suggested the qualitative study of the three-body problem. Poincare maps

1.2 Japanese-Finnish dish with some Russian taste

- Chazy (1922), Classification of jnitial and final motions

  H, HP,HE, HB, B, OS --- 36 combinations
- 1940s – 1960s,The Russian school (Merman, Sitnikov, Alekseev),
- late 1960s – , Free-fall problem: Agekyan, Anosova, Orlov, Tanikawa et al.(1995), …
- Toward the end of the 20 century, Tanikawa & Mikkola, …

1.3. Other dishes

- Stability of hierarchical system:
  Harrington(1975), Szebehely&Zare(1977),…,
  Mardling&Aarseth(1999), Valtonen et al.(2008)
- Binaries and single stars: Yabushita(1966),
  Heggie(1975), Heggie&Hut(1993)
- Figure eight and choreography: Moore(1993),
  Chenciner&Montgomery(2000),Simo(2002)
- Escape criterion: Yoshida(1974), Marchal et al.(1984),
- Black hole triples: Iwasawa et al.(2006).

- Oscillatory motions, …

2. Techniques in SD

1) The definition of symbols leads to collision orbits in the planar problem (internal symbols).

2) Search for POs using the symmetric mass configuration (1-D problem).

3) Find sub-configurations in the planar problem.

4) Change mass parameters.

3. Planar symbolic dynamics

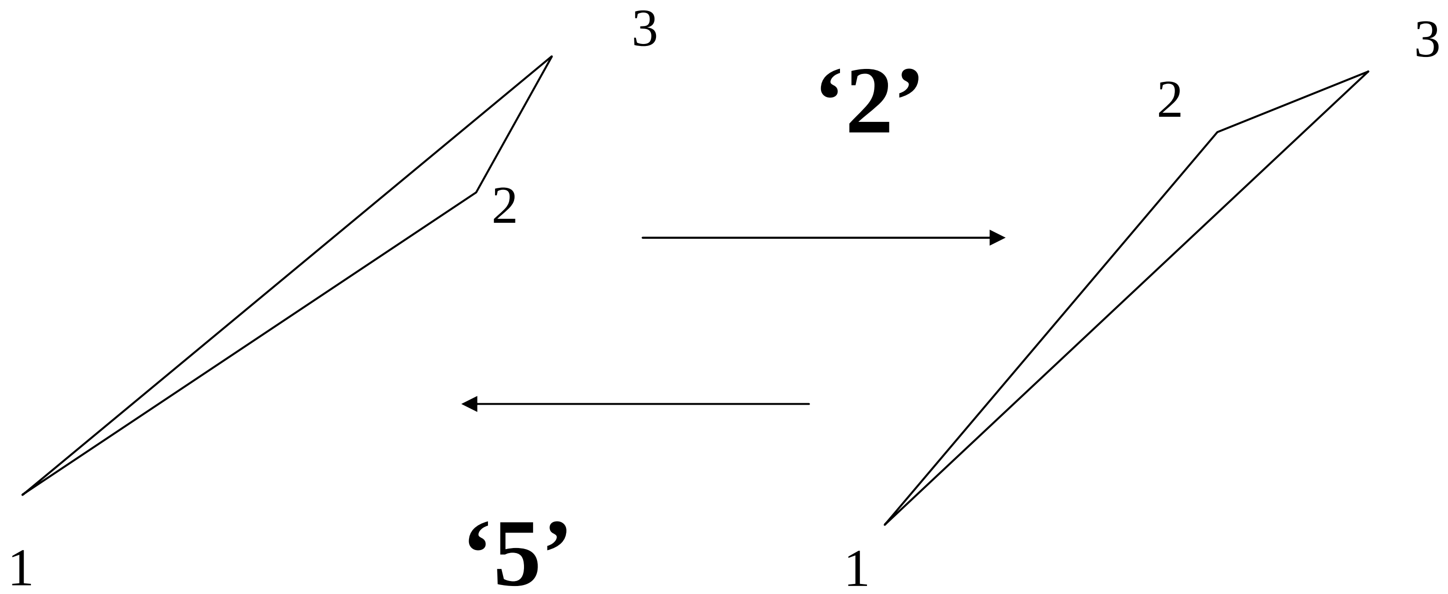
3.1. The Planar three-body problem

Three degrees of freedom.

The method of PM is not generally applicable.

The method of SD with internal symbols may be useful.

