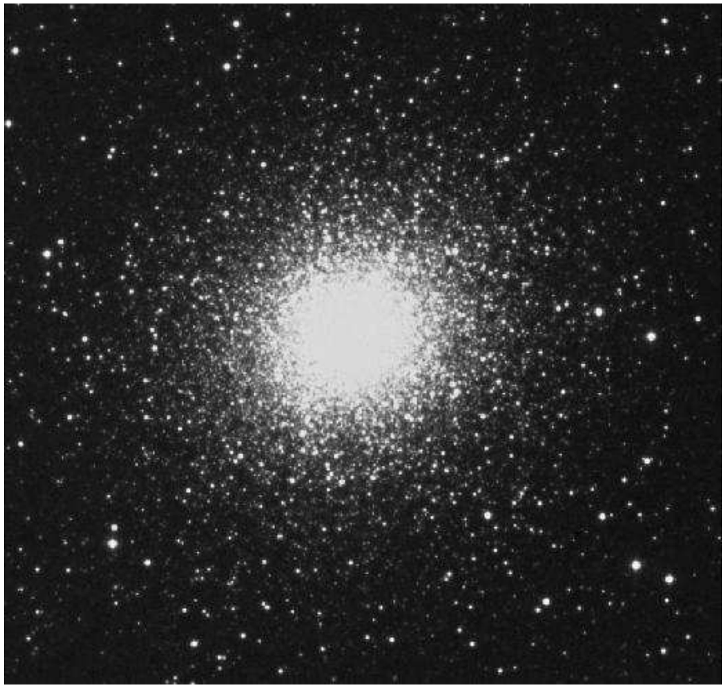
N-Body 2008 "N-body problem: numerical methods and applications"

Turku, Finland, August 10 - 14, 2008

– Dynamical Evolution of Star Clusters –



Lecturer: Holger Baumgardt

Date: 13.08.2008

Schedule:

- Important processes for the dynamical evolution of star clusters
- Core-collapse and mass segregation
- Dissolution of star clusters
- Intermediate-mass black holes in star clusters

Topic I

Important processes for the evolution of star clusters

Important processes for the evolution of clusters

(1) Primordial gas expulsion

(2) Stellar evolution

(3) Two-body relaxation

(4) External tidal fields

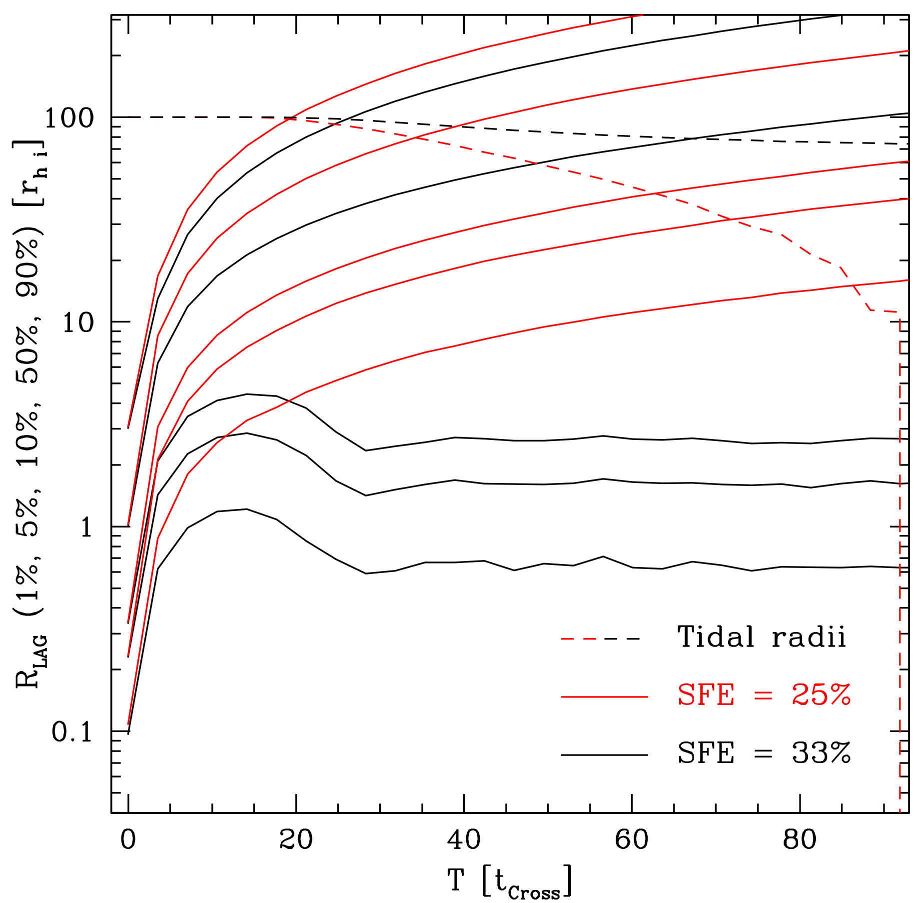
(5) Stellar binaries

Important processes for the evolution of clusters

(1) Gas expulsion:

- Gas expulsion is important since typically only 30% of the mass of a molecular cloud core is transformed into stars.
- The rest of the gas is lost within a few Myrs due to radiation pressure and stellar winds from massive stars and supernova explosions (energy of a single supernova $\approx 10^{51}$ erg is larger than the binding energy of most molecular clouds).
- The resulting sudden loss of cluster potential destroys most clusters. Observations show that about 90% of all clusters are destroyed in this phase (e.g. Lada & Lada 2003).

Impact of gas expulsion on the evolution of star clusters:



Important processes for the evolution of clusters

The impact of gas expulsion on the evolution of a star cluster will mainly depend on three quantities:

- The star formation efficiency SFE:

  low SFE: Clusters are destroyed or lose a large mass fraction
  high SFE: Most clusters will survive

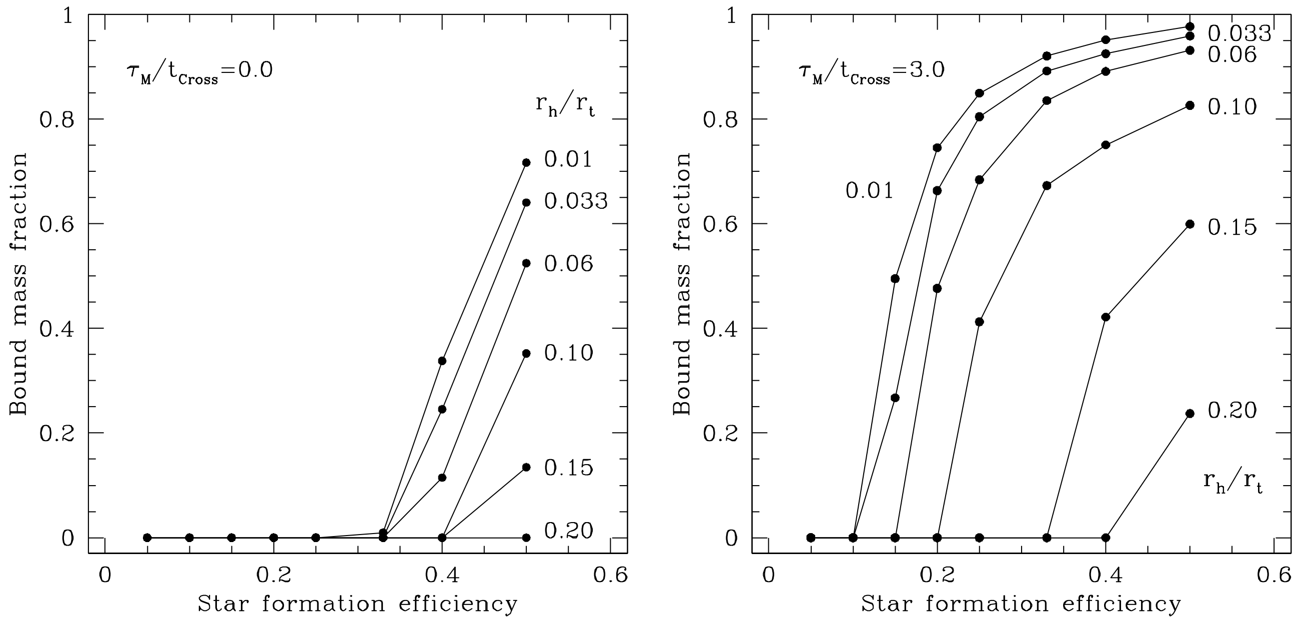
- Ratio of the gas expulsion timescale to the crossing time τ_M/t_{Cross}:

  small τ_M/t_{Cross}: ”Instantaneous gas loss”, leading to large mass loss
  high τ_M/t_{Cross}: Clusters expand adiabatically

- The ratio of the half-mass radius to the tidal radius r_h/r_t:

  high r_h/r_t: Expanding clusters are easily destroyed
  small r_h/r_t: Clusters are nearly isolated and more likely to survive

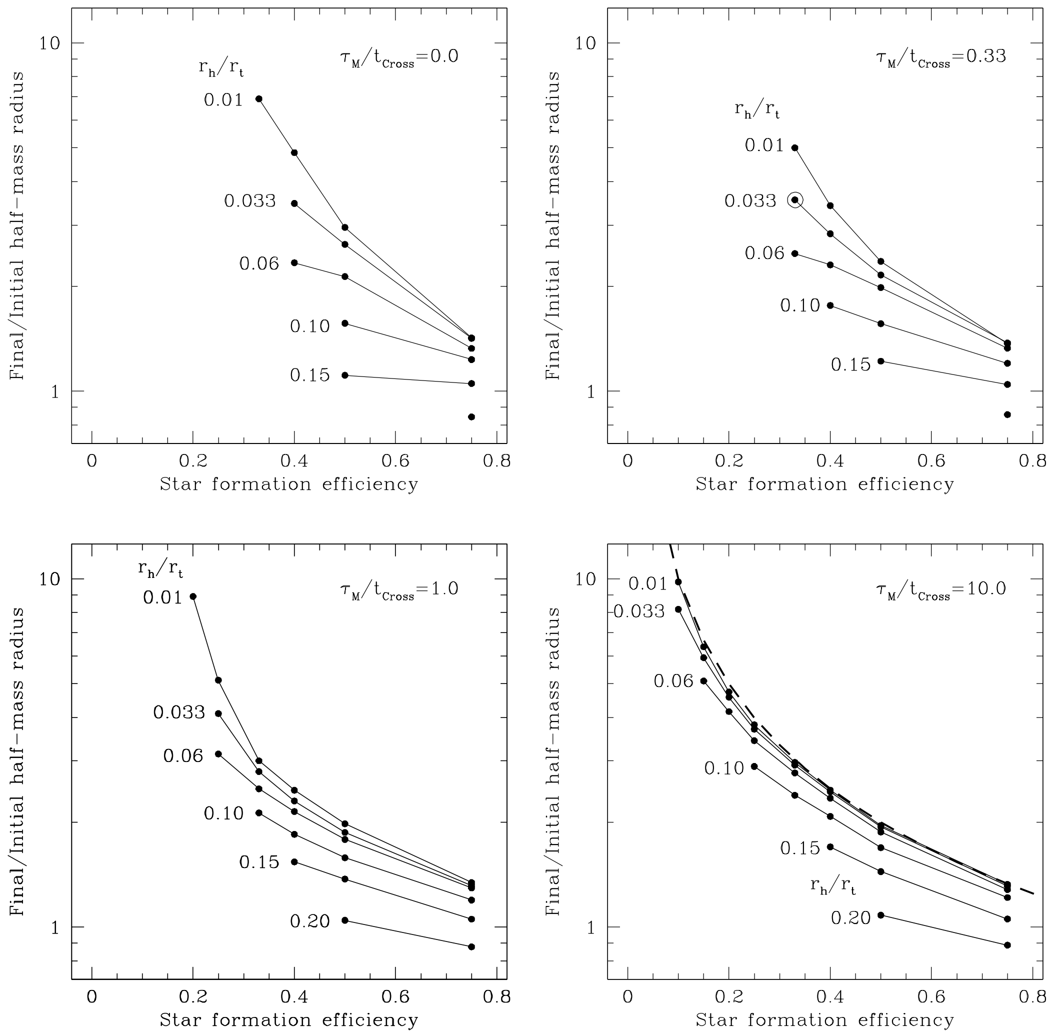
❒ Results of surviving mass fraction as a function of r_h/r_t and SFE:



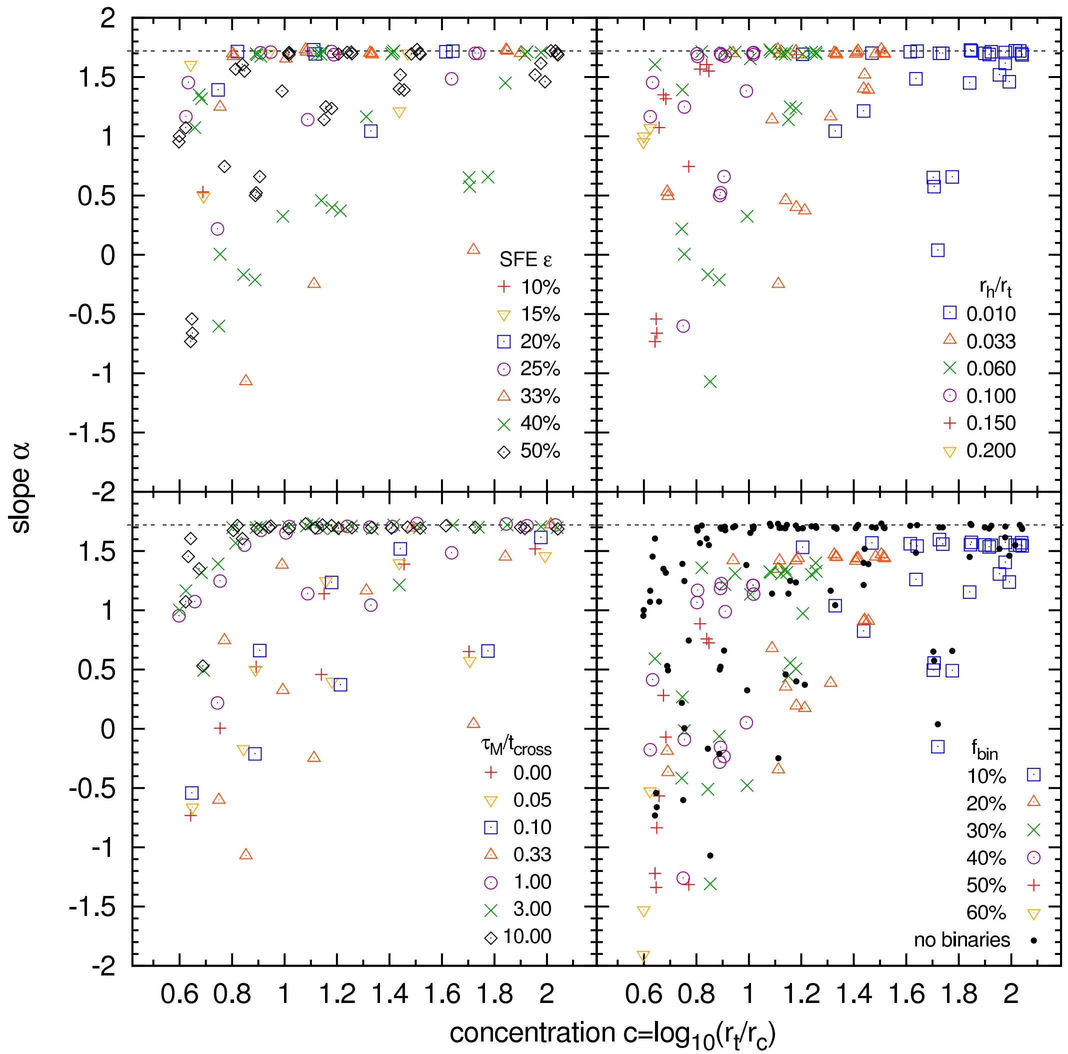
from Baumgardt & Kroupa (2007)

❒ For instantaneous gas removal, clusters need more than 33% SFE to survive. This limit is significantly lowered for adiabatic gas removal.

