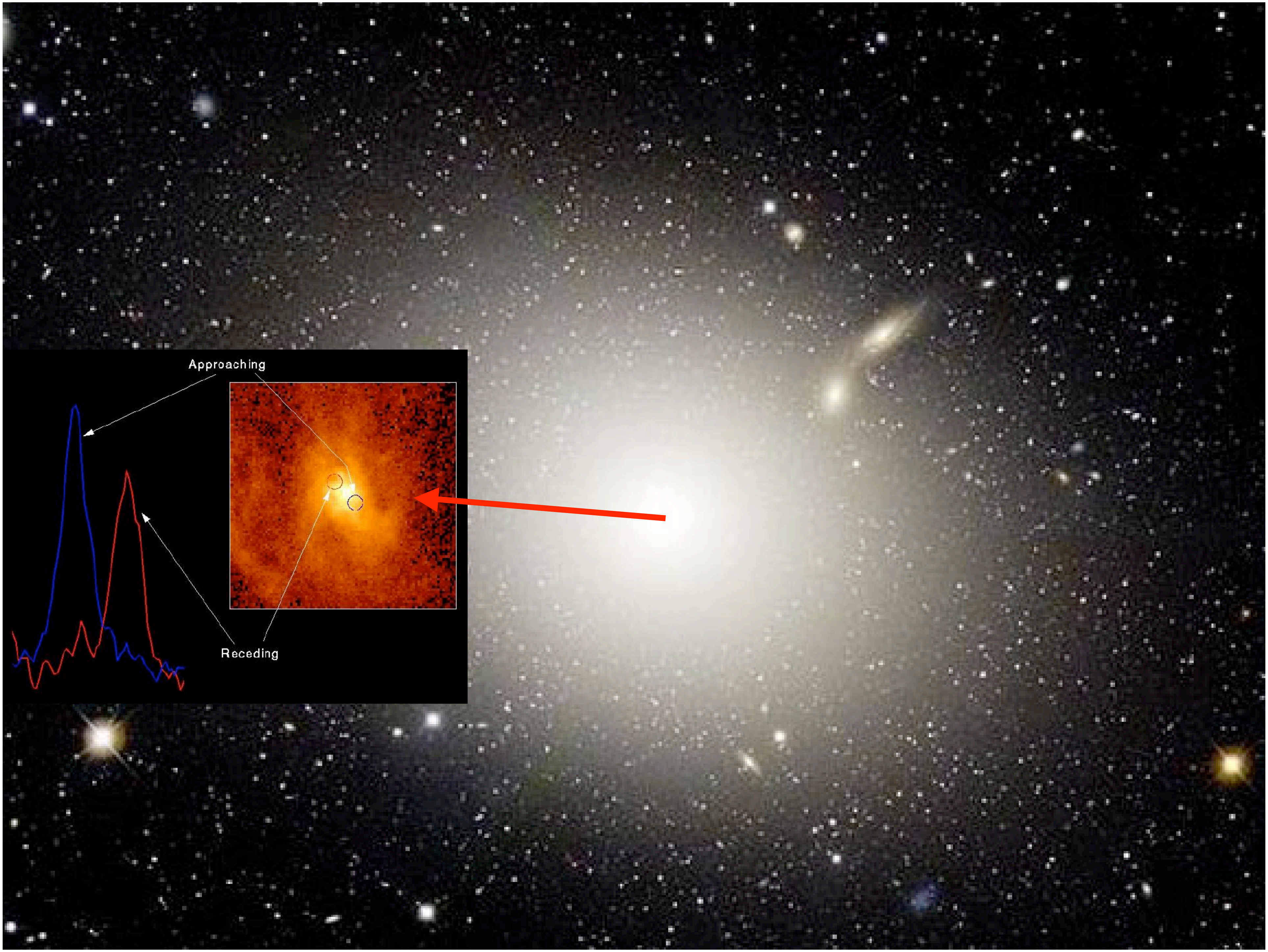