3.2. Introduction of symbols

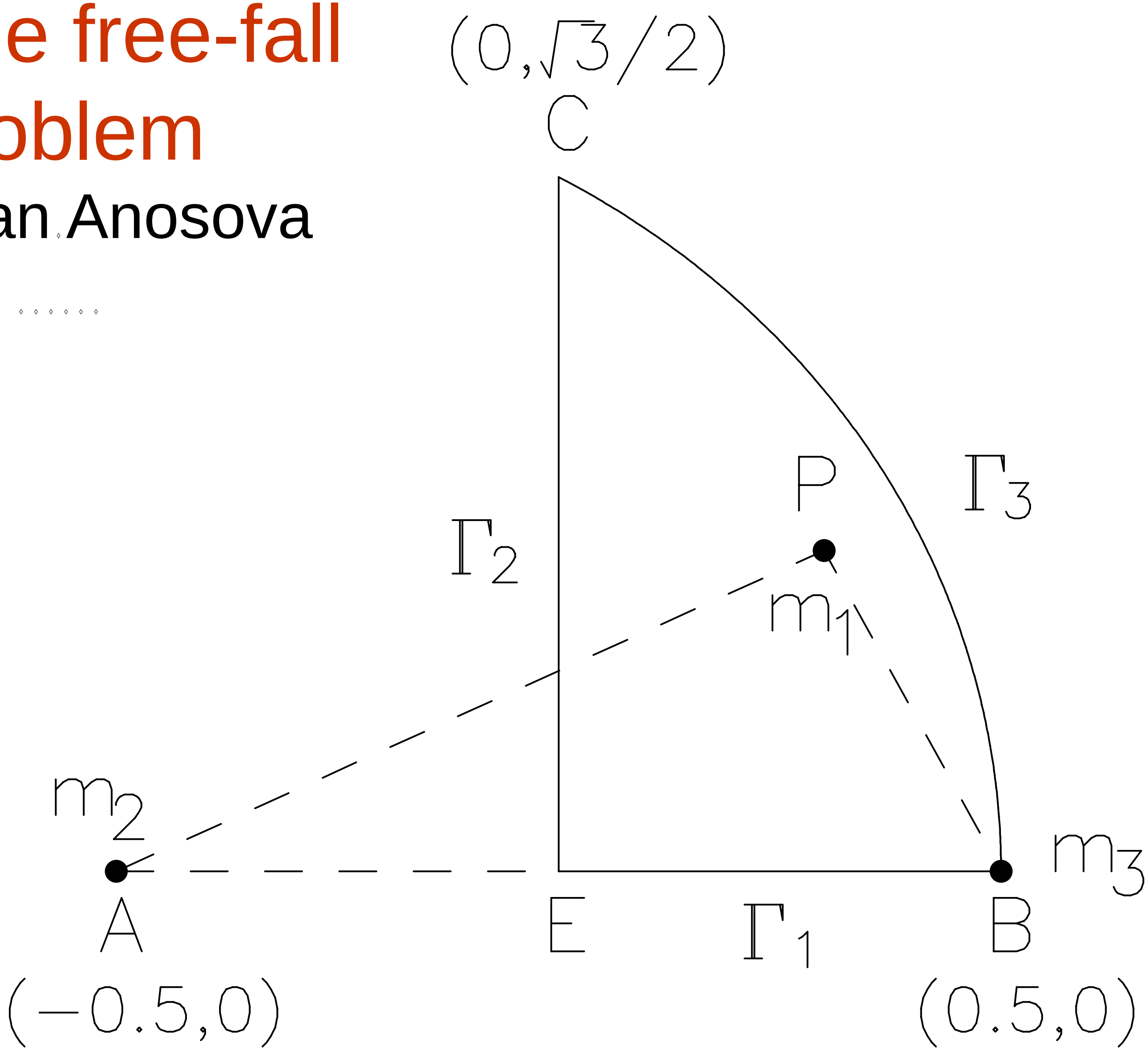


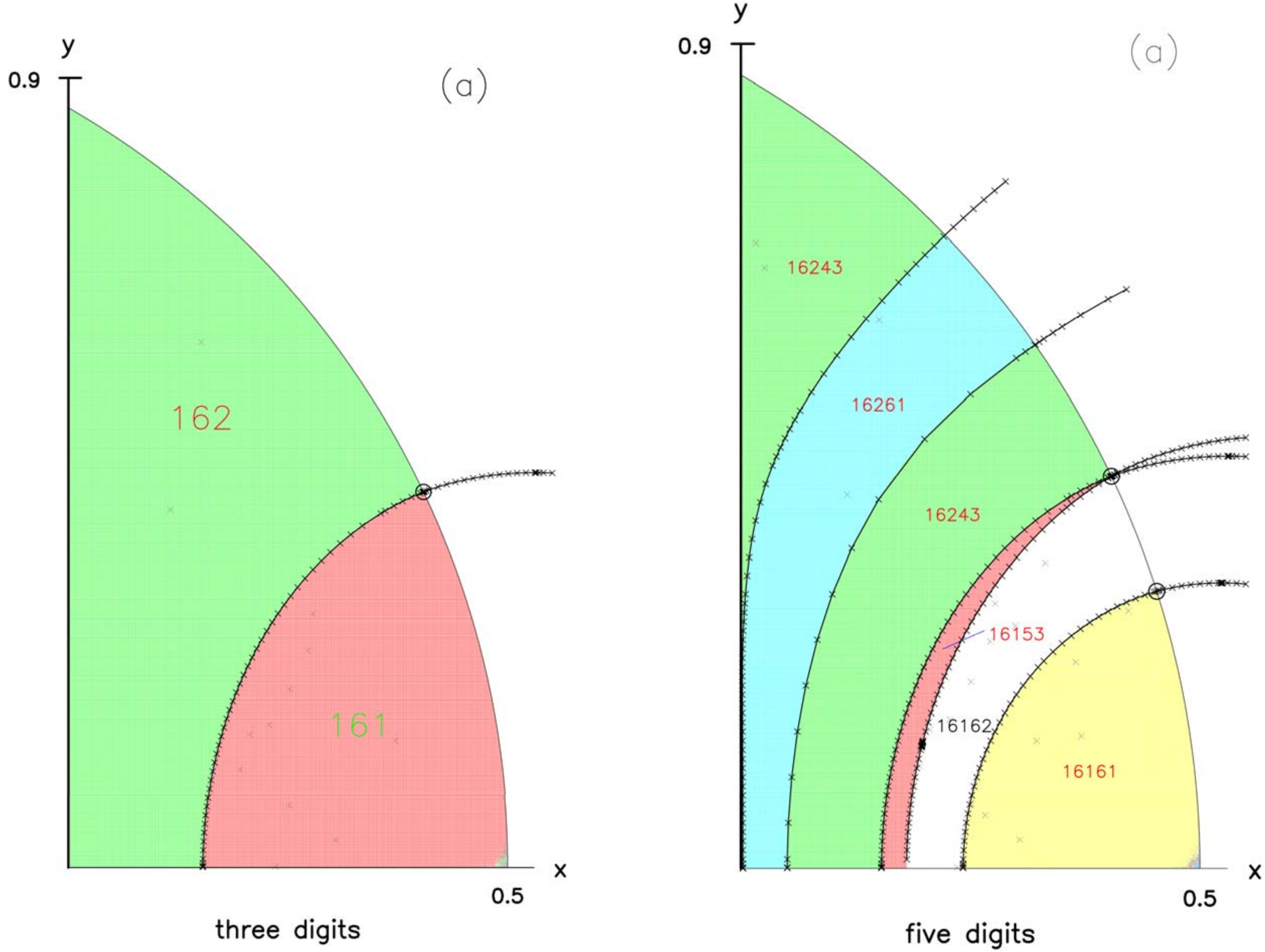
Positive area Negative area

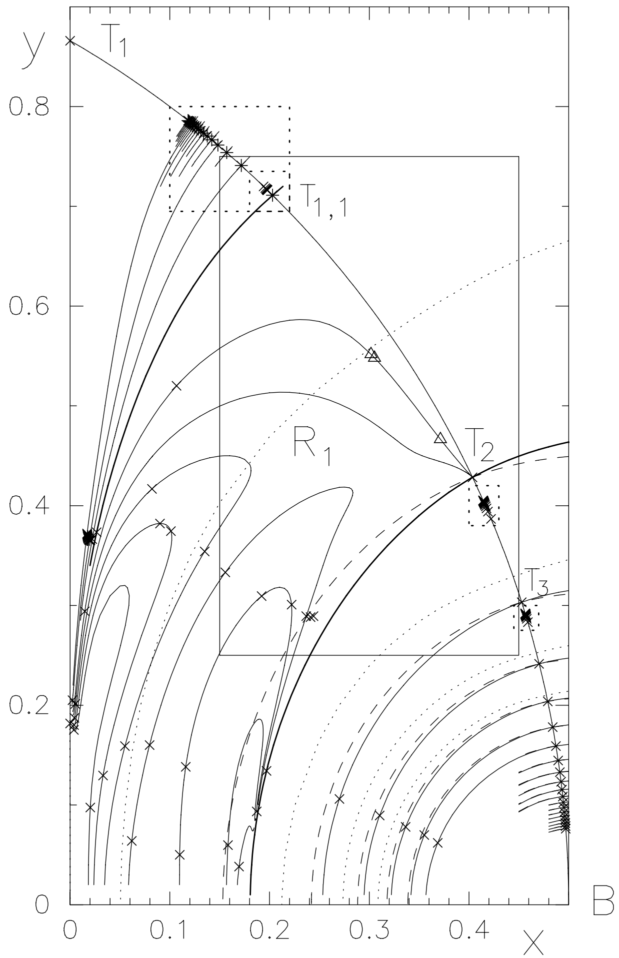
Symbol sequence: $.s_1 s_2 s_3 \ldots$

3.3.The free-fall problem

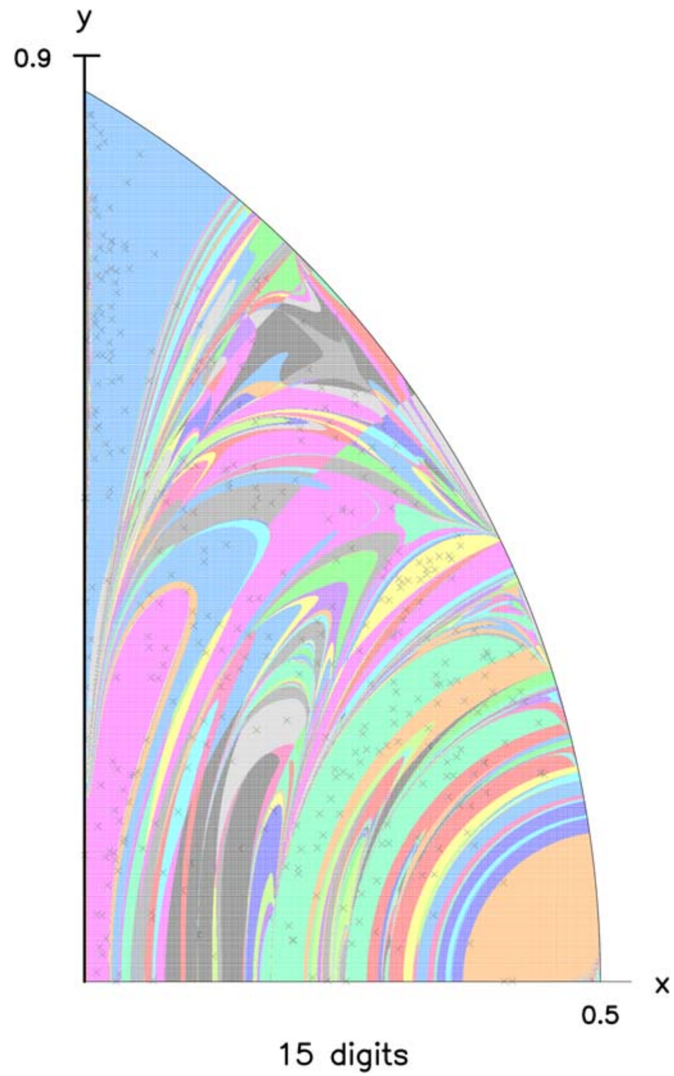
Agekyan Anosova




y
0.9
(a)
162
161
x
0.5
three digits
y
0.9
(a)
16243
16261
16243
16153
16162
16161
x
0.5
five digits