Gas expulsion leads to a cluster expansion, which explains why embedded clusters have sub-pc radii while open/globular clusters have radii of a few pc.



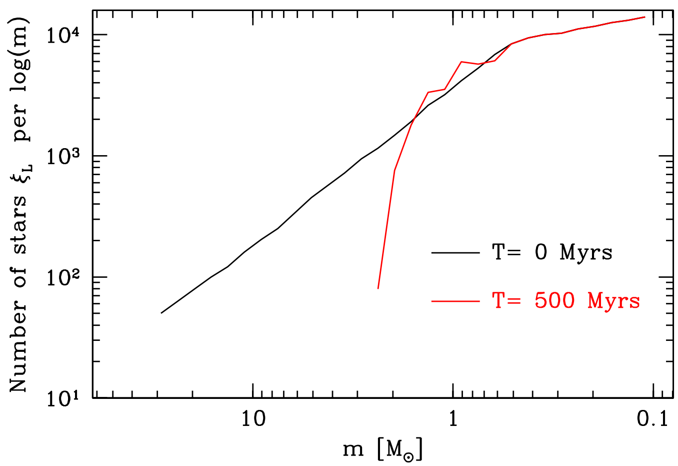
- If clusters form mass segregated, gas expulsion would also lead to a change of the stellar IMF in surviving clusters, as was shown by Marks et al. (2008):

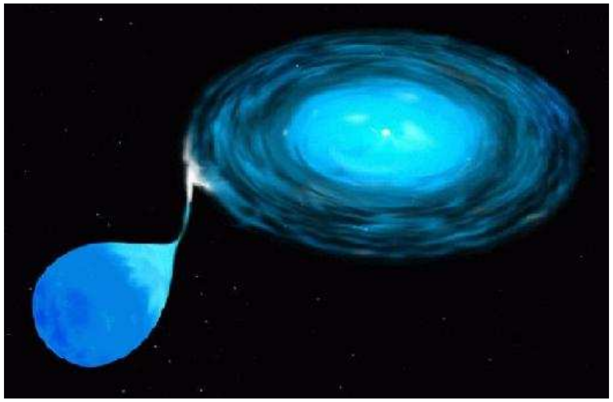


Important processes for the evolution of clusters

(2) Stellar evolution:

- Stellar evolution is responsible for the ejection of the gas out of which the cluster formed
- It changes the stellar mass-function of star clusters and therefore also the total cluster mass.
- It is also important for binaries in star clusters.

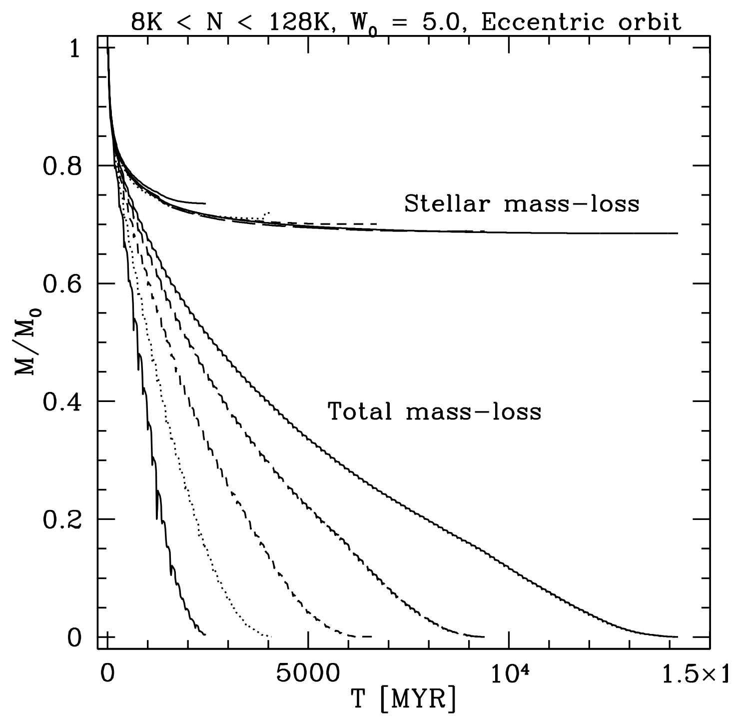




Important processes for the evolution of clusters

(2) Stellar evolution:

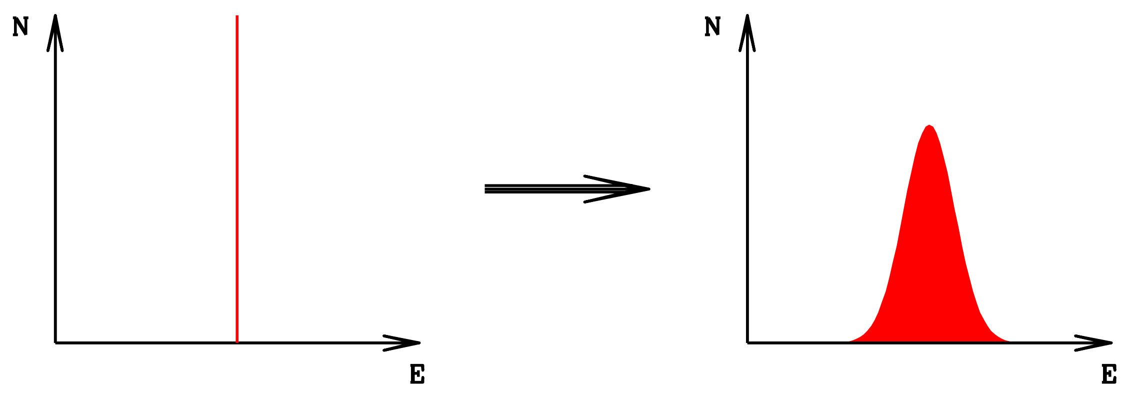
- For a standard IMF (e.g. Kroupa), about 30% of the mass of any cluster is lost due to stellar evolution.
- Most of this mass loss happens within the first Gyr.
- Stellar evolution might be able to destroy clusters with top-heavy IMFs or high r_h/r_t



Important processes for the evolution of clusters

(3) Two-body relaxation:

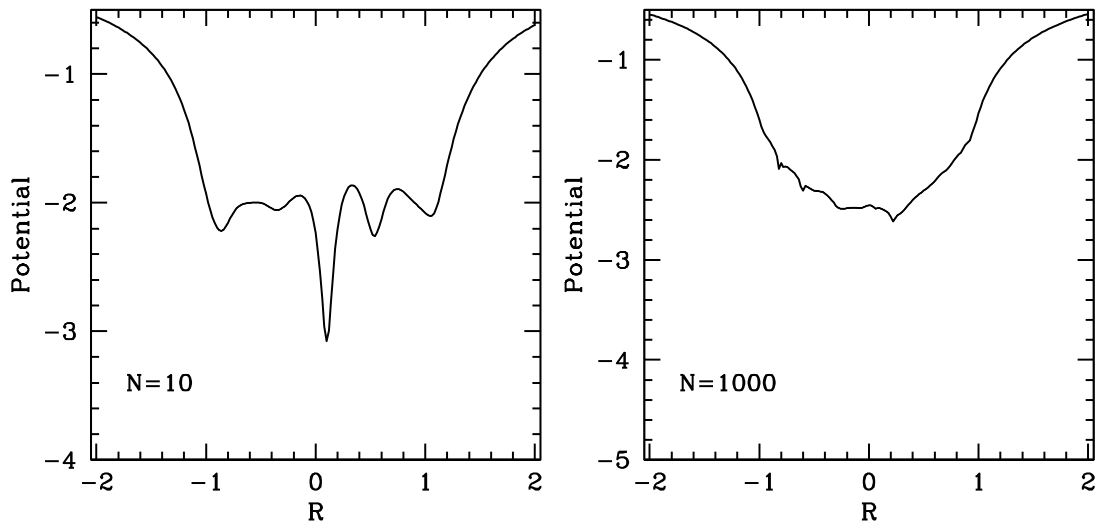
- Relaxation is due to encounters between the cluster stars and leads to a drift in energy space of the cluster stars.



Characteristic timescale: $t_{RH} \sim \frac{N}{\log \gamma N} t_{CR}$

(3) Two-body relaxation:

- The relaxation time increases with the number of cluster stars since the cluster potential gets smoother, so a star has to pass through the system more often in order to change its energy:



- Typical values for the relaxation time are:
  - Open clusters: $10^7 - 10^8$ yrs
  - Globular clusters: $10^9 - 10^{10}$ yrs
  - Galaxies: 10^{16} yrs

Important processes for the evolution of clusters

(3) Effects of two-body relaxation:

- ❒ Mass segregation: Massive stars sink into the centre of a star cluster as a result of dynamical friction and energy equipartition.
- ❒ Core collapse: Together with mass segregation there is also a change in the central density profile
- ❒ Cluster dissolution: From time to time a star gains enough energy to be able to leave the cluster

Important processes for the evolution of clusters

(4) The external galaxy:

In a rotating reference frame which moves with a star cluster around its parent galaxy such that the direction from the galactic centre to the cluster is constant, the equation of motion of a single star is given by:

$$\ddot{\vec{r}} = -\nabla\phi - 2\left(\vec{\Omega}x\dot{\vec{r}}\right) - \vec{\Omega}x\left(\vec{\Omega}x\vec{r}\right)$$

Here the second and third terms on the right hand side are the Coriolis and centrifugal forces. Taking the product of the above equation with $\dot{\vec{r}}$, we see that we can write this as:

$$\frac{dE_J}{dt} = 0$$

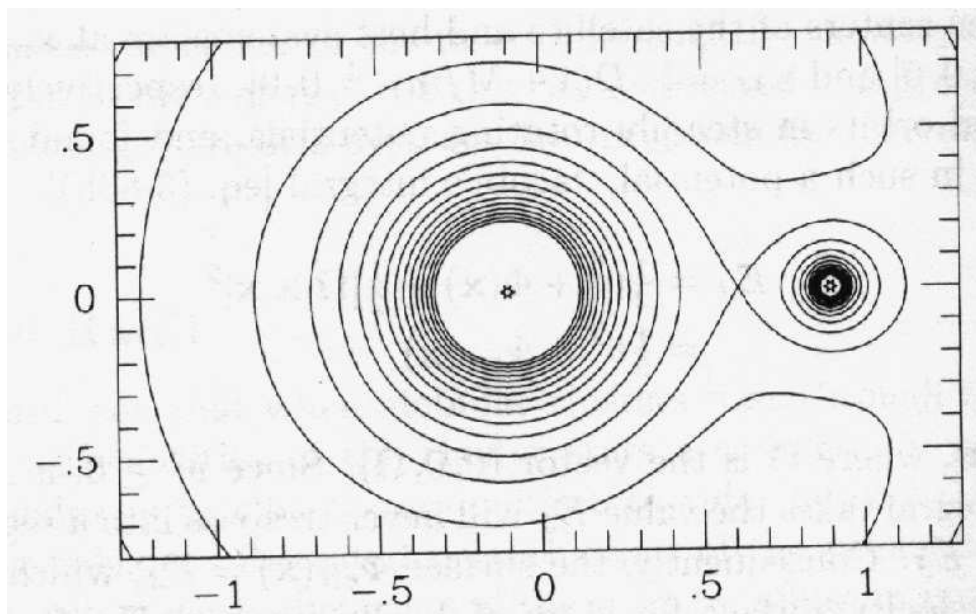
where

$$E_J = \frac{1}{2}\dot{\vec{r}}^2 + \phi + \frac{1}{2}\left|\vec{\Omega}x\vec{r}\right|^2$$

Influence of an external galaxy

E_J is called the Jacobi integral. It plays the same role as the energy for a particle in a non-rotating reference frame, i.e. it is conserved. $\phi_{eff} = \phi + \frac{1}{2}\left|\vec{\Omega}x\vec{r}\right|^2$ is known as the effective potential.

Contour lines of constant ϕ_{eff} for two point masses:



Influence of an external galaxy

The tidal radius is the maximum distance a star can have from the cluster while still being bound to it. It is given by:

$$r_t = \left(\frac{M_C}{3M_G}\right)^{1/3} r_C$$

where M_C is the mass of the cluster, M_G the mass of the galaxy and r_C the distance of the cluster from the galactic center. An external tidal field speeds up cluster dissolution since the energy needed for escape is lowered.