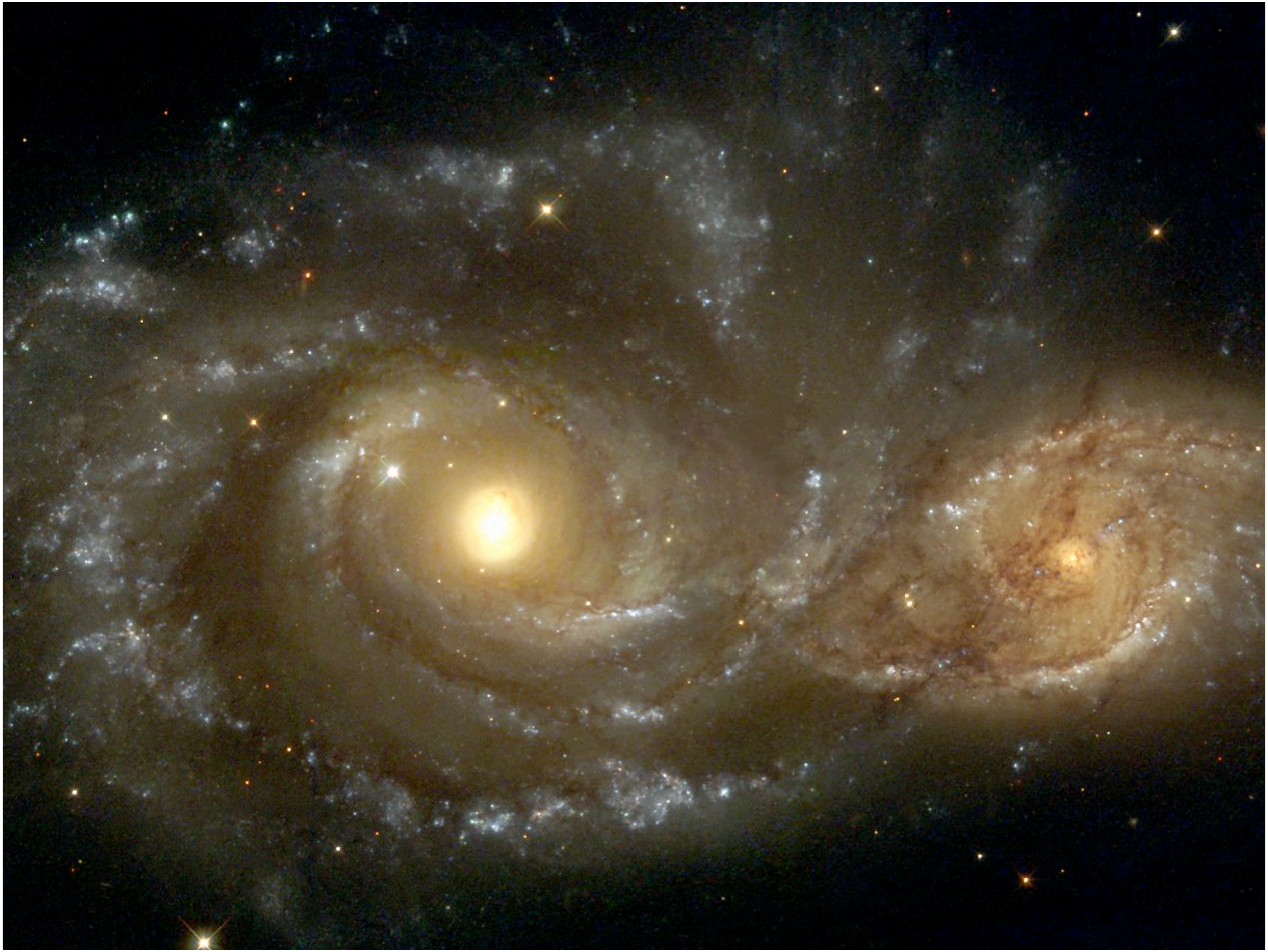
Galactic Nuclei