y
T1
0.8
T1,1
0.6
R1
T2
0.4
T3
0.2
0
0
0.1
0.2
0.3
0.4
B
x



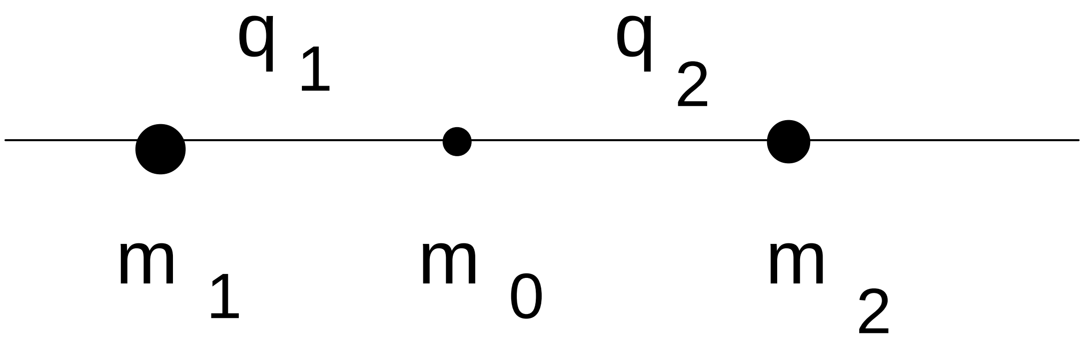
y
0.9
x
0.5
15 digits

4. Symmetry of the problem

Symmetry affects the form of symbol sequences.

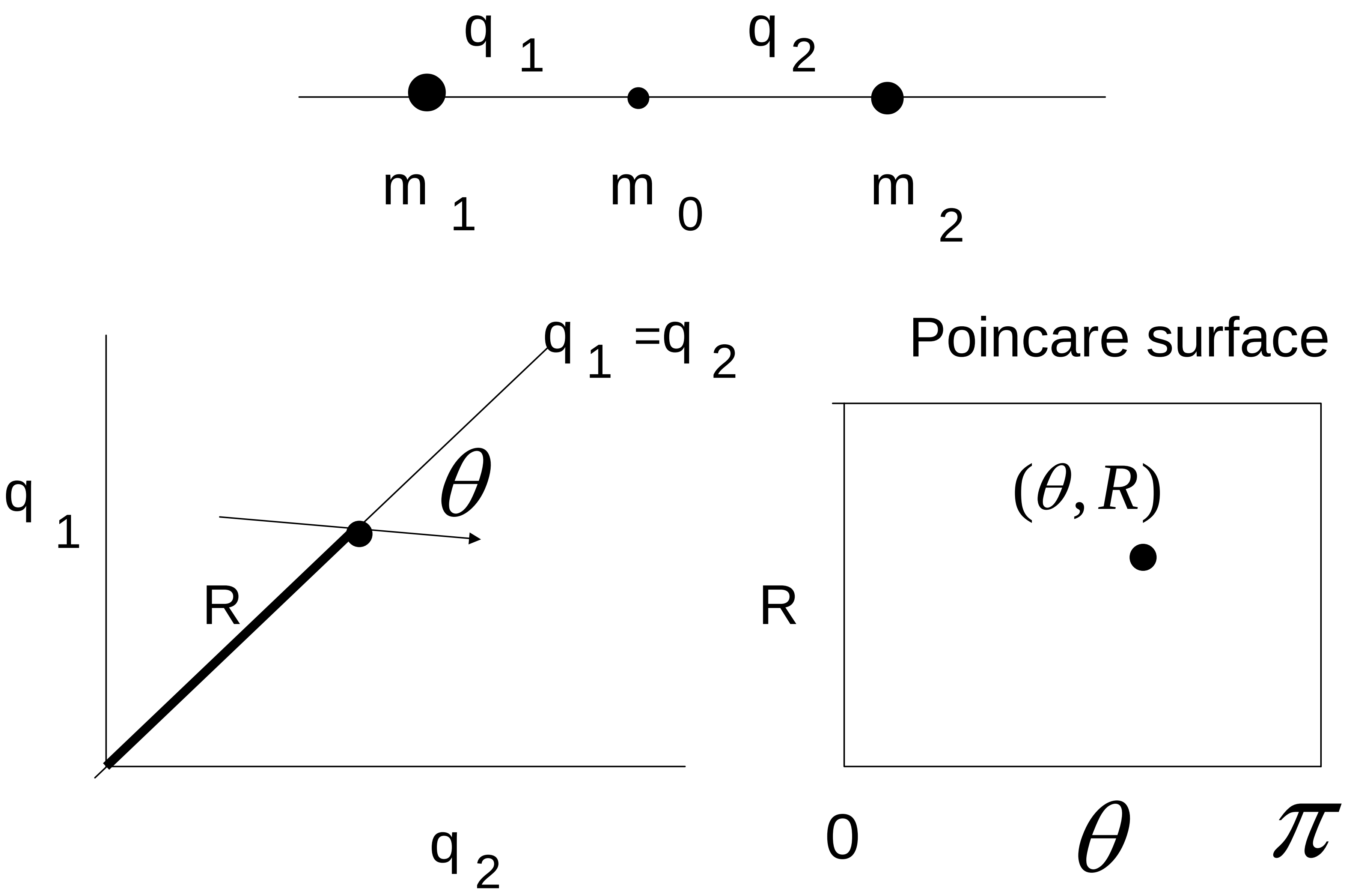
4.1. The rectilinear (collinear) problem

1) Three masses move on a fixed line.

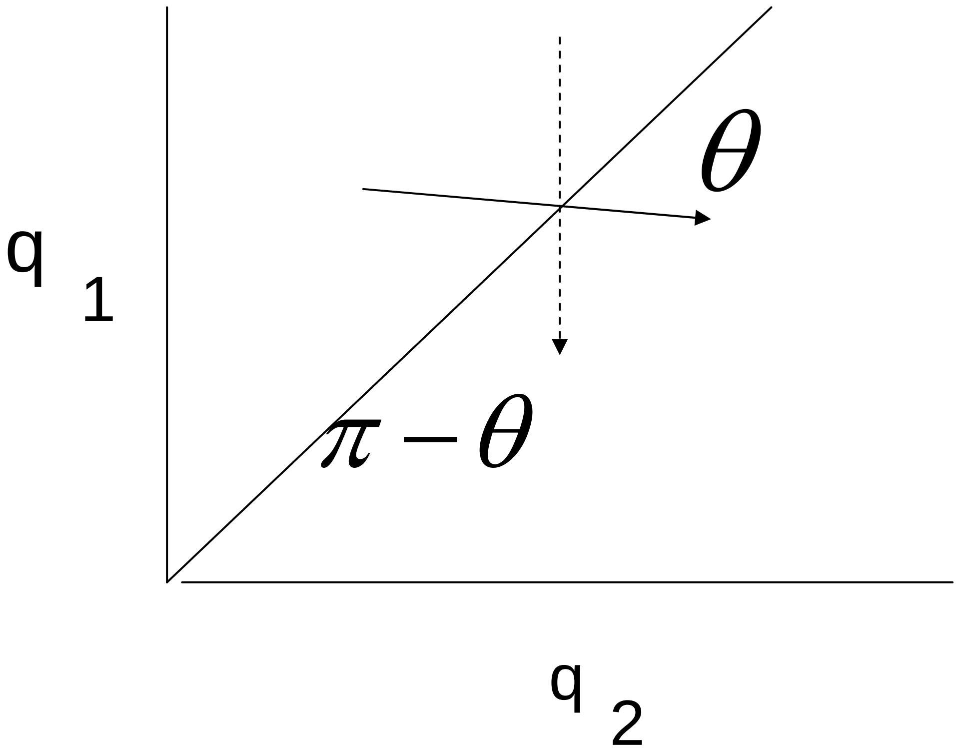


symmetric case: $m_1 = m_2$

2) They repeat binary collision.