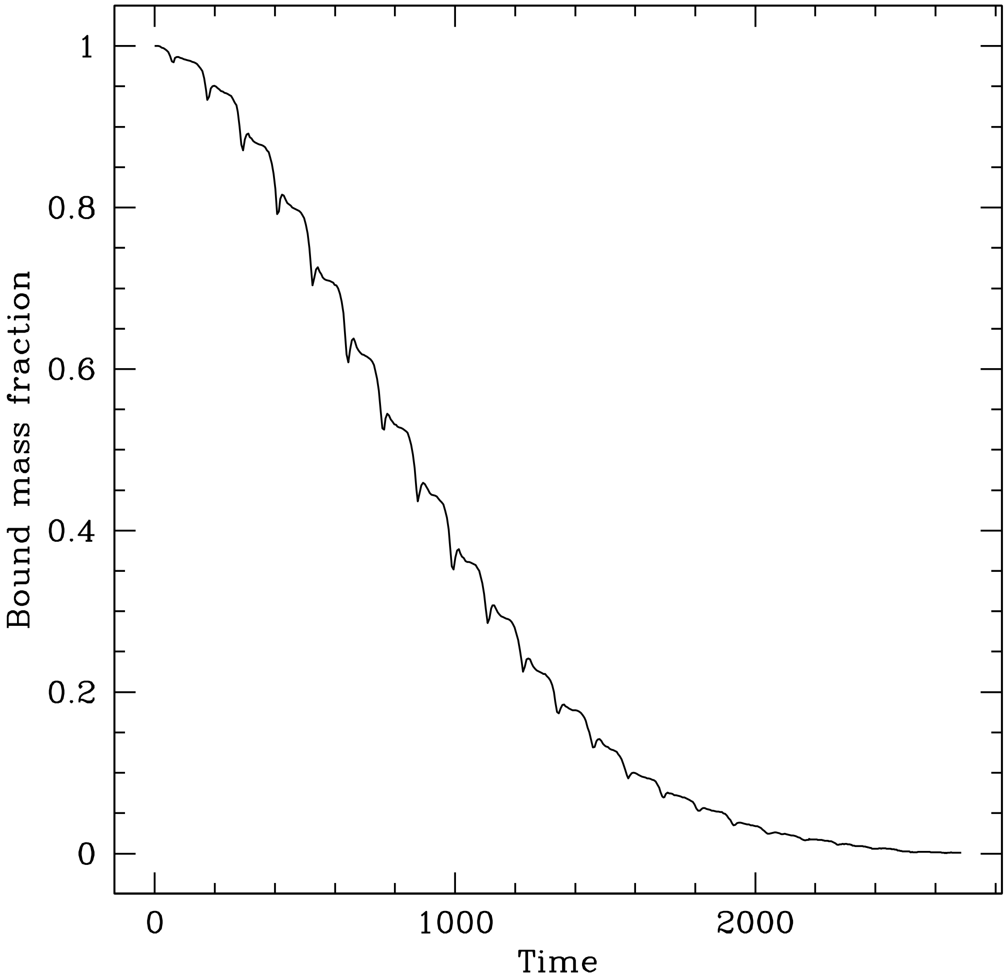
Influence of an external galaxy - Elliptic orbits

- For clusters on elliptic orbits, the tidal force due to the galaxy varies along the orbit and is strongest at pericenter. Ω becomes time-dependent and no conserved quantity similar to the Jacobi energy exists.

- Since the strength of the external tidal field and the size of the tidal radius is varying along the cluster orbit, stars can be removed from the cluster by the varying tidal field and the energy of the cluster is increased **(disruptive tidal shocks).**

- Similarly, passages of star clusters through galactic discs speed up cluster destruction by **compressive tidal shocks.**

Example of the dissolution of a cluster on an elliptic orbit. Each time the cluster passes near the pericenter, stars are stripped away from it:



Important processes for the evolution of clusters

(5) Binaries:

- Binaries are important due to the energy stored in them:

$$E_{Bin} = -G\,\frac{m_1\,m_2}{2\,a} \qquad (1)$$

- Assume $m_1 = m_2 = 1M_\odot$ and $a = 1$ AU.

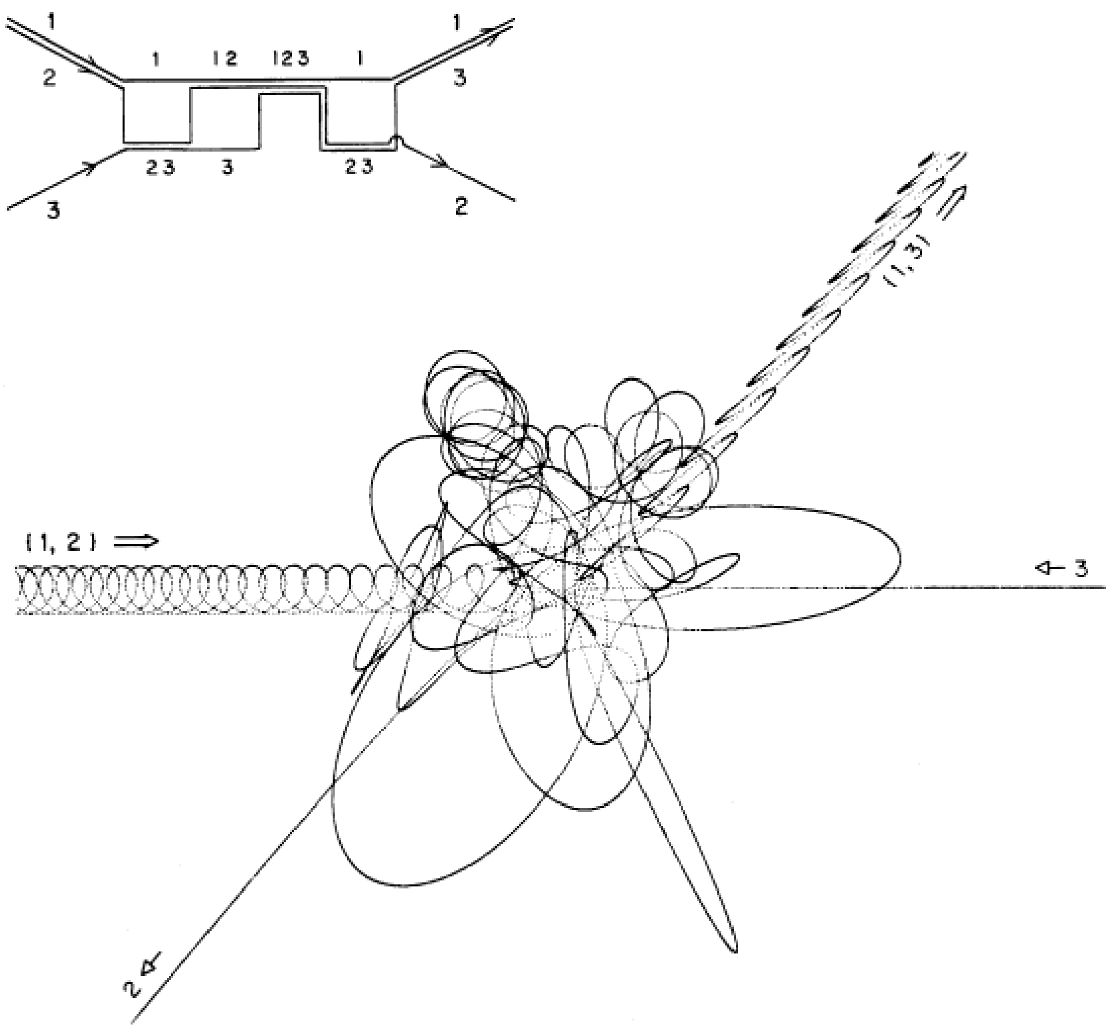
  Energy of this binary: $|E_{Bin}| \approx 10^3\;\frac{M_\odot km^2}{sec^2}$

  Energy of a star in a cluster: $E = \frac{1}{2}mv^2 \approx 2\;\frac{M_\odot km^2}{sec^2}$

  Total energy of a $10^3 M_\odot$ star cluster: $E_{Cl} \approx 0.4\frac{GM_C^2}{r_h} = 10^3\;\frac{M_\odot km^2}{sec^2}$

$\Rightarrow$ Binaries can be very important for the dynamics of clusters

Example of the interaction of a binary with another cluster star



Topic II

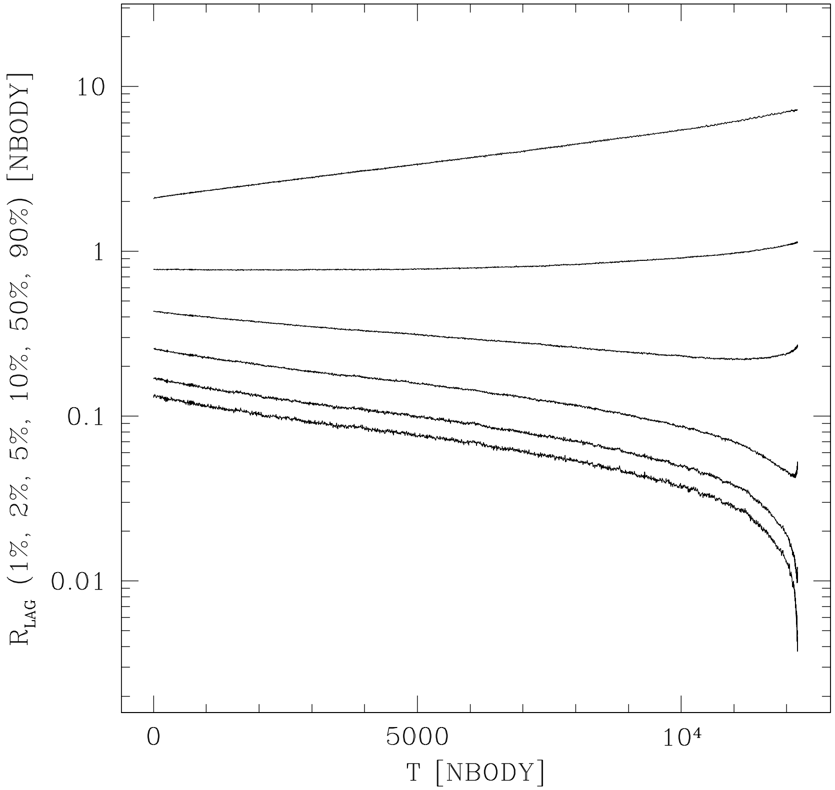
Core collapse and mass segregation

Core collapse of star clusters

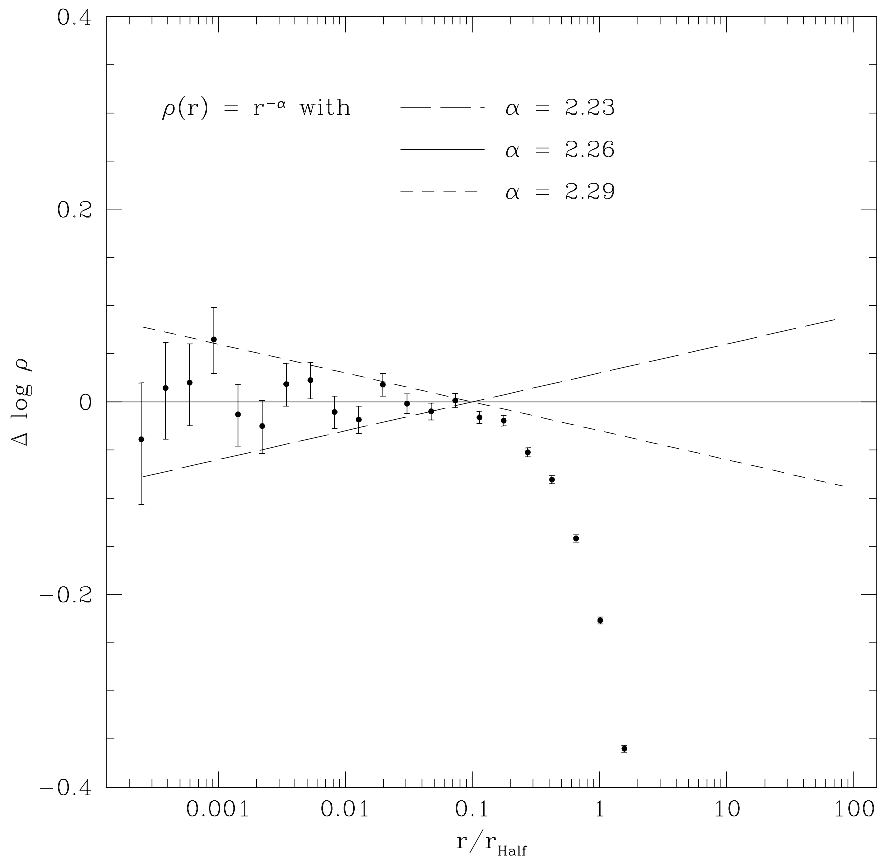
Due to stellar encounters, energy is transferred from the core to the cluster halo. As a result, the cluster core shrinks and stellar encounters become even more frequent.

The end-point is that a power-law density profile $\rho(r) \sim r^{-\alpha}$ with exponent $\alpha = 2.23$ is established in the centre.

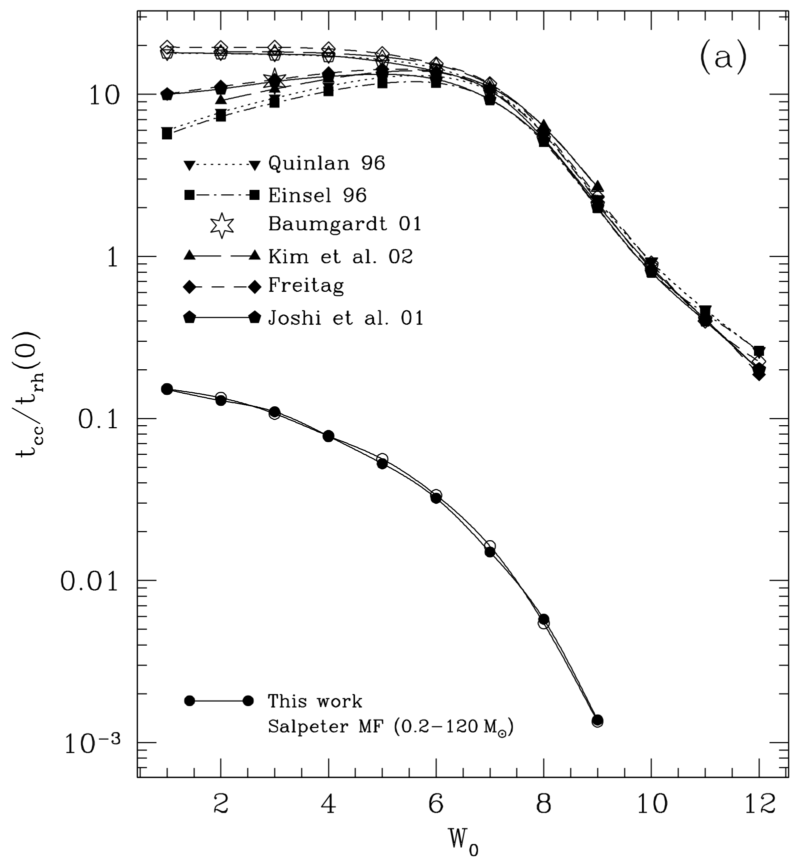
It takes several initial half-mass relaxation times to reach core-collapse.