1. Phenomenology
2. Single-Black-Hole Solutions
3. Loss Cones
4. Computational Approaches
5. Binary Black Holes and Cores
6. The “Final Parsec” Problem
7. Gravitational Wave Recoil

N-Body Problem: Numerical Methods and Applications
Turku, Finland
August 2008

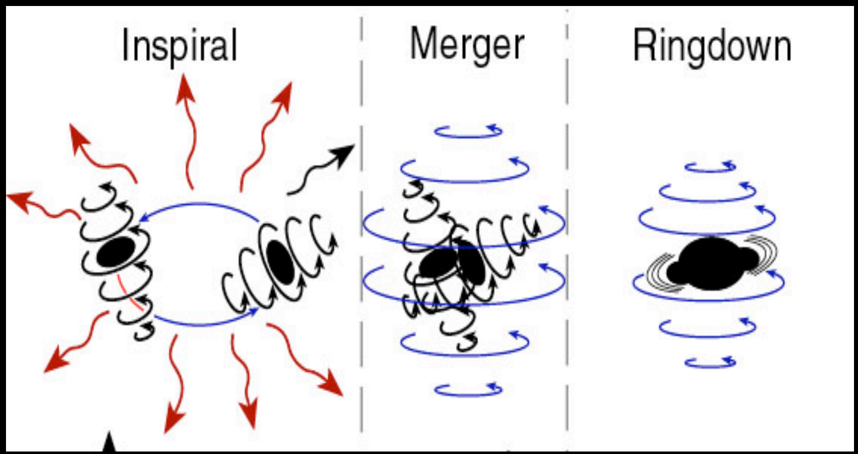

Approaching
Receding

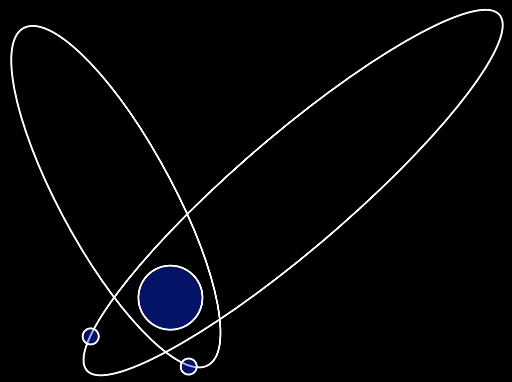


Gravitational Waves from Black Holes

Two of the strongest potential sources in the low-frequency (LISA) regime are

- Coalescence of binary supermassive black holes
- Extreme-mass-ratio inspiral into supermassive black holes





Influence radius:

$r_h = G\, M_\bullet/\sigma^2$

$= 11 \text{ pc } (M_\bullet/10^8 M_\odot)(\sigma/200 \text{ km s}^{-1})^{-2}$

$M_\bullet$-σ^2 relation:

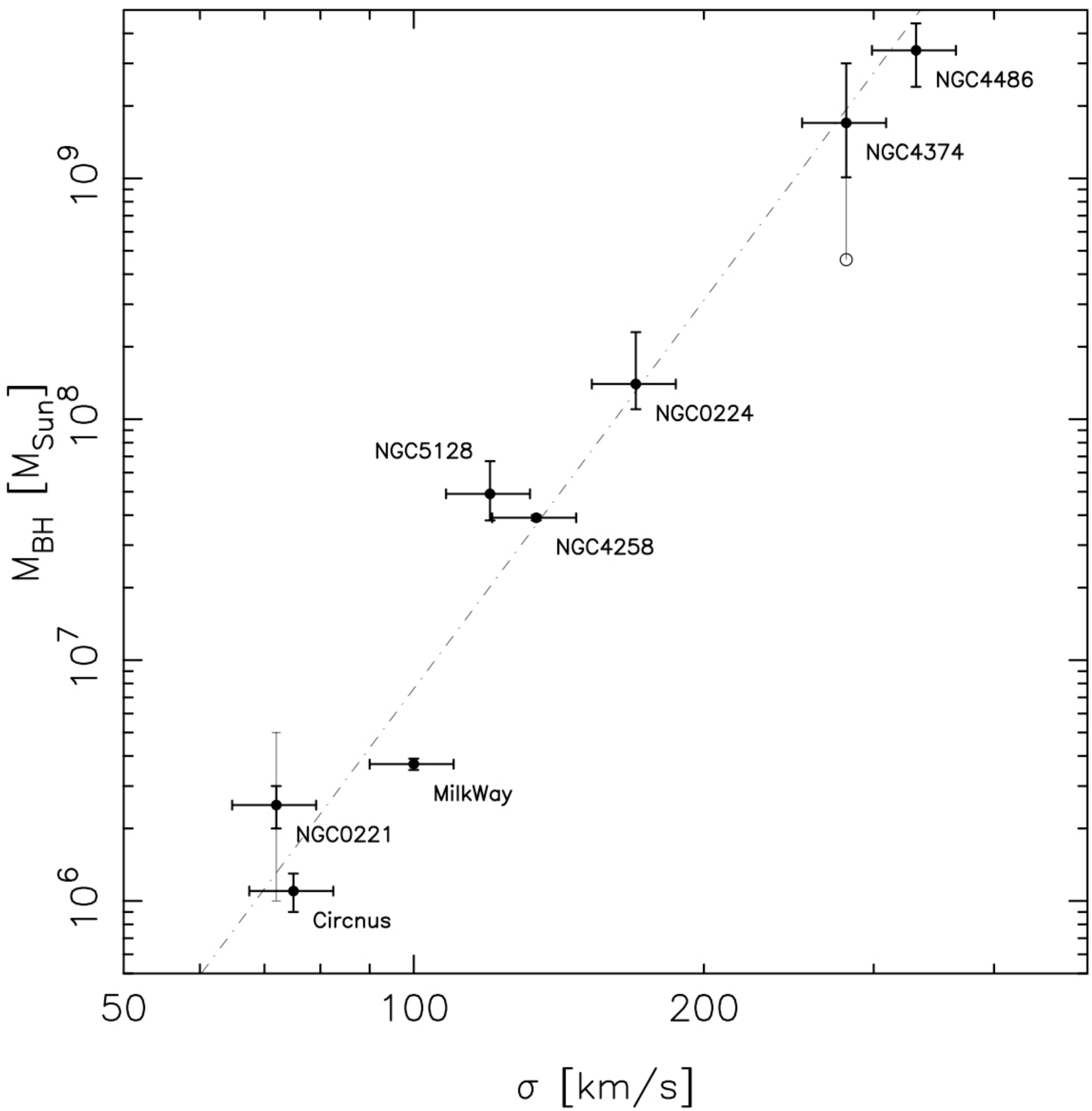
$M_\bullet / 10^8 M_\odot \approx 1.6\, (\sigma / 200 \text{ km s}^{-1})^\alpha\,, \quad 4 \leq \alpha \leq 5$