q 1
q 2
m 1
m 0
m 2
q 1 =q 2
θ
q 1
R
q 2
Poincare surface
(θ, R)
R
0
θ
π

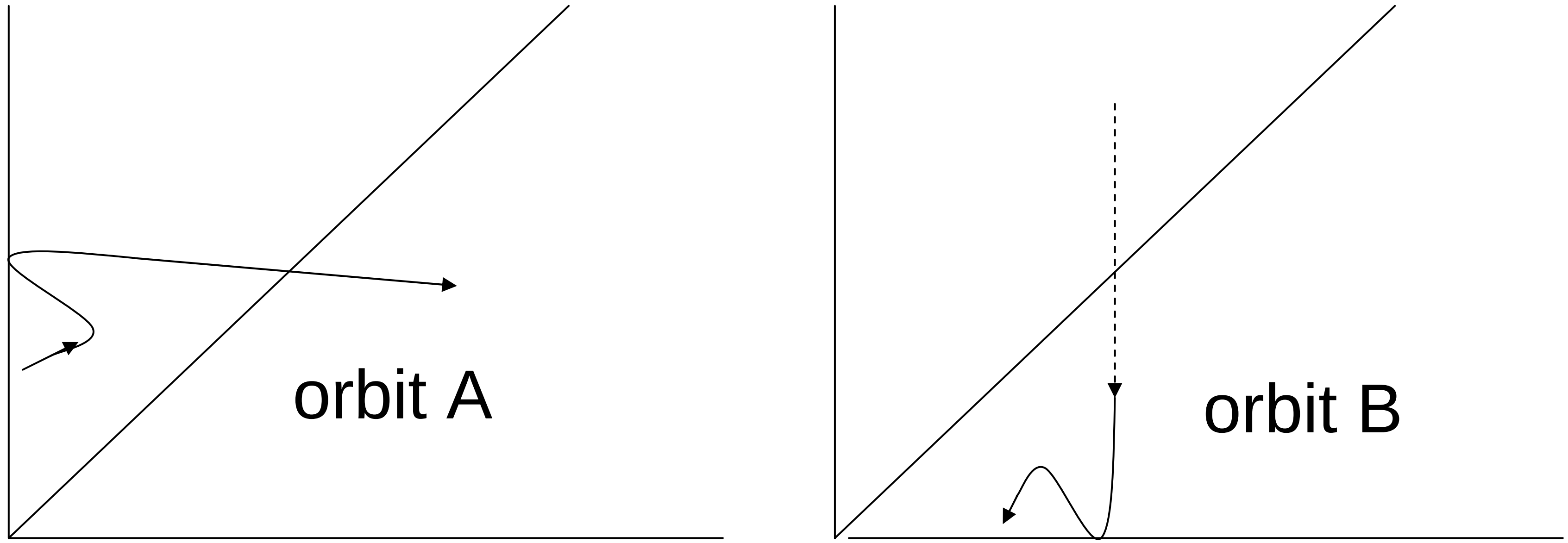
4.2. Symmetry due to $m_1 = m_2$



$\dot{q}_1 \Rightarrow -\dot{q}_2$

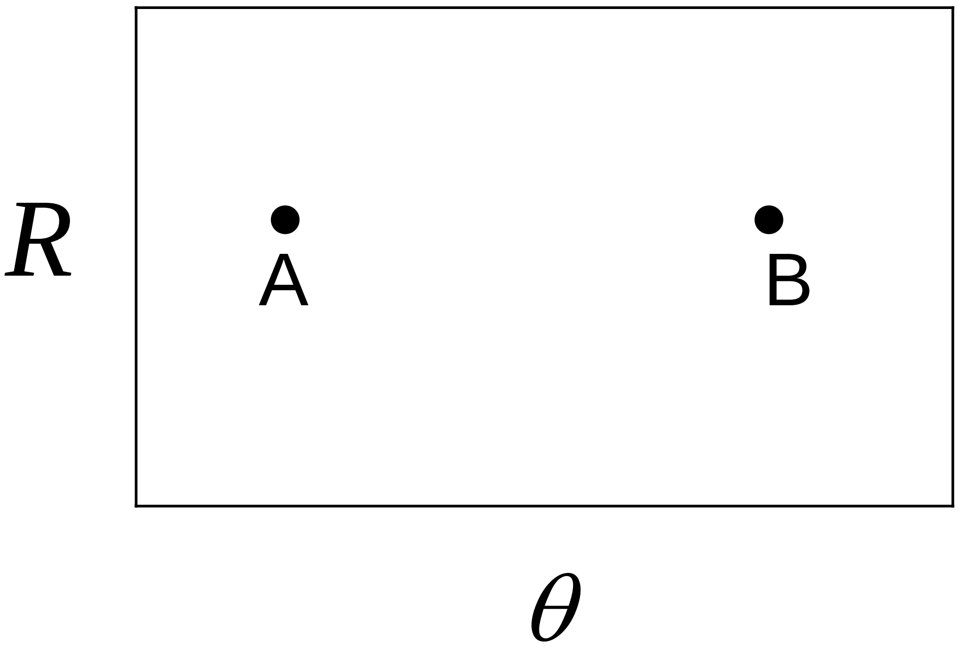
$\dot{q}_2 \Rightarrow -\dot{q}_1$

Symmetry due to $m_1 = m_2$



The past of **orbit A** is realized by the future of **orbit B**.

4.3. How to get bi-infinite symbol sequences



A (θ, R)