Core collapse of star clusters



(from Baumgardt et al. 2003)

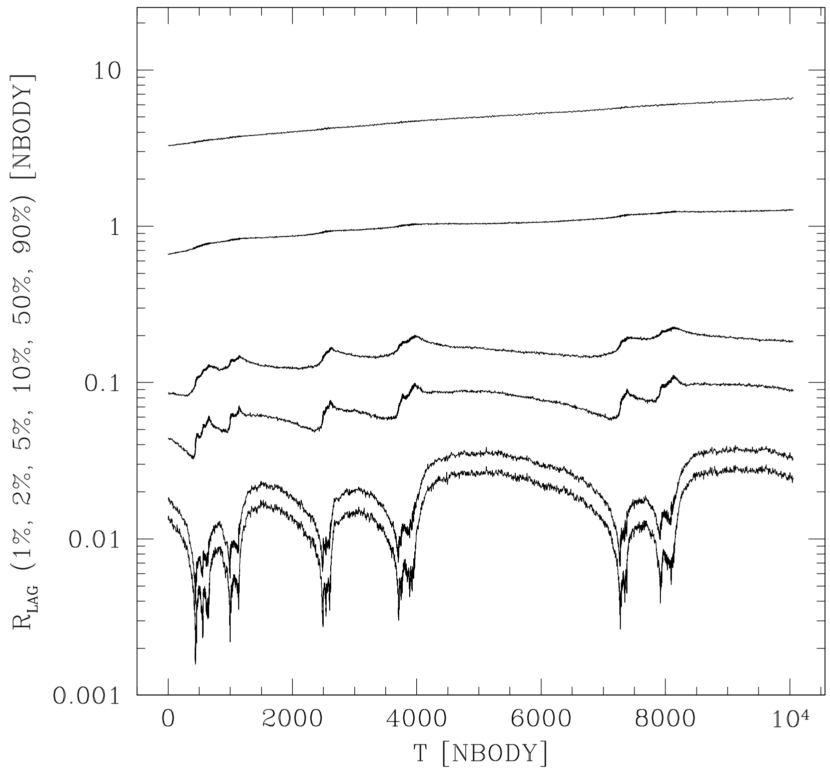


(from Gürkan et al. 2004)

Core collapse of star clusters

Core collapse is halted once binaries form in the cluster centre as a result of close three and four-body interactions. These release energy through encounters with field stars.

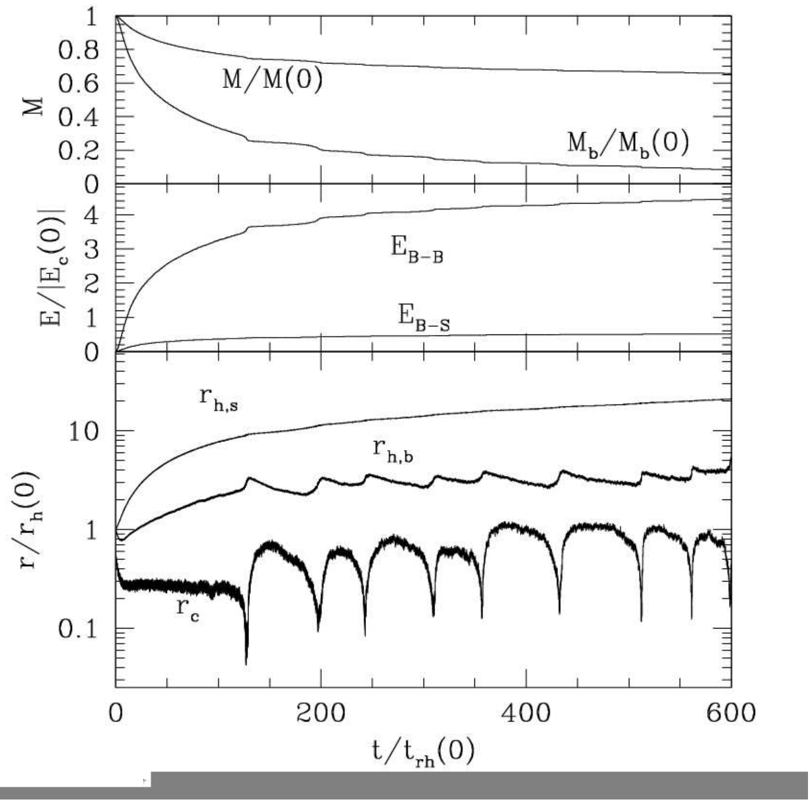
After core collapse, massive clusters go through a series of **core oscillations**: Each time active binaries are ejected from the cluster centre, the core collapses again.



Core collapse of star clusters

If primordial binaries are taken into account, it takes significantly longer until a high-density core has formed since primordial binaries stabilise the cluster against collapse.

Realistic clusters, which also undergo dissolution, might not be able to reach core collapse since they dissolve before all primordial binaries are destroyed.

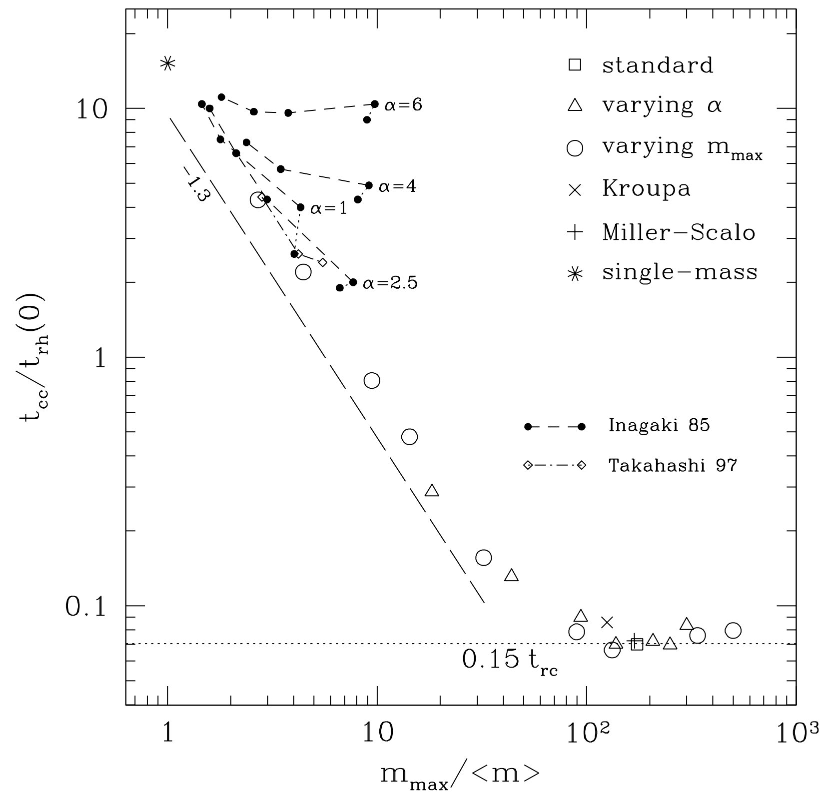


(from Fregeau et al. 2003)

Mass segregation in star clusters

For clusters with an initial mass spectrum, the time to core collapse is decreased since massive stars quickly spiral into the cluster center.

Gürkan et al. (2004) found that the core-collapse time varies roughly like $t_{cc} \sim (<m>/m_{max})^{1.3}\, t_{rh}$ over a wide range of $<m>/m_{max}$.

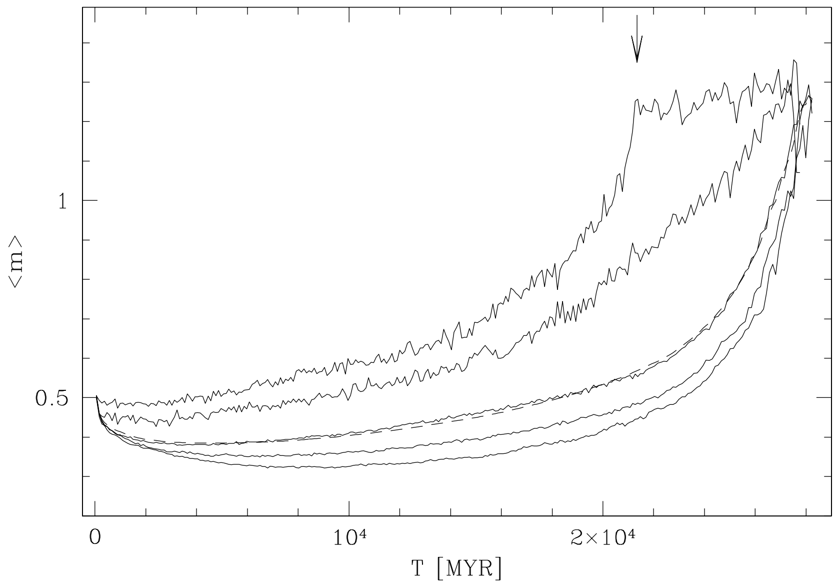


(from Gürkan et al. 2004)

Mass segregation in star clusters

In open clusters, mass segregation leads to the accumulation of massive main sequence stars in the center.

In globular clusters, the most massive stars are compact remnants. Any globular cluster which went into core collapse should therefore contain a compact core of these remnants in its centre.

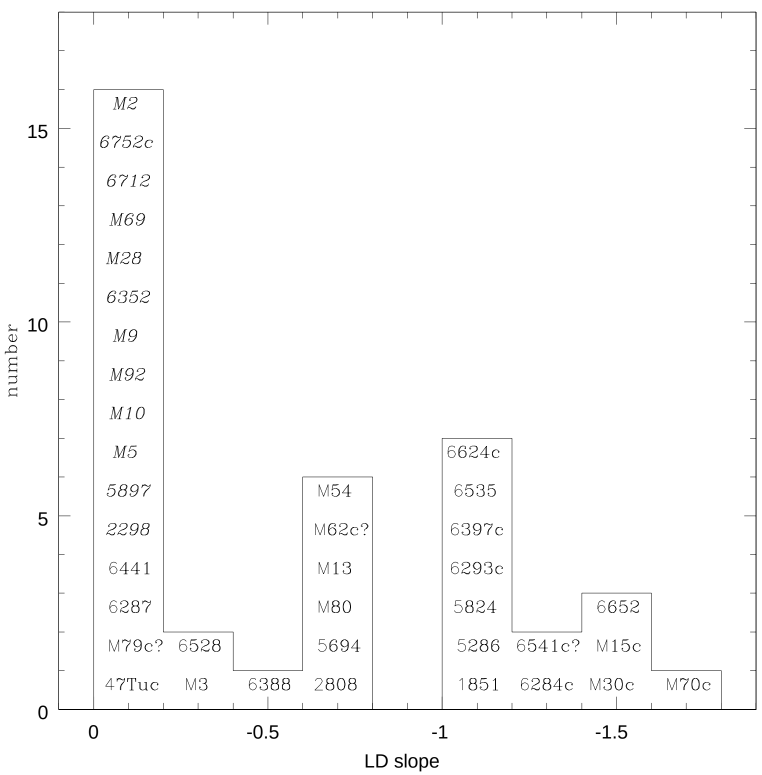


(from Baumgardt & Makino 2003)

Observational evidence for core collapse

In a recent study, Noyola & Gebhardt (2006) investigated central surface brightness profiles of globular clusters.

According to their analysis, the galactic globular clusters cover a full range of density profiles. About 1/3 have central density slopes steeper than $\alpha = 1$, which could indicate that they are in core collapse.



Topic III

Dissolution of star clusters

Cluster dissolution is caused by two-body relaxation:

Through stellar encounters, any star cluster tries to establish an equilibrium density and velocity profile. In one dimension, the equilibrium velocity profile is a Gaussian, so the 3D velocity profile should follow a Maxwellian velocity distribution:

$$N(E) \sim v^2 exp(-mv^2/2kT)$$

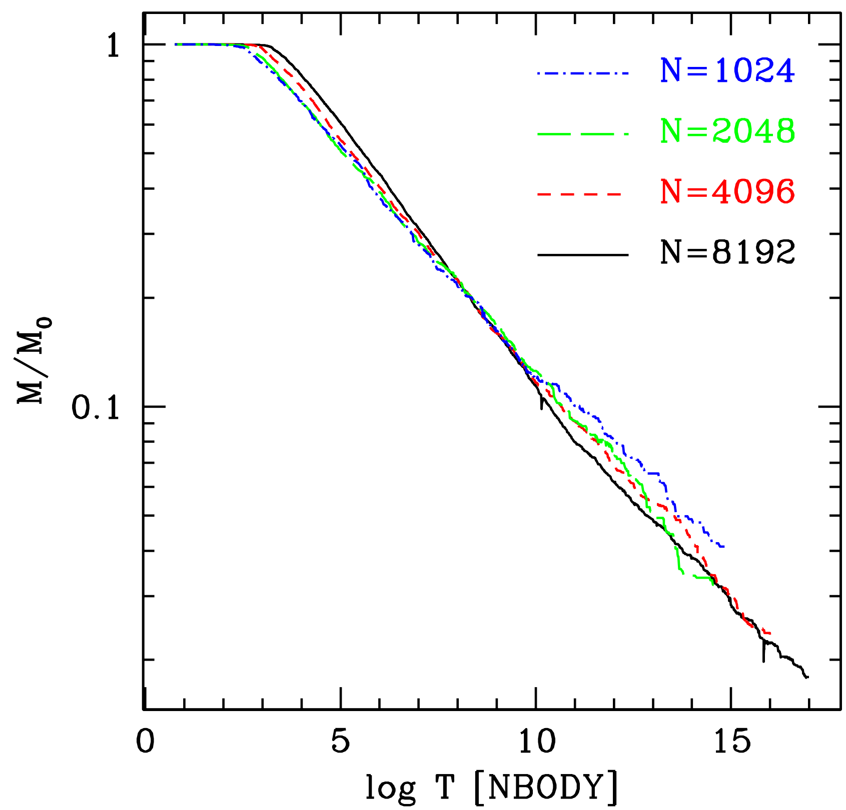
Since a Maxwellian velocity distribution contains stars with positive energies, a certain fraction of stars gains positive energy each relaxation time and leaves the cluster. A **(naive)** prediction is therefore that the lifetimes of star clusters should be multiples of their relaxation times.

Relaxation and cluster dissolution:

Dissolution of *isolated* clusters:

Stars which gain energy move to the outer cluster parts, where they rarely encounter other stars. Relaxation therefore does not expel stars from a cluster but causes a strong cluster expansion instead.