Combining the two:

$r_h \approx 18 \text{ pc } (\sigma/200 \text{ km s}^{-1})^{-2.5}$

$\approx 13 \text{ pc } (M_\bullet/10^8 M_\odot)^{-0.55}$

A (roughly) equivalent definition of r_h is the radius containing a mass in stars equal to 2 $M_\bullet$.



NGC4486
NGC4374
NGC0224
NGC5128
NGC4258
MilkWay
NGC0221
Circnus
10^9
10^8
10^7
10^6
M_{BH} [M_{Sun}]
50
100
200
σ [km/s]

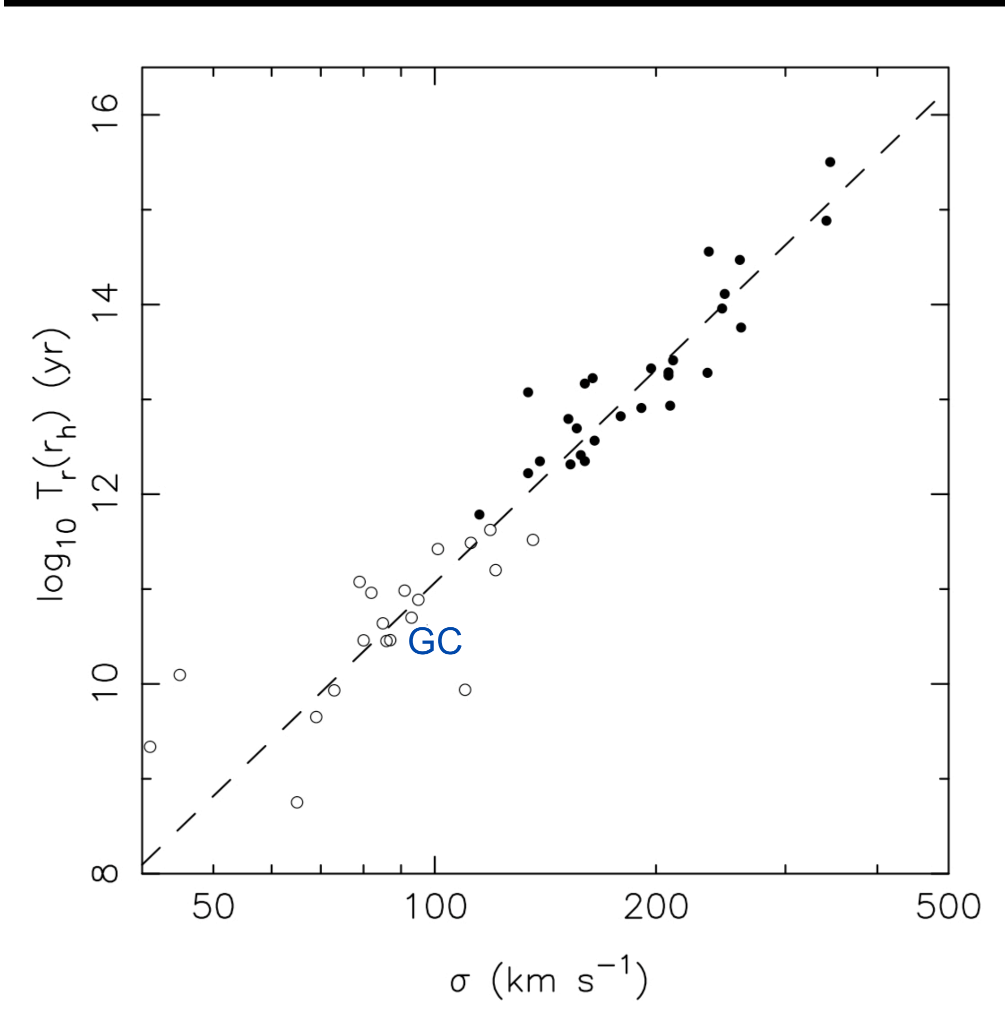
Characteristic Times

$$t_D \approx \frac{r}{v} \approx 2\pi\sqrt{\frac{r^3}{GM(<r)}}$$