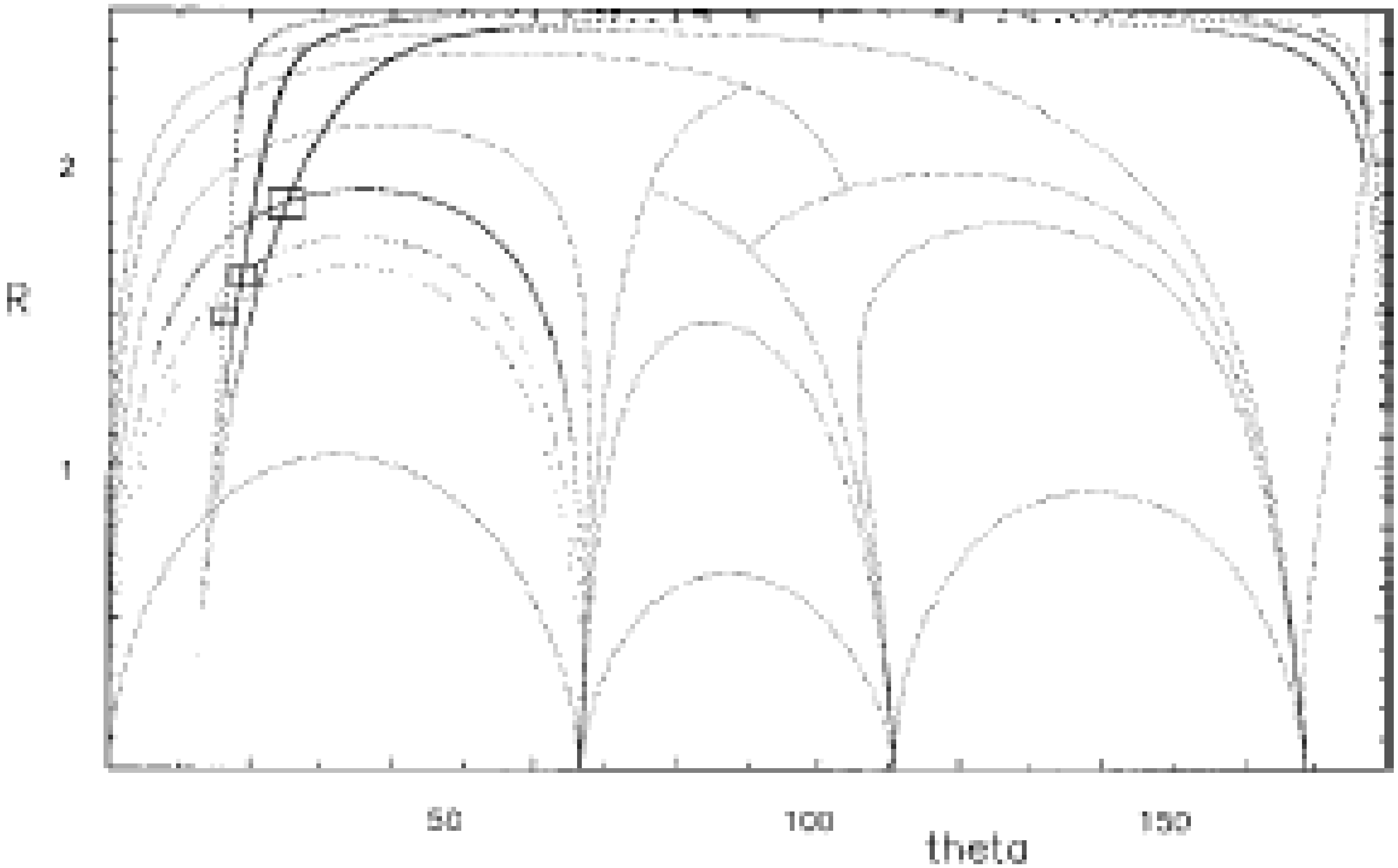
B $(\pi - \theta, R)$

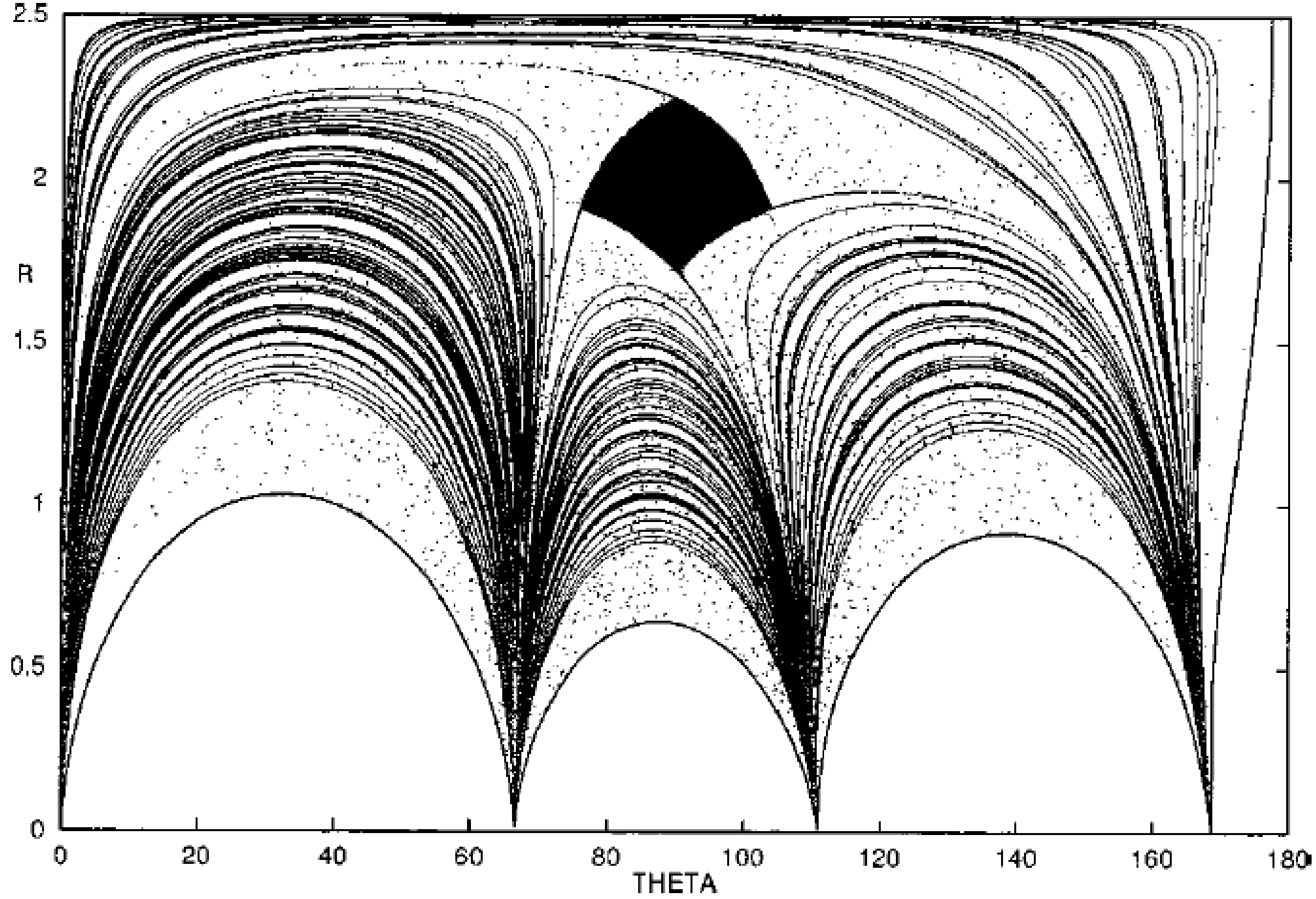
The future symbol sequence of orbit A $\bullet s_1 s_2 s_3 \cdots$