As a result, isolated clusters **never** dissolve ! (Henon 1960)

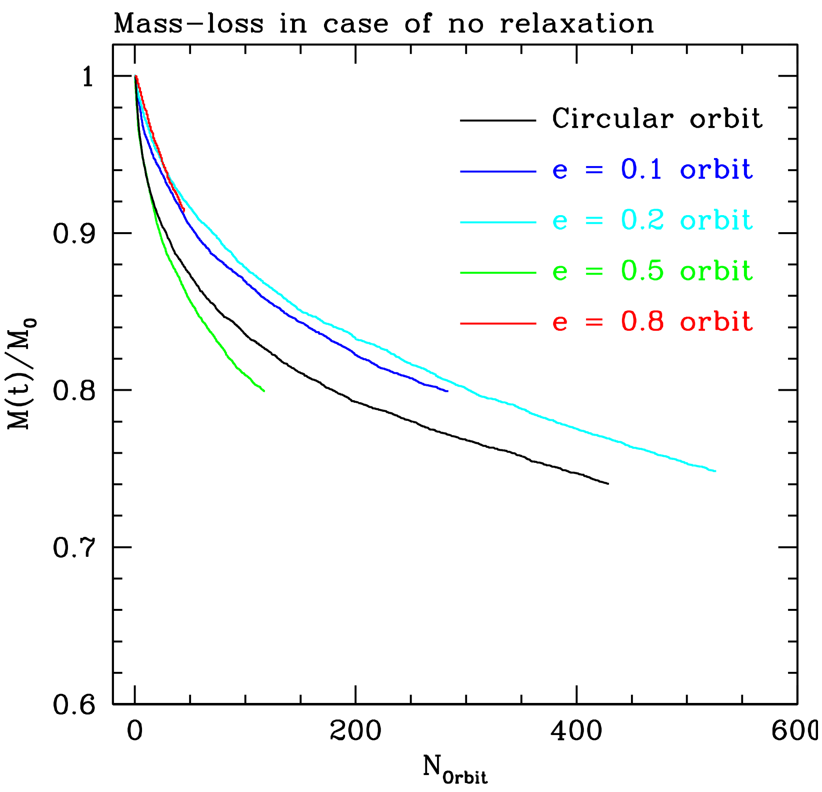


Dissolution in tidal fields

Without relaxation, tidal fields alone are also inefficient in destroying clusters:

Outer envelope gets stripped away but bound core remains.

⇒ No dissolution !

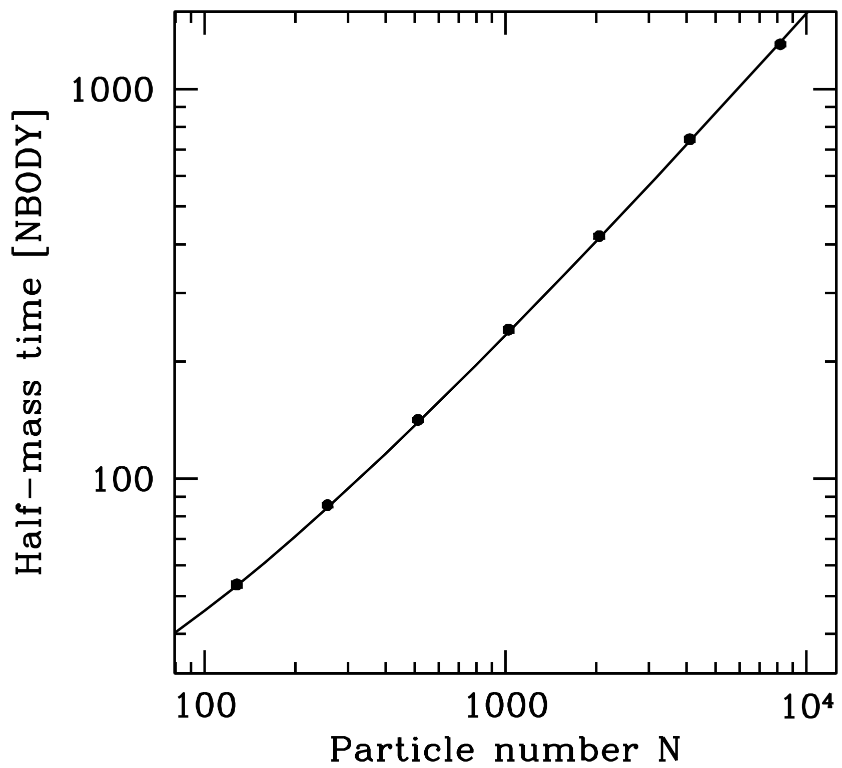


Relaxation and cluster dissolution:

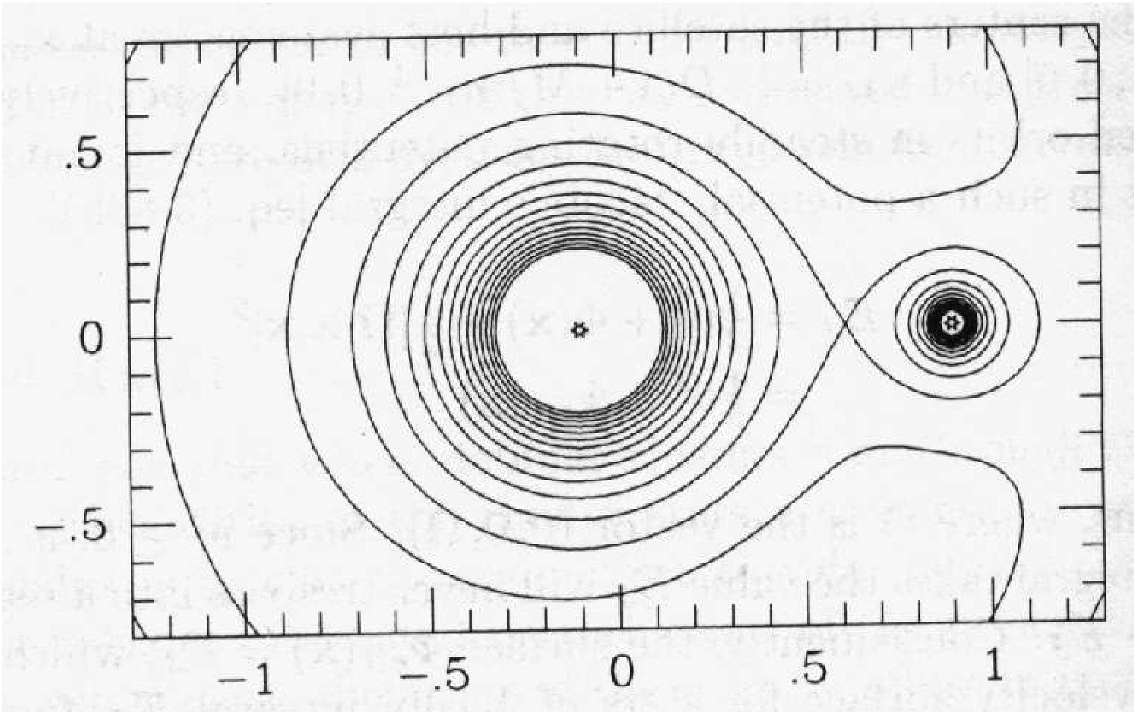
Dissolution of *tidally limited* clusters:

Tidal boundary prevents cluster expansion and one obtains a scaling of the lifetime with the relaxation time.

The external tidal field and relaxation have to work together to dissolve clusters.



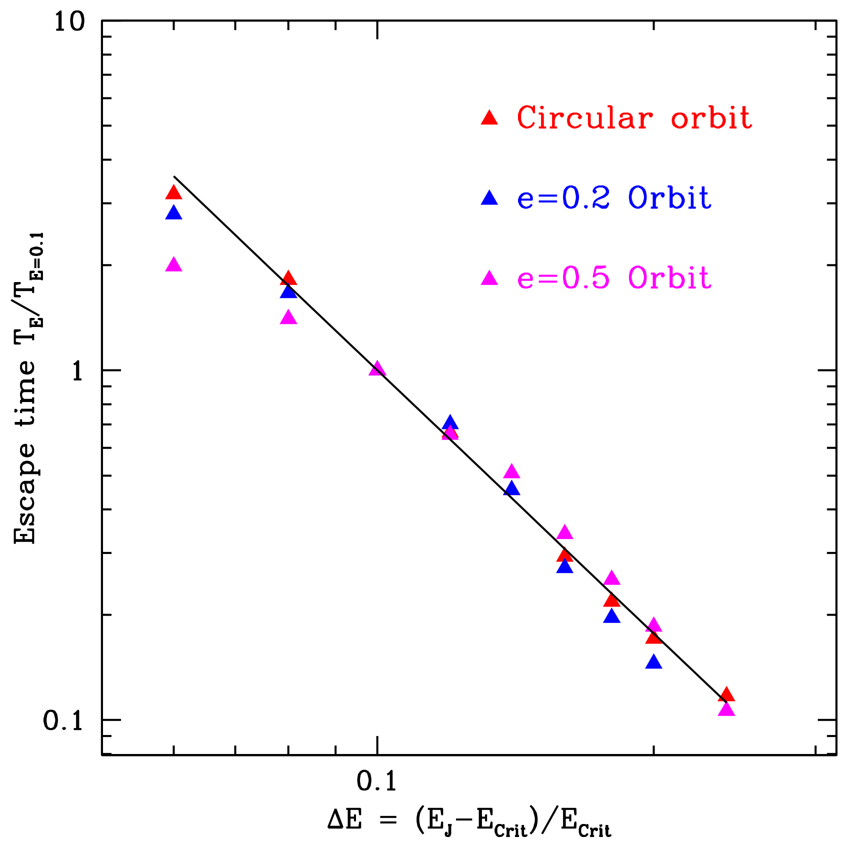
In a tidal field, stars with energies only slightly above the critical one can only escape through small apertures around the Lagrangian points:



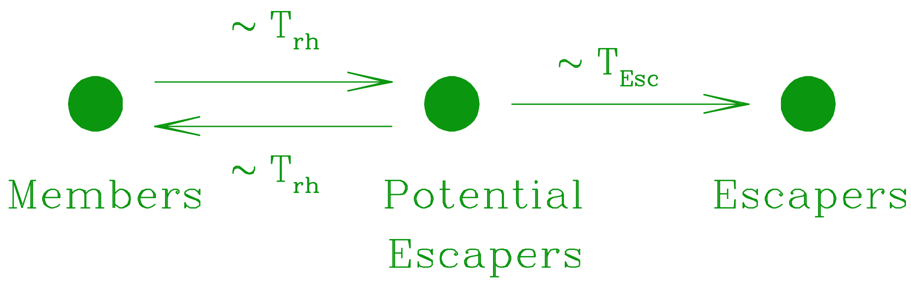
Dissolution in tidal fields

Simulations by Fukushige & Heggie (2000) have shown that the time needed for escape drops as $T_{Esc} \sim 1/(E_J - E_{Crit})^2$.

This causes a complication of the escape process since the escape time can become as large as the relaxation time !

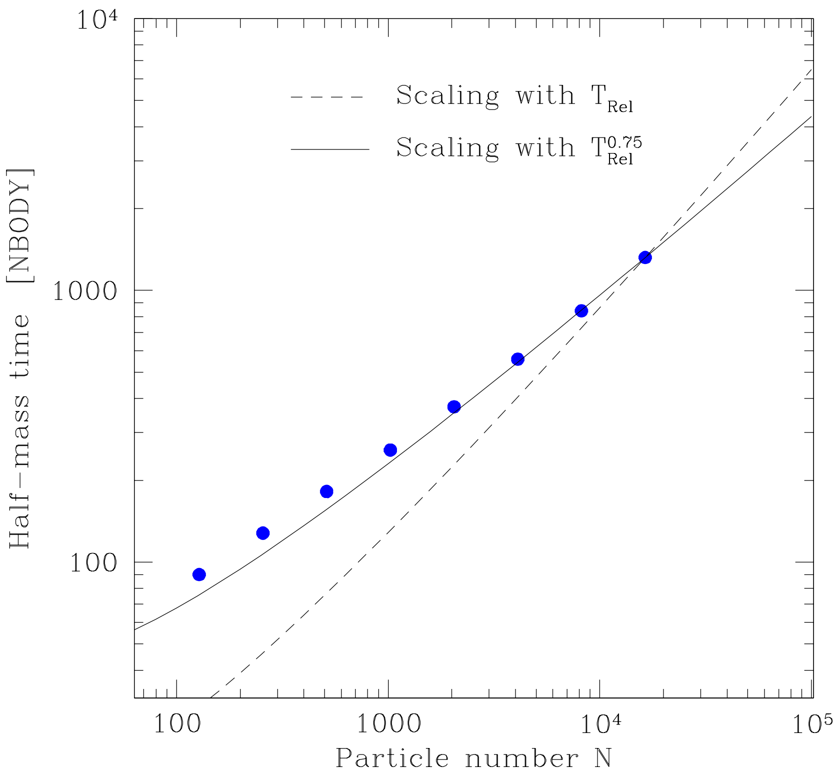


As a result, potential escapers can still be retained in the cluster:



This in turn leads to a different scaling of the lifetimes for star clusters in external tidal fields:

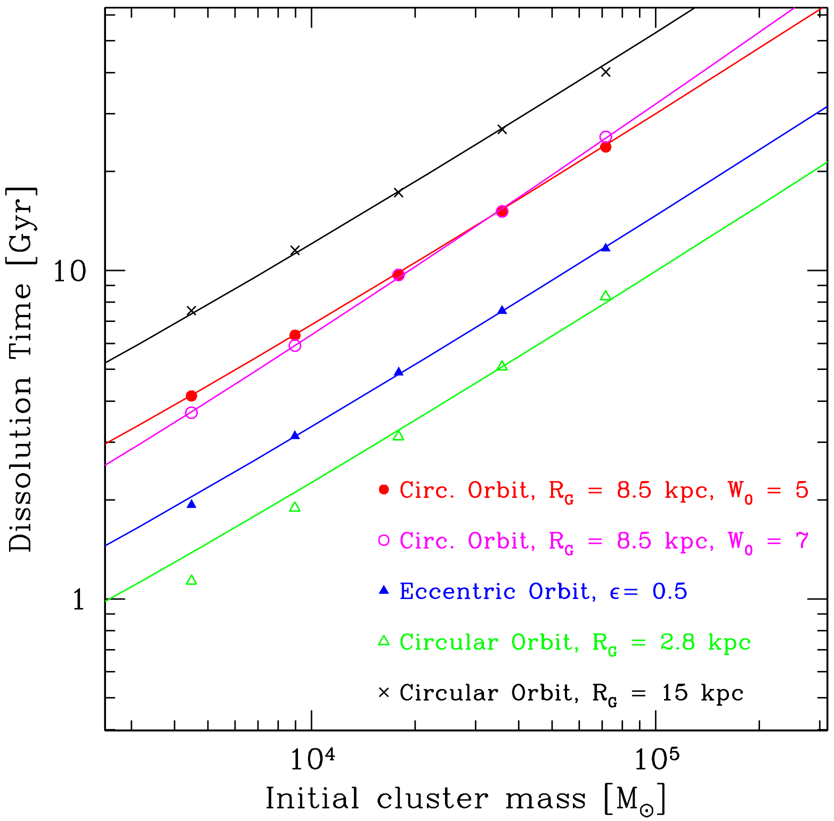
The plot to the right shows half-mass times for single-mass clusters in circular orbits.



Dissolution in tidal fields:

Realistic clusters show a similar scaling than single-mass ones:

Lifetimes scale with $T_{RH}^{3/4}$, nearly independent of galactocentric distance, density profile and orbital type.