$$\approx 2\times 10^5 \text{yr at 3 pc}$$

$$t_r \approx \frac{0.34\sigma^3}{G^2 m\rho \ln\Lambda}$$
$$\approx 10^{10} \text{yr at 1 pc}$$

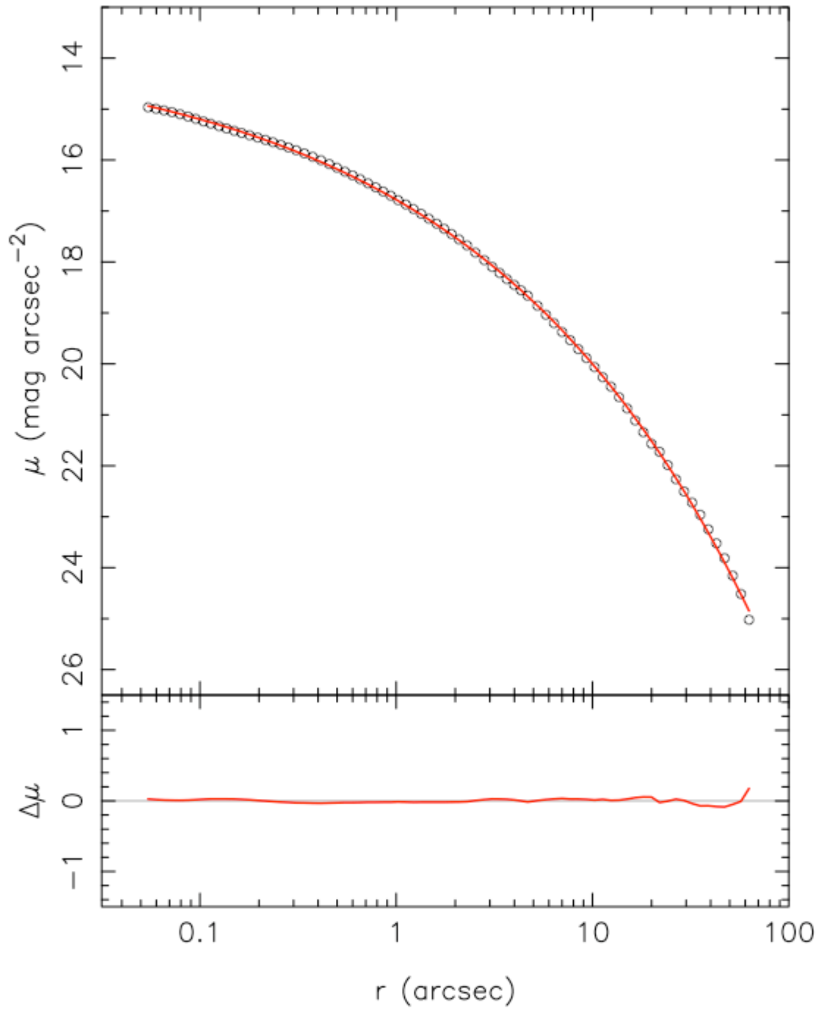
$$t_{\rm coll} \approx \left[16\sqrt{\pi} n\sigma r_\star^2 (1+\Theta)\right]^{-1}, \quad \Theta = \frac{Gm_\star}{2\sigma^2 r_\star}$$
$$\approx 10^{11} \text{yr at 1 pc}$$

Nuclear Relaxation Times



…in a sample of galaxies, measured at the SMBH's influence radius.