The future symbol sequence of orbit B $\bullet r_1 r_2 r_3 \cdots$

$$\cdots r^t_{-3} r^t_{-2} r^t_{-1} \bullet s_1 s_2 s_3 \cdots$$

4.4. Look for periodic orbits: superpose the reversed surface on the original surface





2.5
2
R
1.5
1
0.5
0
0
20
40
60
80
100
120
140
160
180
THETA

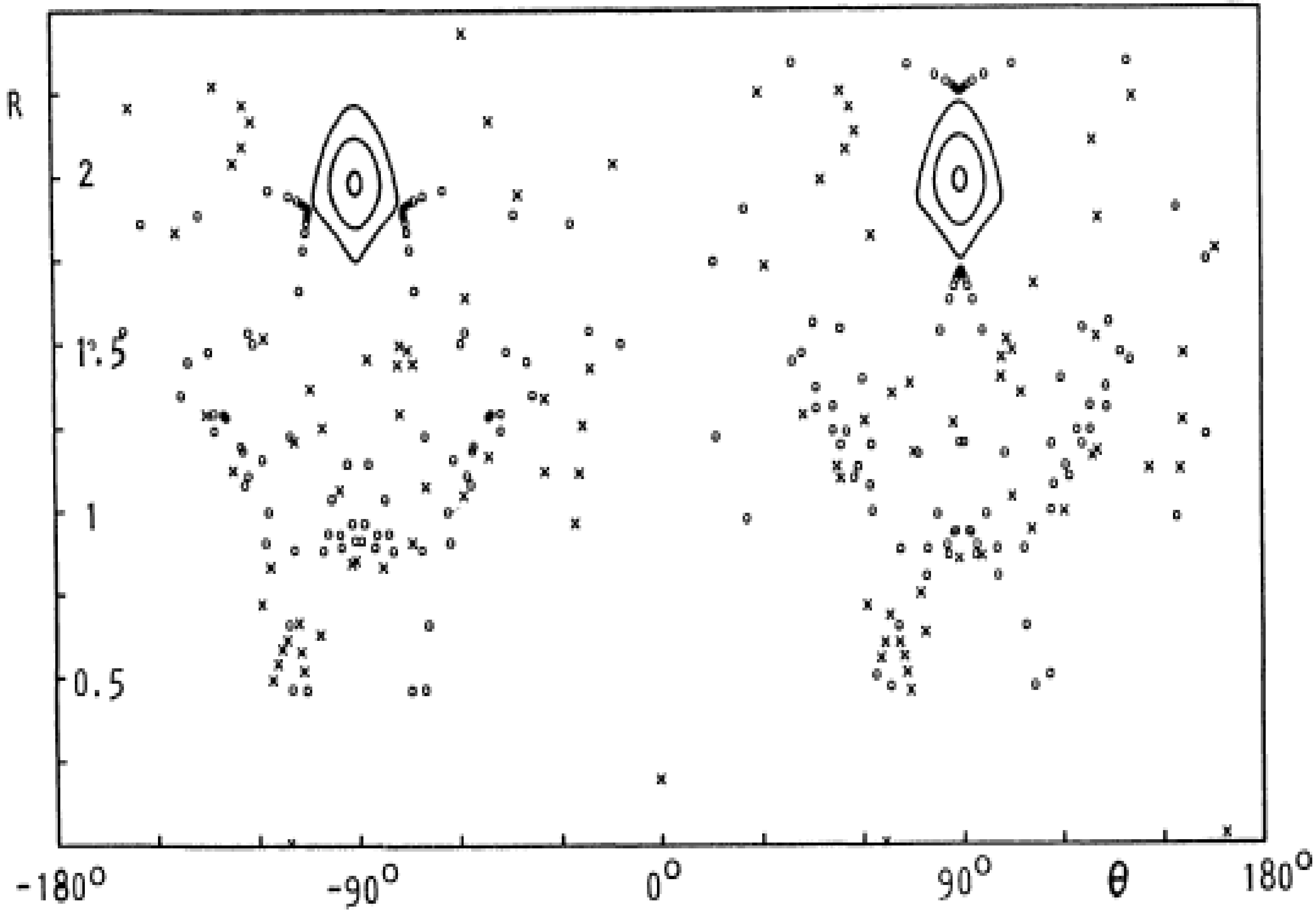


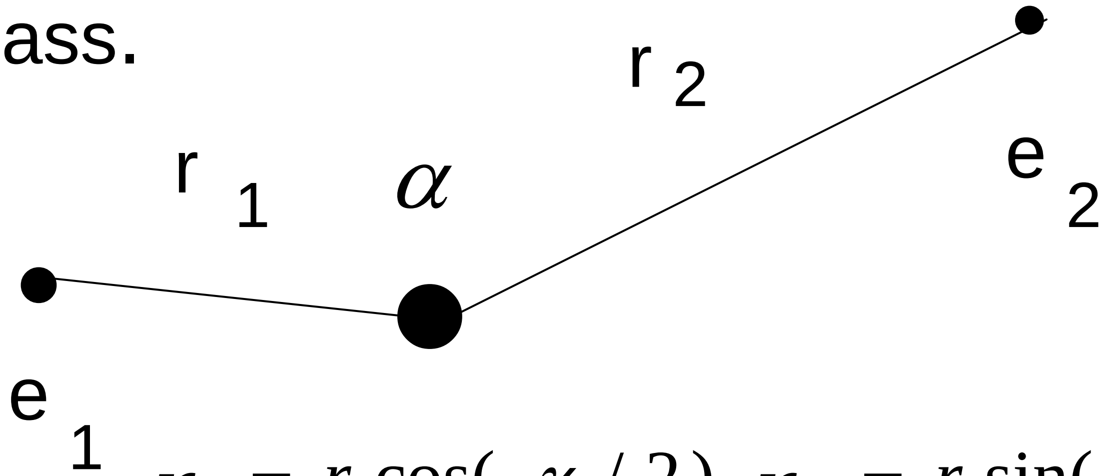
Figure 7: Poincaré sections of some typical trajectories. For details see the text.

5. Sub-configurations

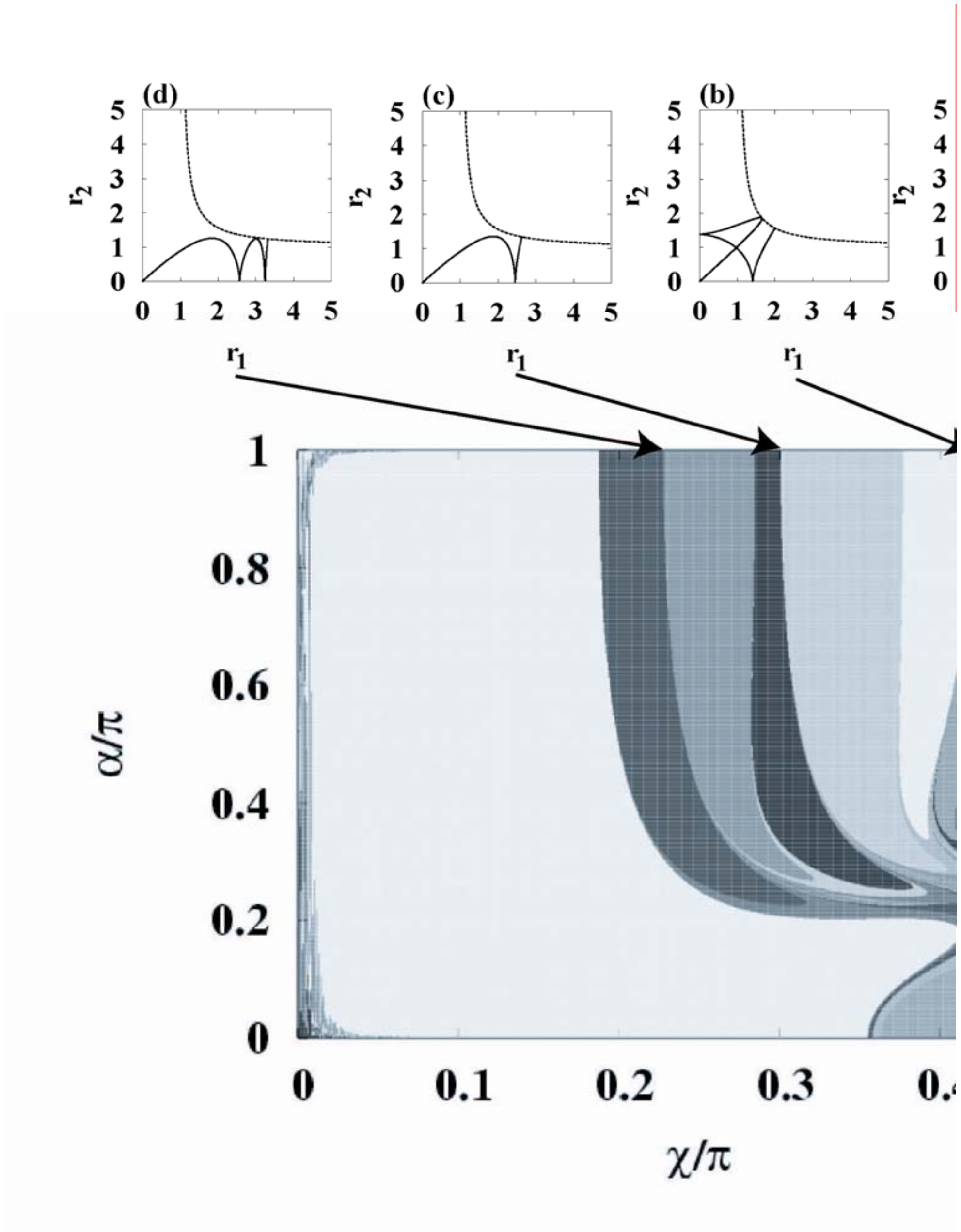
5.1 The coulomb three-body problem

The free-fall case

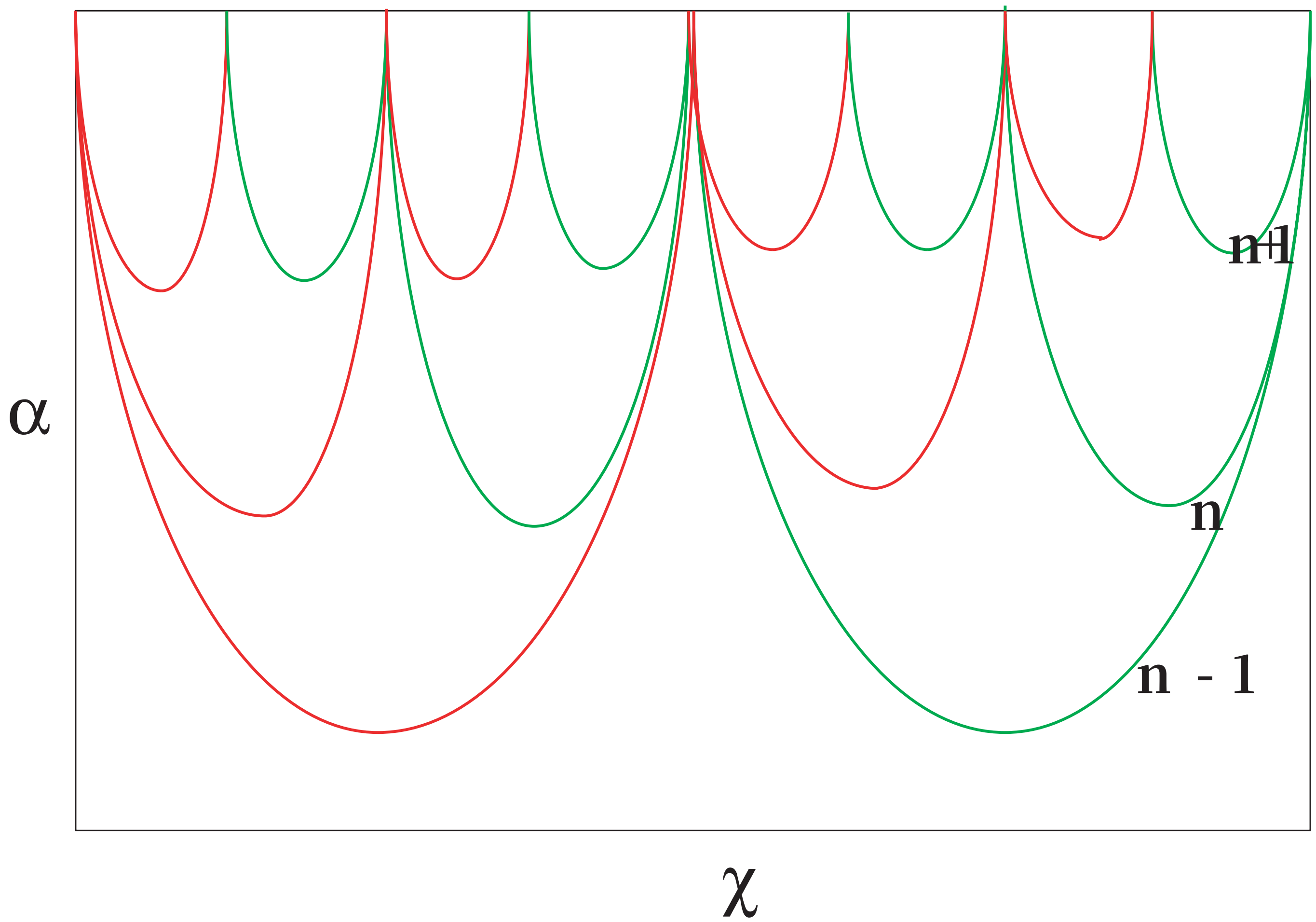
Helium nucleus with
infinite mass.