(from Baumgardt & Makino 2003)

Dissolution in tidal fields:

A scaling of lifetime flatter than with the relaxation time was also derived by Boutloukos & Lamers (2003) and Lamers et al. (2005) from fits to the observed luminosity distribution of open clusters:

Table 1. The parameters of the disruption time: $t_{\rm dis} = t_4 \times (M_{\rm cl}/10^4)^\gamma$

Galaxy	$r_{\rm Gal}$ kpc	Nr clusters	Age range log (yrs)	Mass range log $M_\odot$	log t_4 log (yrs)[a]	γ	log $\rho_{\rm amb}$ $M_\odot$ pc^{-3}	Remark
M51	0.8 - 3.1	1152	$6.3 - 9.0$	$2.6 - 5.6$	7.85 ± 0.22 (7.6)	0.57 ± 0.10	-0.43 ± 0.30	2
M33	0.8 - 5.0	147	$6.5 - 10.0$	$3.6 - 5.6$	8.80 ± 0.20 (8.1)	0.60 ± 0.15	-0.66 ± 0.30	3
Galaxy ($d \leq$ 1kpc)	7.5 - 9.5	184	$7.2 - 9.5$	—	8.75 ± 0.20 (9.0)	0.60 ± 0.12	-1.00 ± 0.04	1,4
SMC	0 - 4	314	$7.6 - 10.0$	—	9.90 ± 0.20 (9.9)	0.61 ± 0.08	-2.10 ± 0.30	1
					Mean	0.60 ± 0.02		

The $T_{Dis} \sim T_{RH}^{0.75}$ scaling also complicates scaling of results from low-N clusters to high-N ones since the timescales of different processes show very different N-dependence:

- Stellar evolution: $T_{SEV} \sim N^0$
- Mass segregation & Core-collapse: $T_{CC} \sim N/log(\gamma N)$
- Cluster dissolution: $T_{Dis} \sim (N/log(\gamma N))^{0.75}$
- Influence of binaries: Various N-dependencies

$\Longrightarrow$ For simulations including all effects, one needs realistic N.

Dissolution in tidal fields:

Cluster lifetimes can be fitted by the following formula:

$$\frac{T_{Diss}}{[Myr]} = \beta \left(\frac{N}{ln(\gamma N)} \right)^x \frac{R_G}{[kpc]} \left(\frac{V_G}{220\ km/sec} \right)^{-1} (1-\varepsilon)$$

where N is the number of cluster stars, V_G the rotational velocity of the external galaxy, R_G the distance of the cluster from the center.

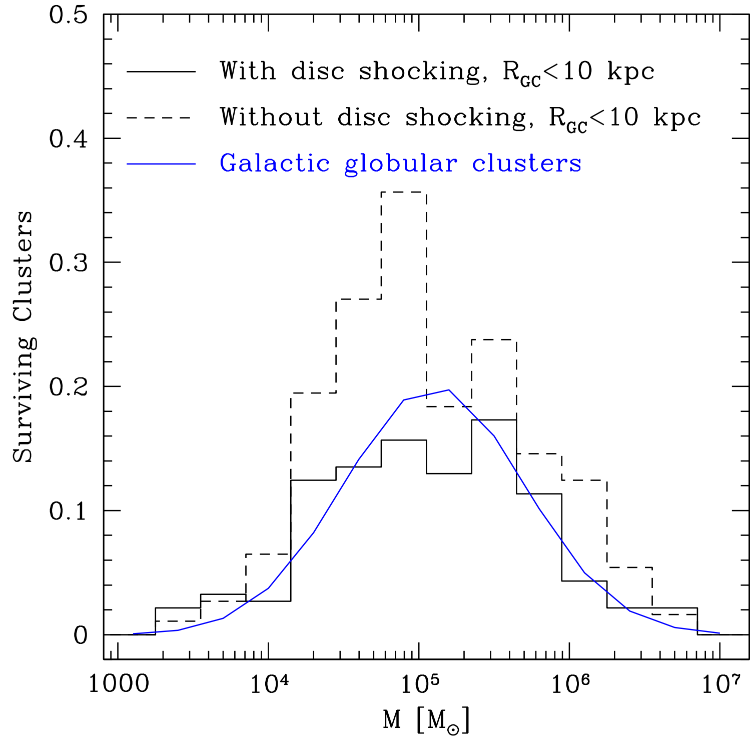
The constants are given by: $\beta = 1.91, x = 0.75, \gamma = 0.02$.

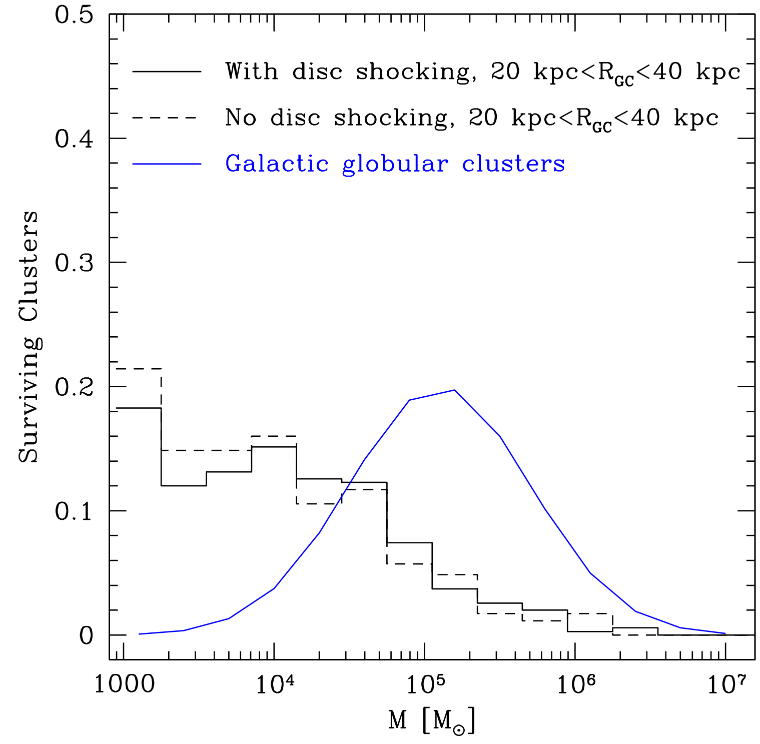
N-body simulations also show that clusters lose mass nearly linearly with time:

$$M(t) = (M_0 - M_{SEV}(t)) \cdot \left(1 - \frac{t}{T_{Diss}}\right)$$

Evolution of globular cluster systems

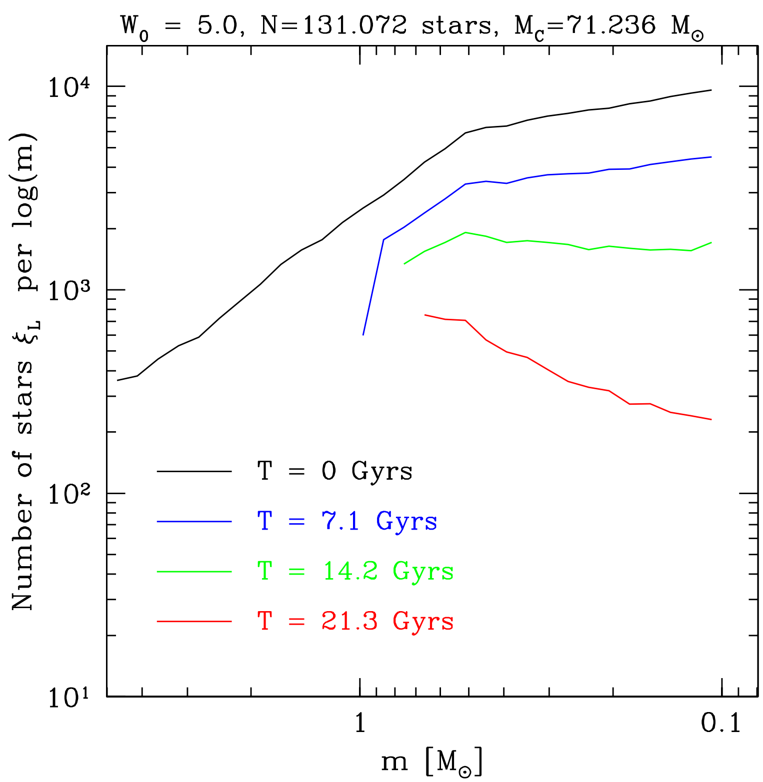
Due to the R_G-dependence of cluster lifetimes, it is not possible to turn an initial power-law mass function into the obsered log-normal mass function in the outer parts of galaxies by dynamical processes alone:



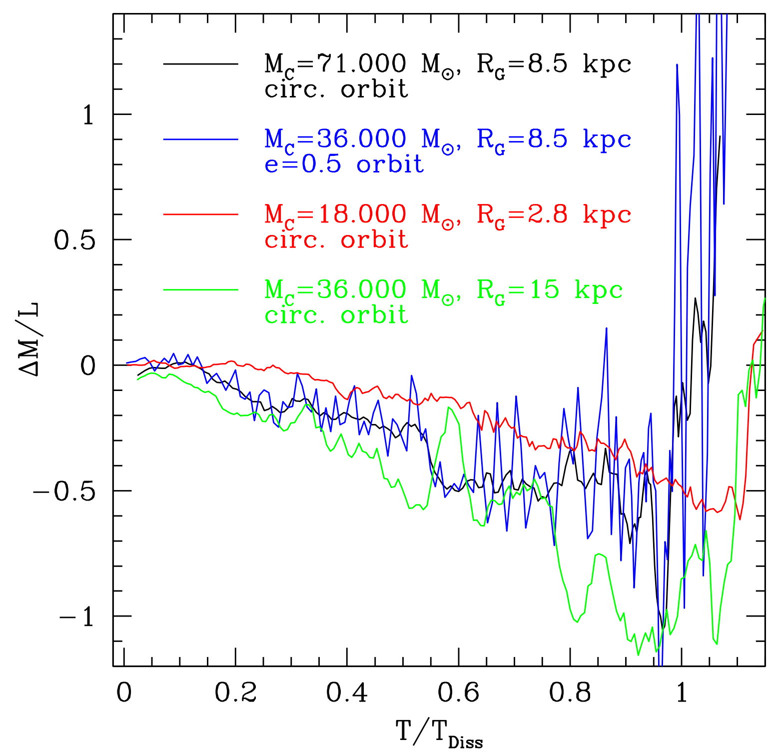


Mass segregation and cluster dissolution

Due to mass segregation, clusters lose mainly low-mass stars:



This changes the mass-to-light ratio of the clusters:

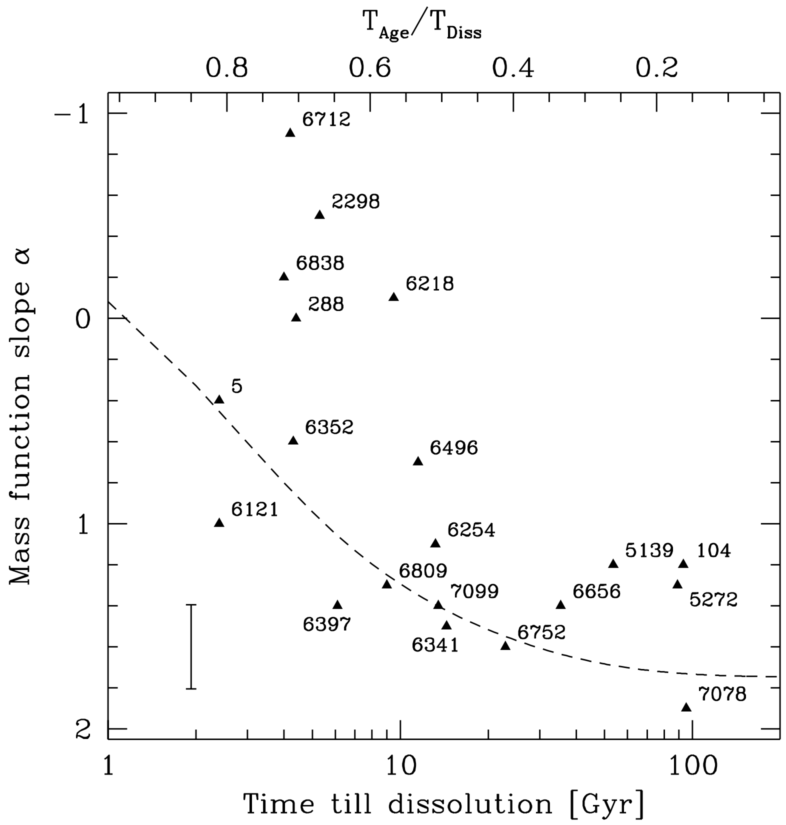


(from Baumgardt & Makino 2003)

Mass segregation and cluster dissolution

A correlation between the mass-function of a star cluster and its time to dissolution is observed for galactic globular clusters.

The mass functions of a number of clusters seem however stronger depleted in low-mass stars than what one would expect.



(from Baumgardt et al. 2008)

Mass segregation and cluster dissolution

A decrease of M/L values with cluster mass is also observed for Milky-Way globular clusters.

This might be due to the preferential loss of low-mass stars (Kruijssen 2008), but there is a need for better modeling and observations to confirm this trend.

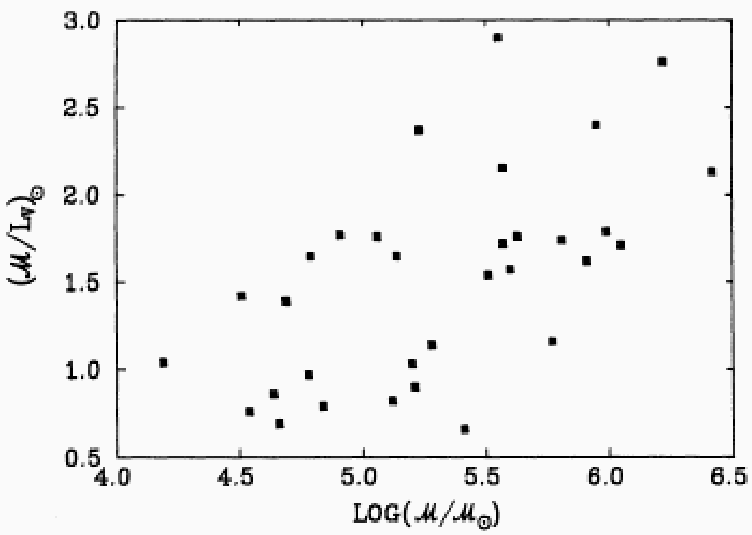


Fig. 3. Cluster mass-to-light ratios as a function of the total mass for the sample of 32 clusters (Table 1)

(from Mandushev et al. 1991)

Topic IV

Intermediate-mass black holes (IMBHs)

Reasons why IMBHs could exist:

- IMBHs could form as the end-product of normal stellar evolution of population III stars.
- They could also form in young and dense star clusters out of the mergers of massive main-sequence stars (Portegies Zwart et al. 1999, 2004, talks by Umbreit, van Bever at this conference).
- The gradual merger of stellar-mass black holes in globular clusters could lead to the formation of an IMBH, although gravitational wave kicks probably prevent this formation channel (Merritt et al. 2004).
- IMBHs could also form in low-mass galaxies in a way similar to the formation of super-massive black holes in large galaxies. Tidal stripping of these galaxies could lead to star clusters with IMBHs. The $M-\sigma$ relation predicts IMBHs for globular clusters.