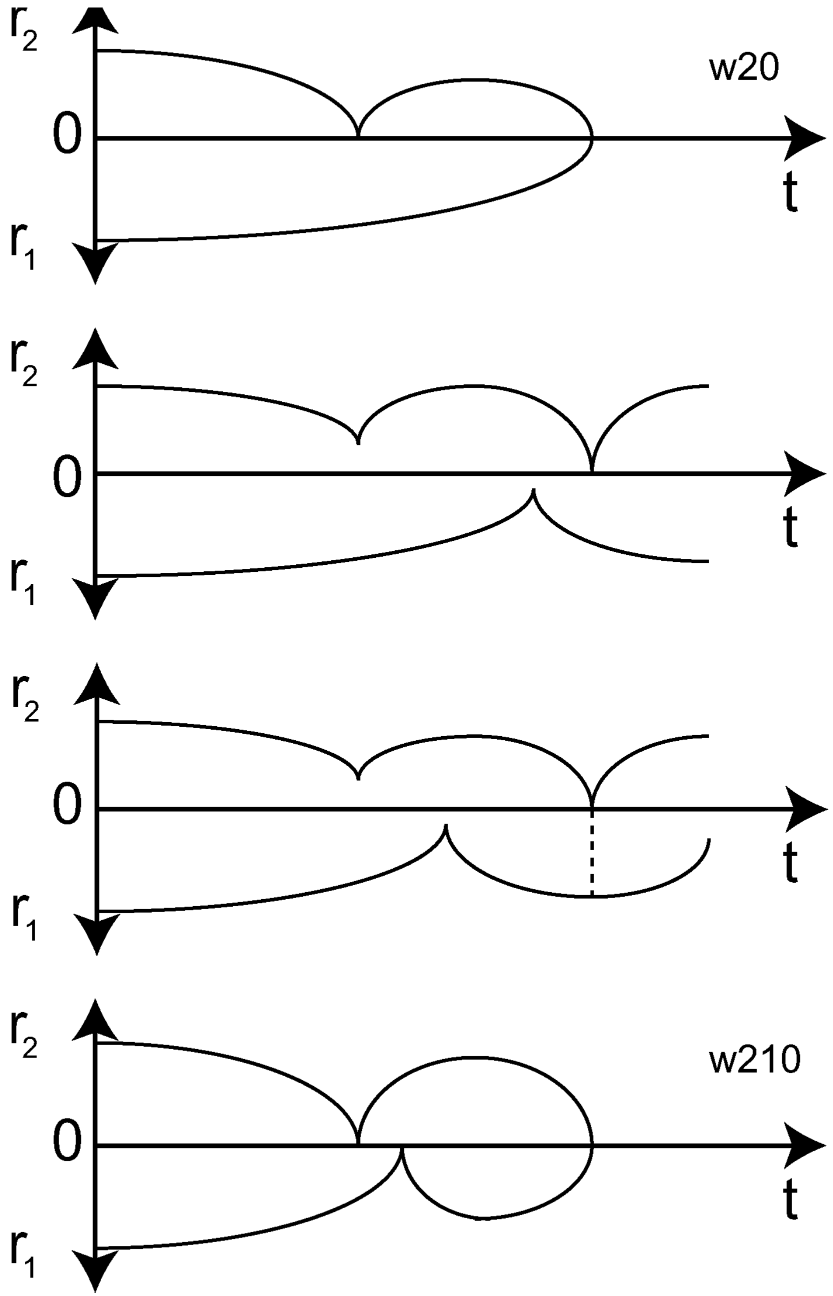
$$r_1 = r\cos(\chi/2),\ r_2 = r\sin(\chi/2)$$



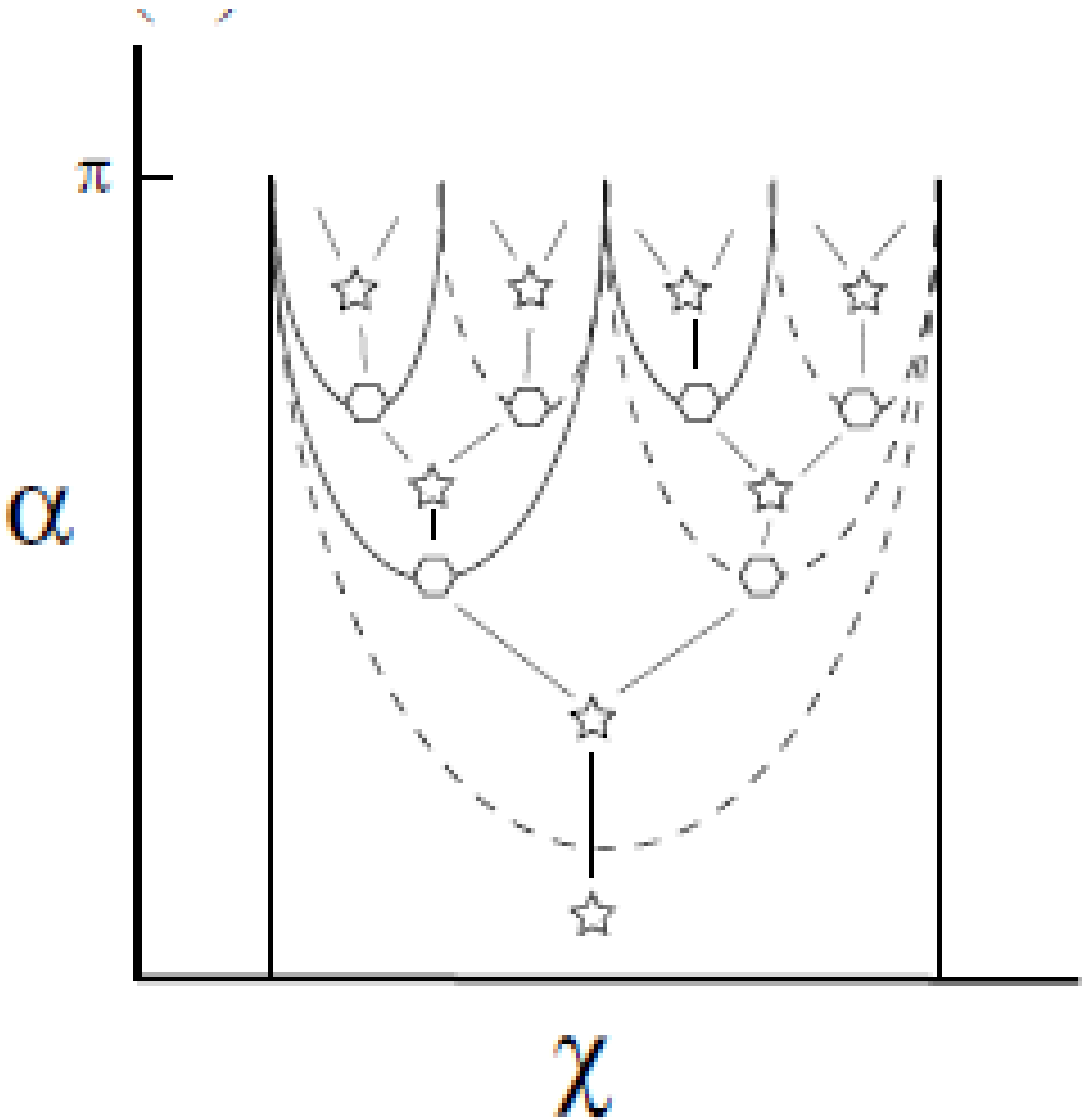
(d)
(c)
(b)
r2
r1
α/π
χ/π



n+1
n
n - 1
α
χ



r_2
w20
0
t
r_1
r_2
0
t
r_1
r_2
0
t
r_1
r_2
w210
0
t
r_1

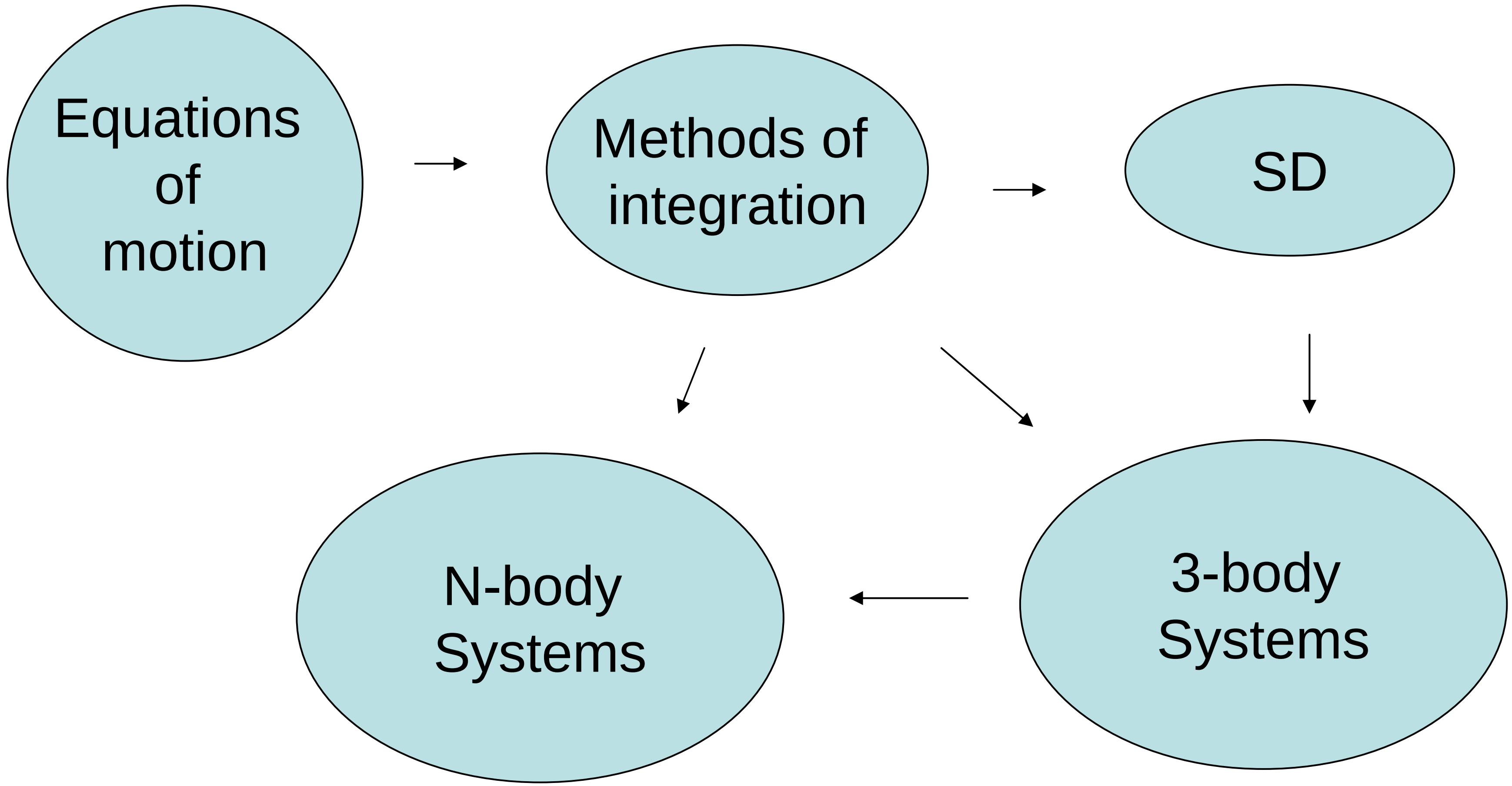

π
α
χ

5. Concluding remark

(1) Symbolic dynamics is a tool with the same level of utility as Poincare maps.

(2) The role of SD is sometimes complementary to that of PM.

(3) SD is not yet fully developed in the three-body problem.


Equations of motion
Methods of integration
SD
N-body Systems
3-body Systems

End

1. Introduction

Japanese-Finnish dish with Russian taste

- Chazy (1922)

  Initial states – Final states of the three-body problem

  Six states: H, HP, HE, HB, OS, B

  -----→ 36 combinations

- 1940s – 1960s, The Russian school, Merman, Sitnikov, Alekseev.
- 1960s – , Numerical studies by Agekyan, Anosova, Orlov, …

-

1.2 Russian taste

- In 1992, J. Yoshida make my attention to the work of Anosova&Zavalov (1989), Anosova(1991)
- This is the start of the Japanese-Finnish dish with Russian taste.

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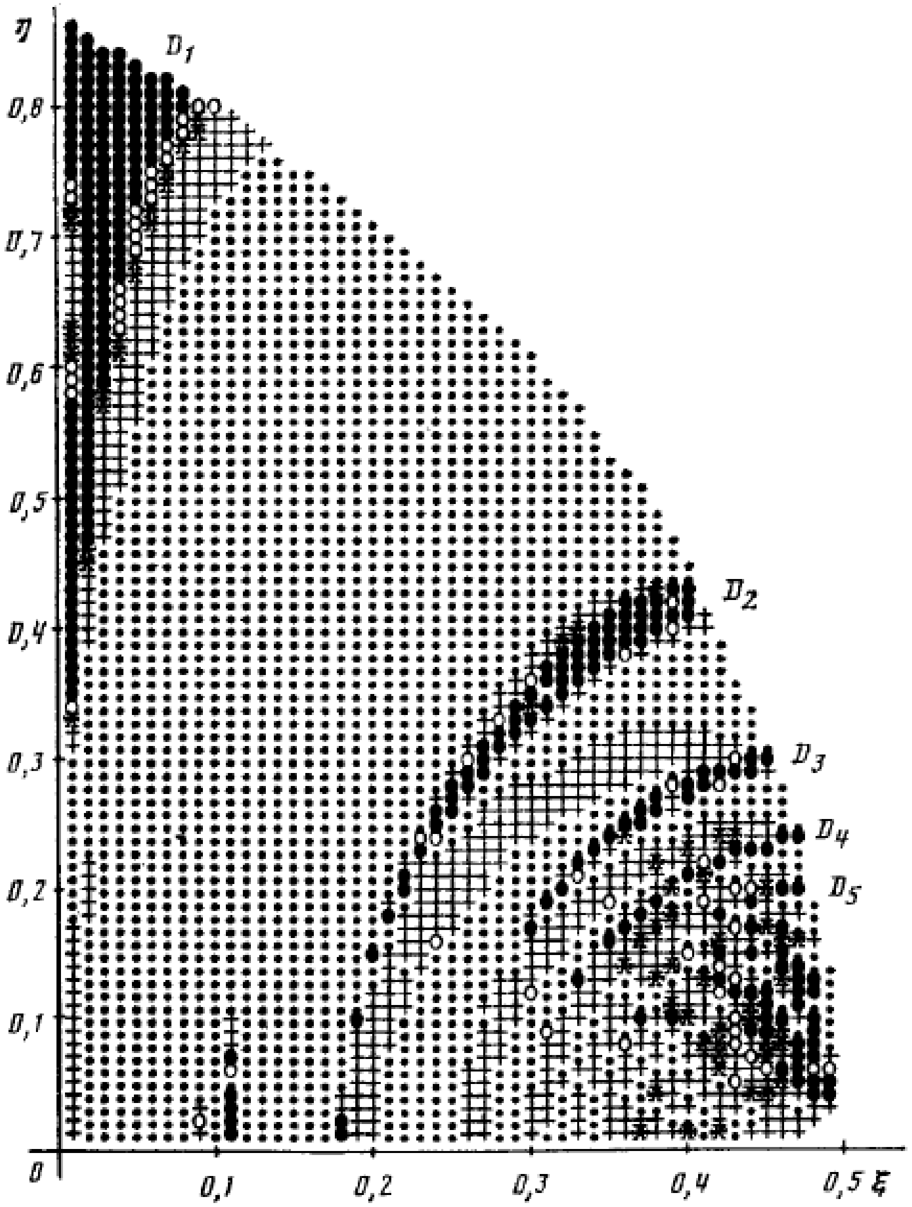

- Escape orbits after the first triple encounter are filled dots.

Impression

- A Fractal
- Where are triple collisions ?