IMBHs in globular clusters

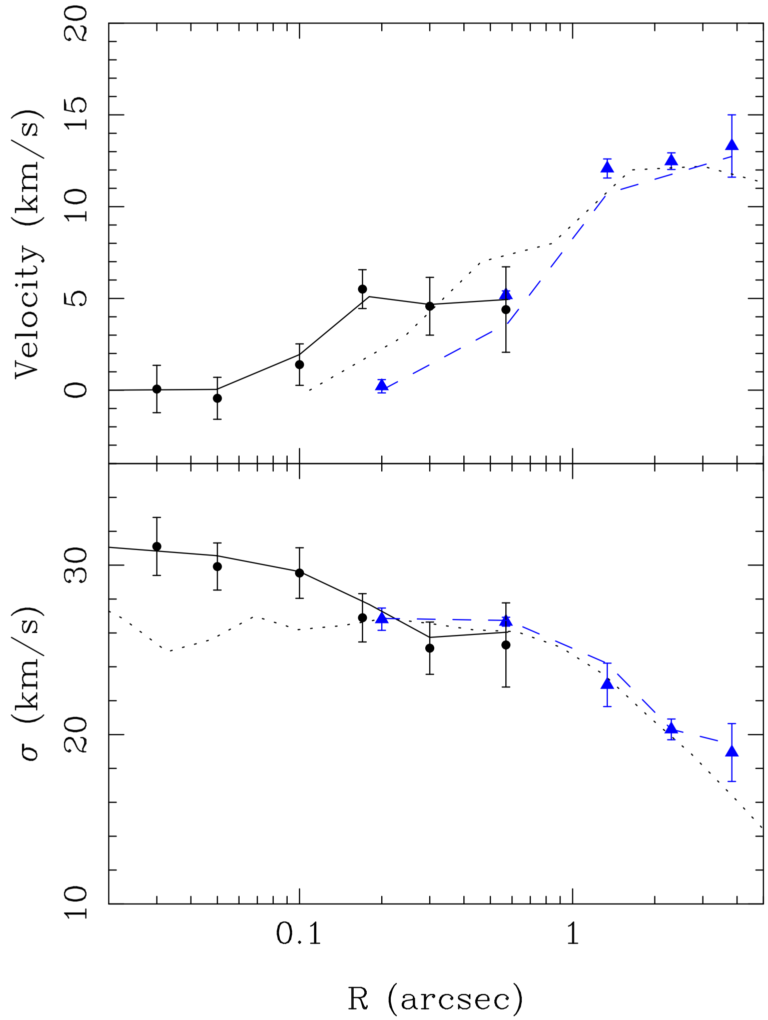
- Currently, IMBHs are discussed to exist in several globular clusters. Early evidence for an IMBH in M15 (Gerssen et al. 2002) has been shown to be due to a dense concentration of compact remnants as a result of mass segregation (Baumgardt et al. 2003)

- There is kinematic evidence for an IMBH in G1 (Gebhardt et al. 2003, 2005). The case for an IMBH in G1 is supported by radio and X-ray data which both show that there is a central source in this cluster (Ulvestad et al. 2007).

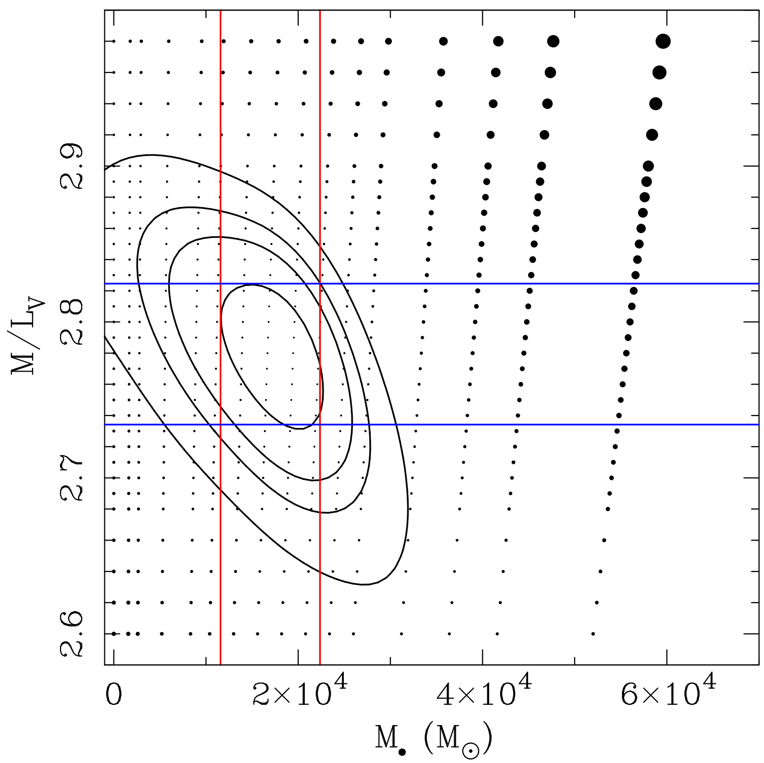
- In addition, ω Cen might also host an IMBH of about $40.000\ M_\odot$ (Noyola et al. 2008).

- The fact that most clusters in their sample did not show central X-ray emission, led Maccarone et al. (2008) to conclude that IMBHs are rare.

Rotational and velocity dispersion profile of G1 (Gebhardt et al. 2005).

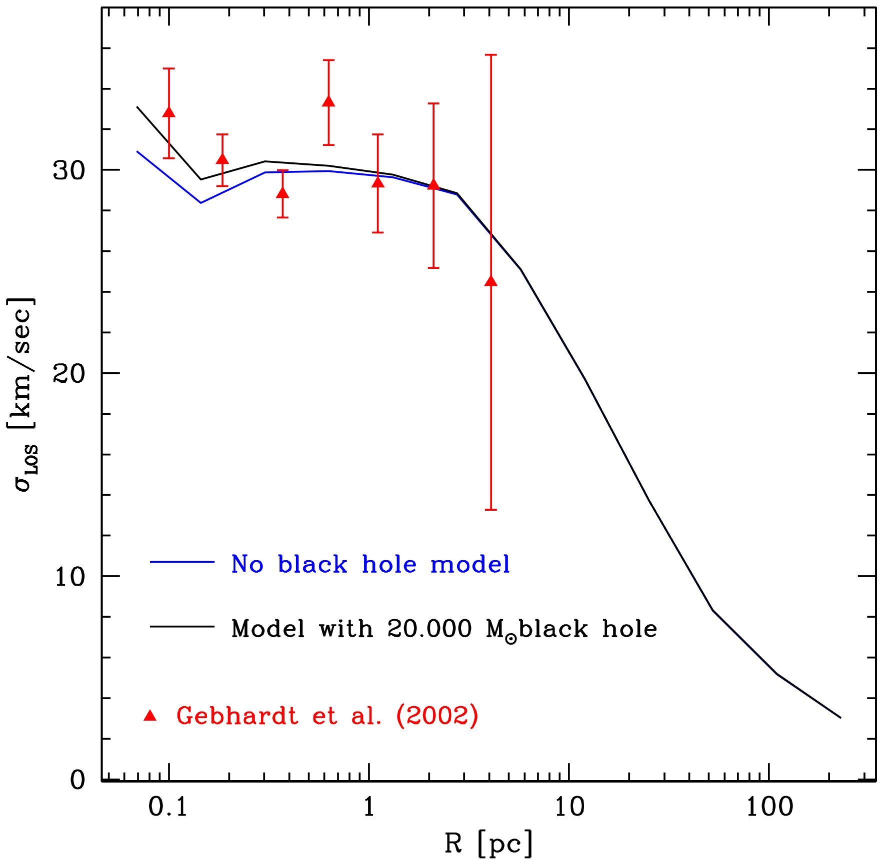


Fit of different dynamical models to G1. An IMBH solution is preferred over the no IMBH case with 99% confidence.

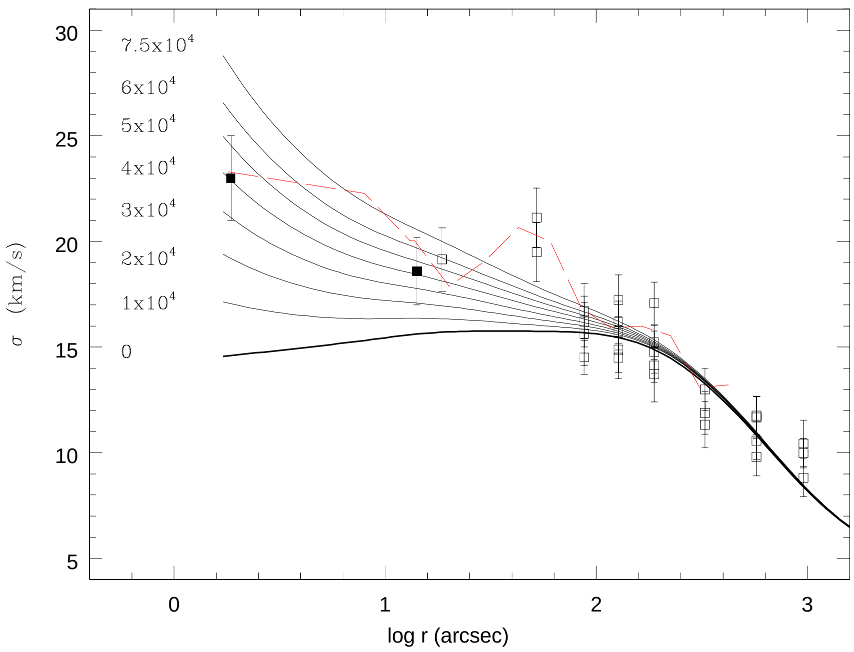


(from Gebhardt et al. 2005)

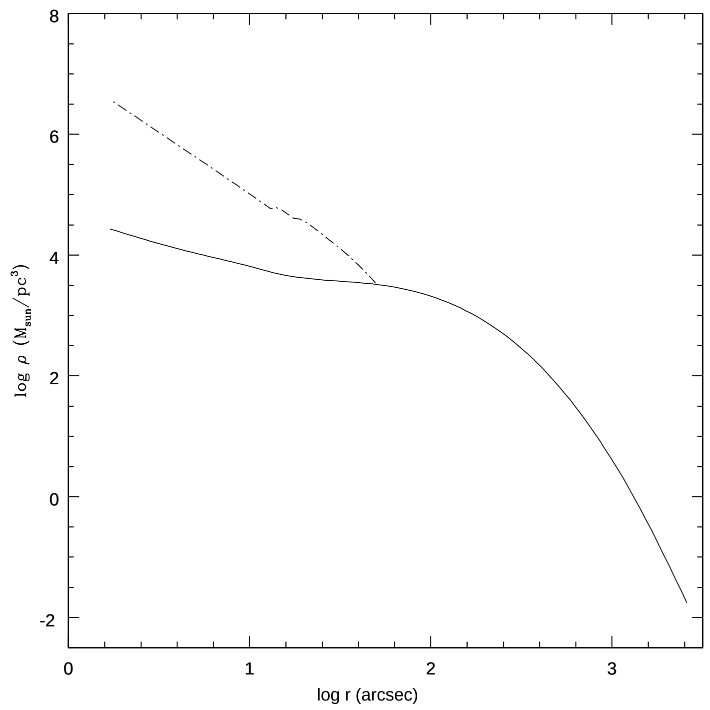
One problem with kinematic data in G1 is the large distance to the cluster, which means that the effect of the black hole hardly shows up in the velocity data:



The velocity dispersion profile of ω Cen also requires an IMBH:

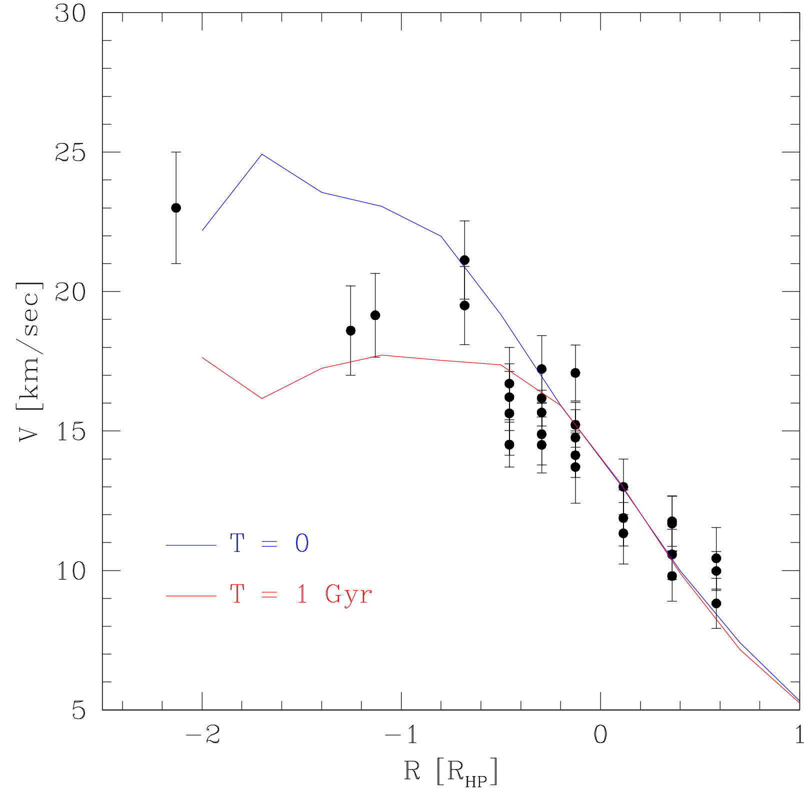


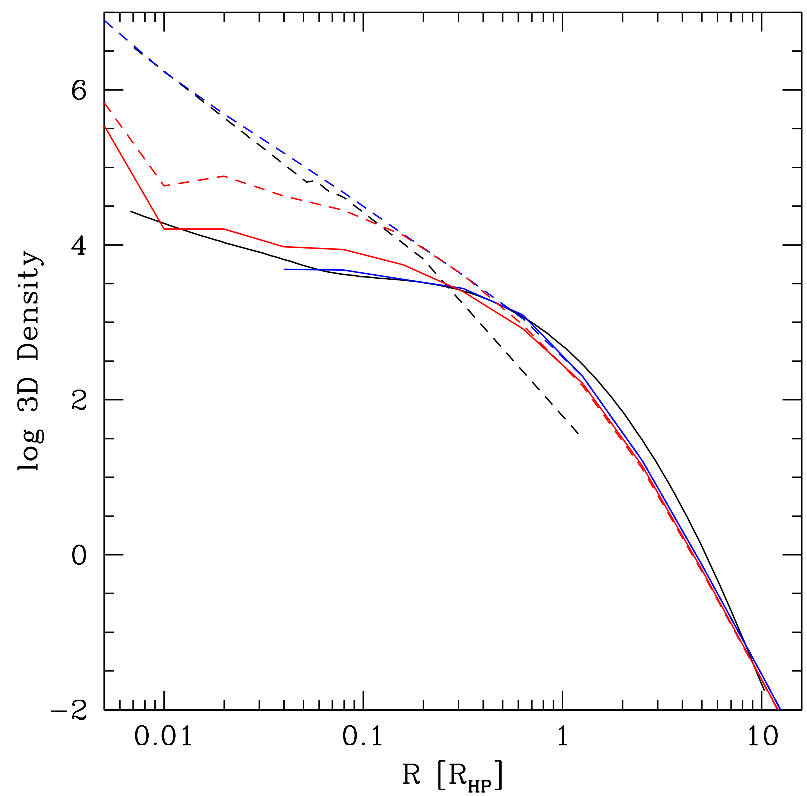
The only alternative to an IMBH would again be a dense cluster of dark remnants:



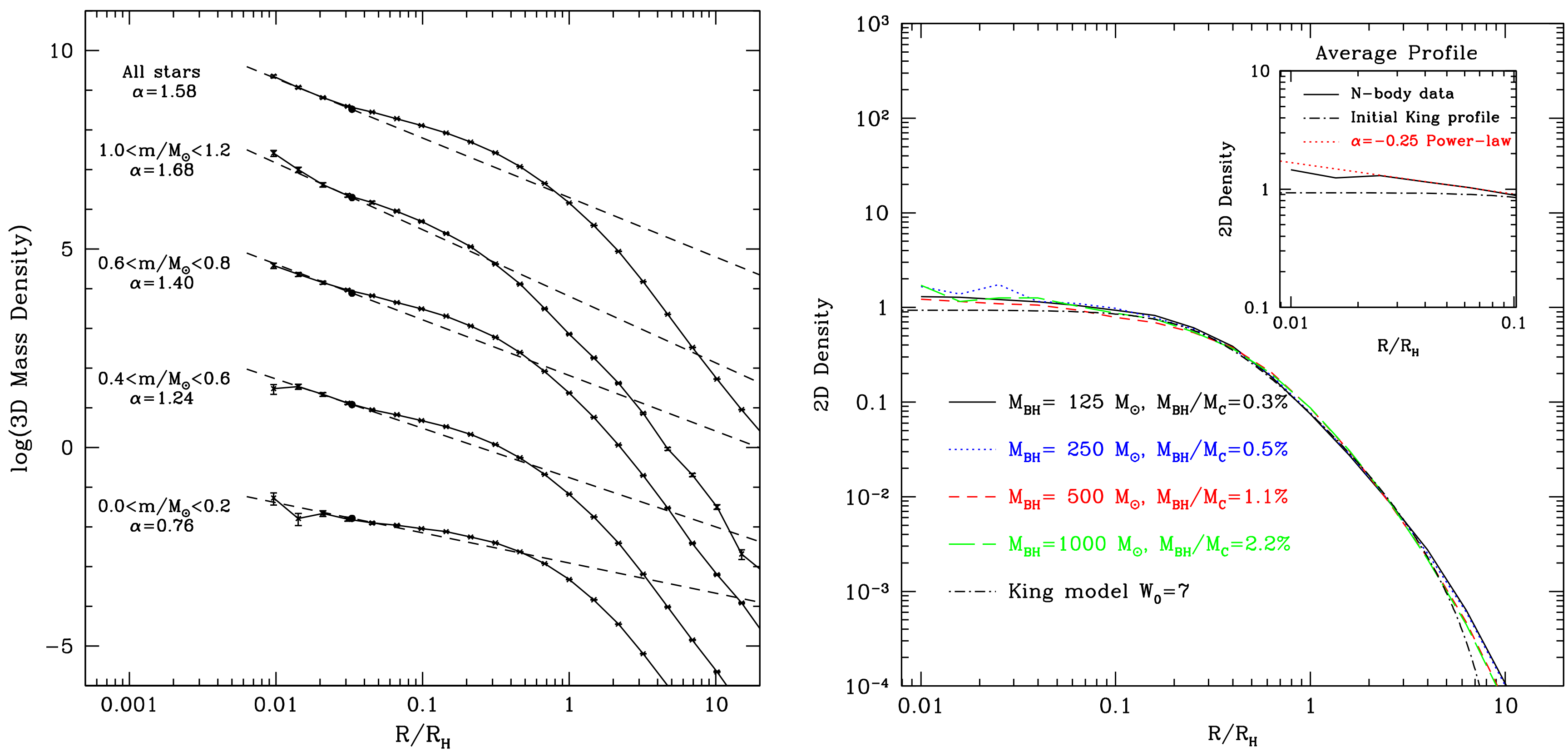
(from Noyola et al. 2008)

Such a cluster would however be dynamically unstable since compact remnants will drift out of the centre and luminous stars drift into it. As a result, the light and velocity dispersion profile of ω Cen seems to be impossible to fit except by an IMBH.





The central density profile develops a **Bahcall & Wolf cusp**. Within the cusp, stars are mass segregated such that higher-mass stars follow steeper density profiles. In projection, the luminosity profile of a cluster with an IMBH should follow a weak cusp profile with $\alpha = 0.25$.



Relaxation around an IMBH

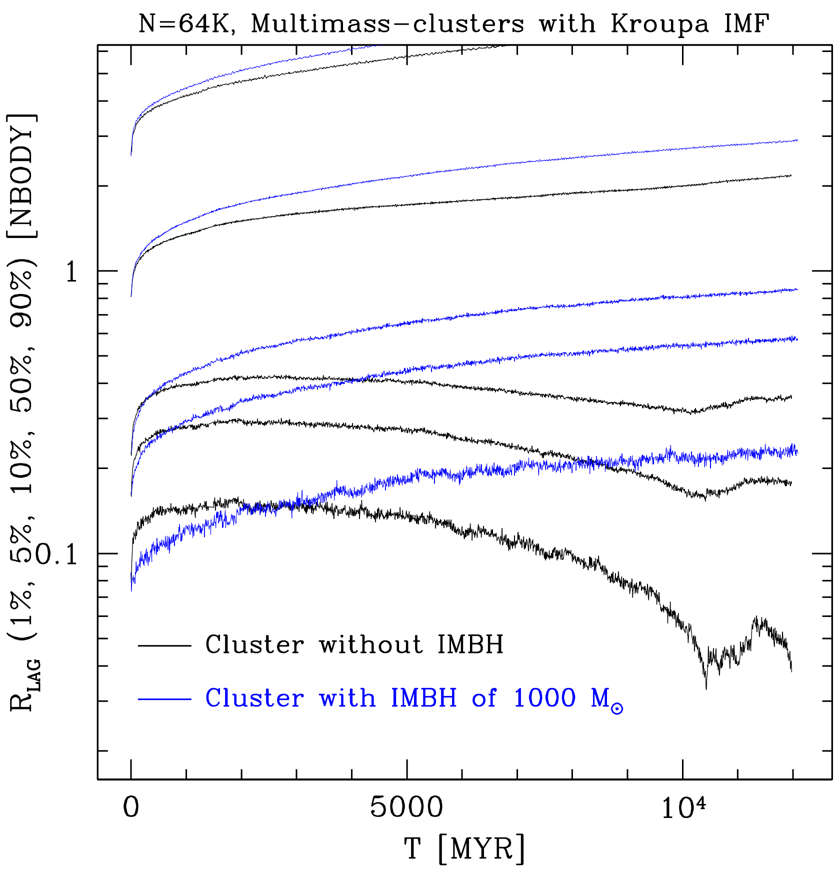
- In the region around the IMBH, encounters of stars cause energy exchange such that some stars lose energy. If stars come close to the black hole, they can be tidally disrupted or swallowed. Their replenishment by stars further out leads to a constant **inward flow of stars.**

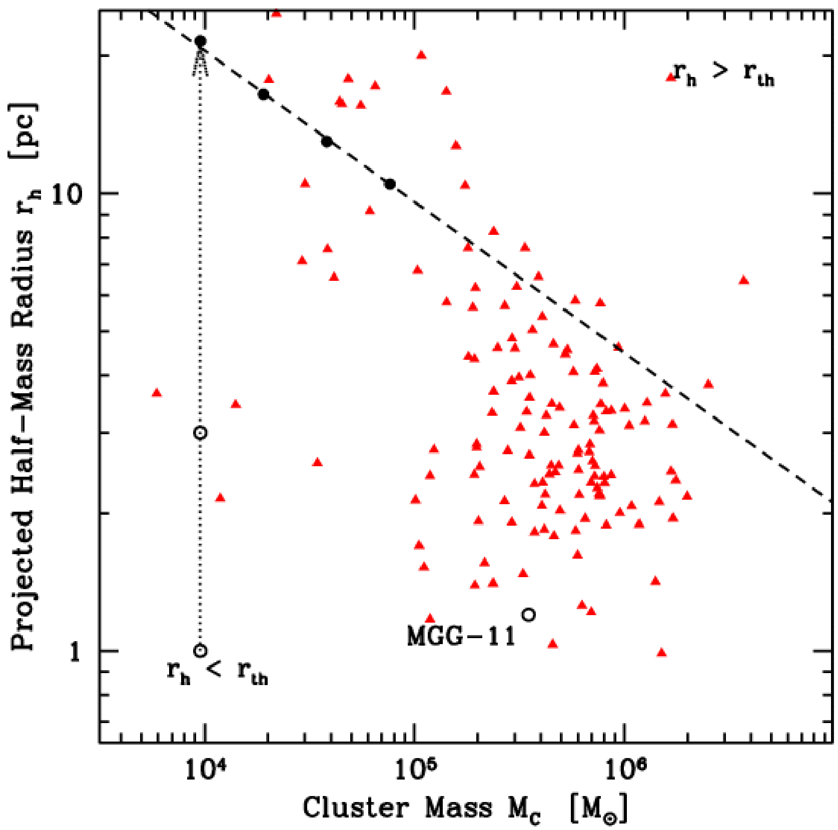
- Since the energy of stars at the time of their disruption is strongly negative ($E_{Pot} = -GM_{BH}/r$), the average stellar energy increases.

- The central black hole therefore provides an energy source for the star cluster and this energy is carried away by relaxation, causing an **outward flow of energy.**

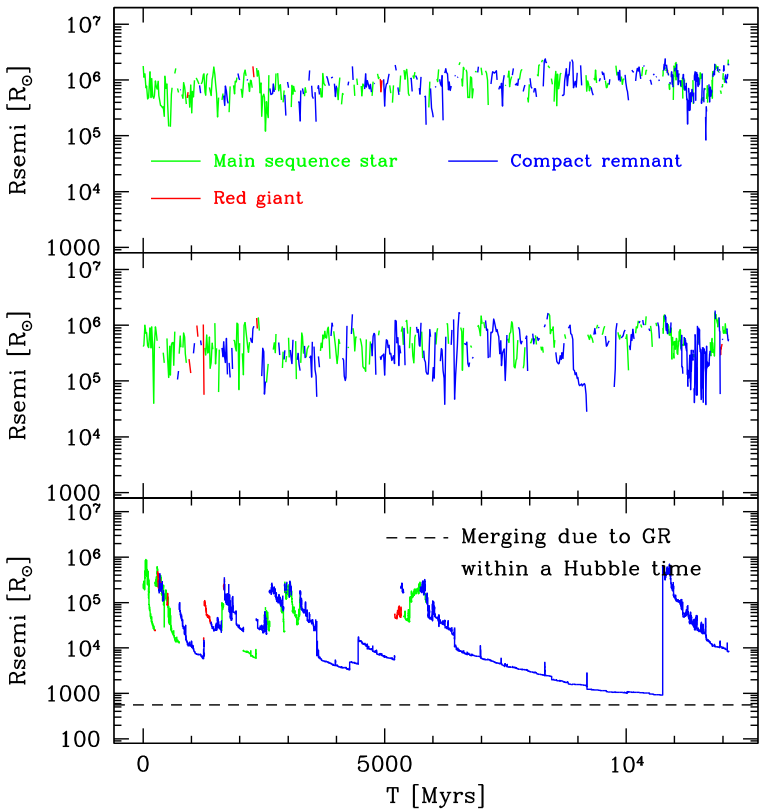
- This causes the cluster to expand.

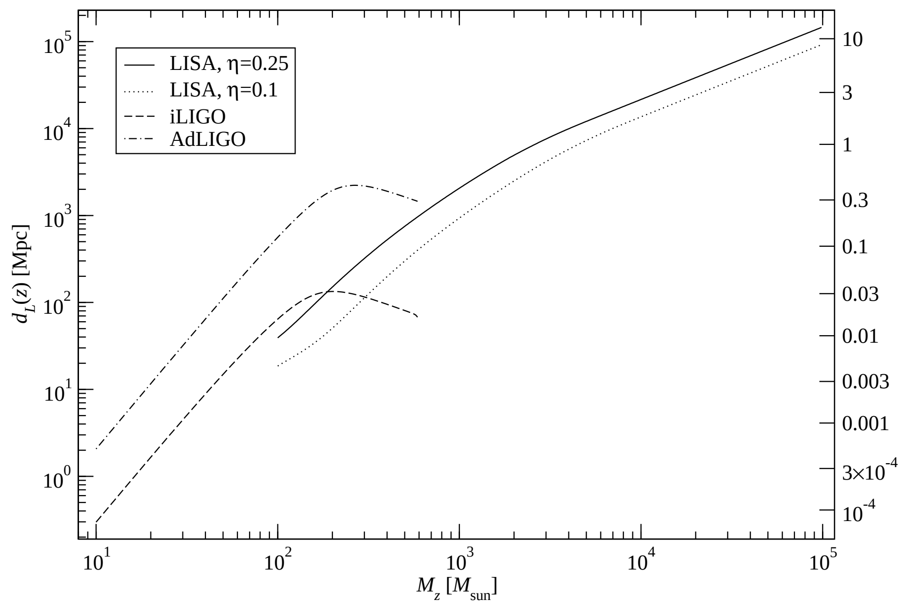
Since an IMBH provides a central heat source, clusters with IMBHs expand. The expansion is strong enough that a cluster with an IMBH would end up among the least concentrated galactic globular clusters.





IMBHs in globular clusters would be important gravitational wave sources due to mergers with compact remnants in their vicinity. Some of them might also spiral into galactic centers and merge with super-massive black holes.





(from Fregeau et al. 2006)