Structure of Spheroids

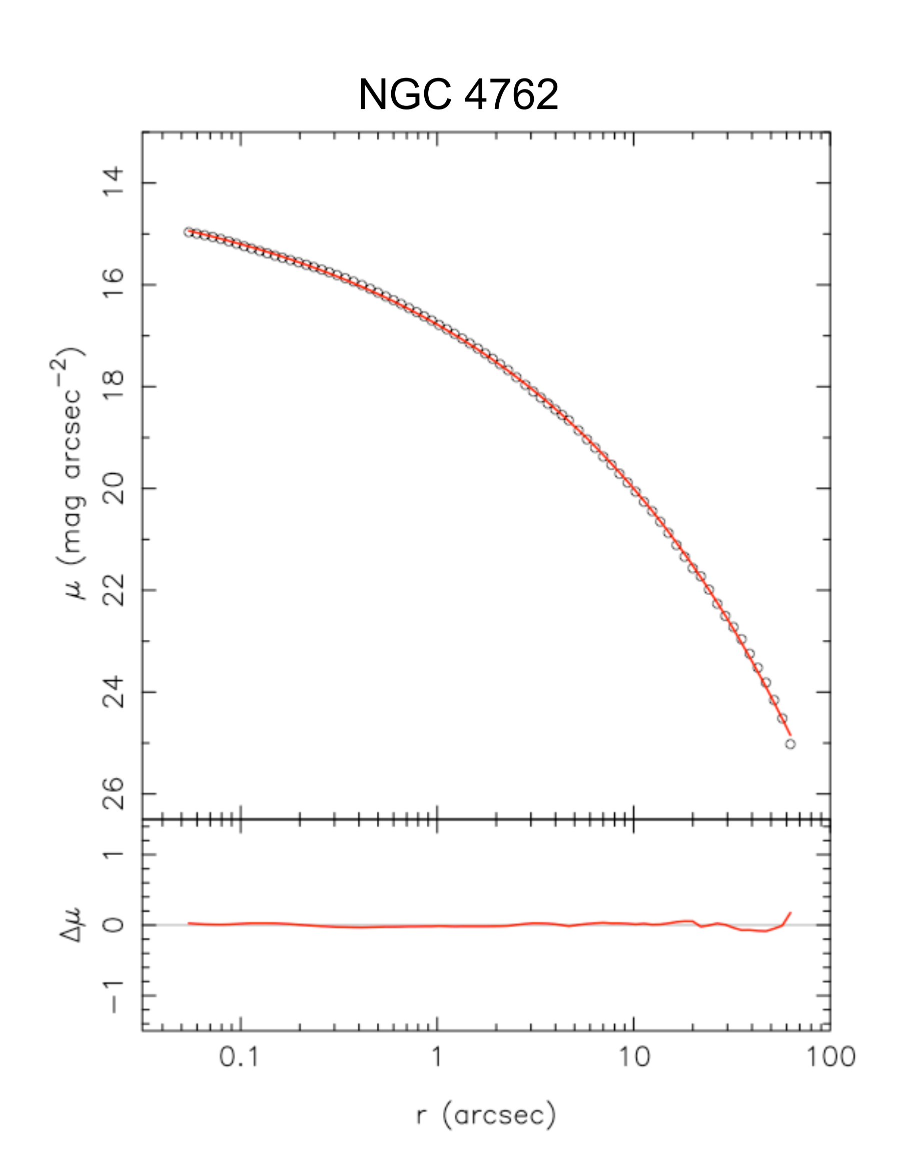


Most spheroids* are well fit by Sersic profiles:

$$\frac{d\ln\Sigma}{d\ln R} = -\frac{b}{n}\left(\frac{R}{R_e}\right)^{1/n}$$

*Elliptical galaxy, or bulge of spiral galaxy.

Structure of Spheroids



Sersic profile:

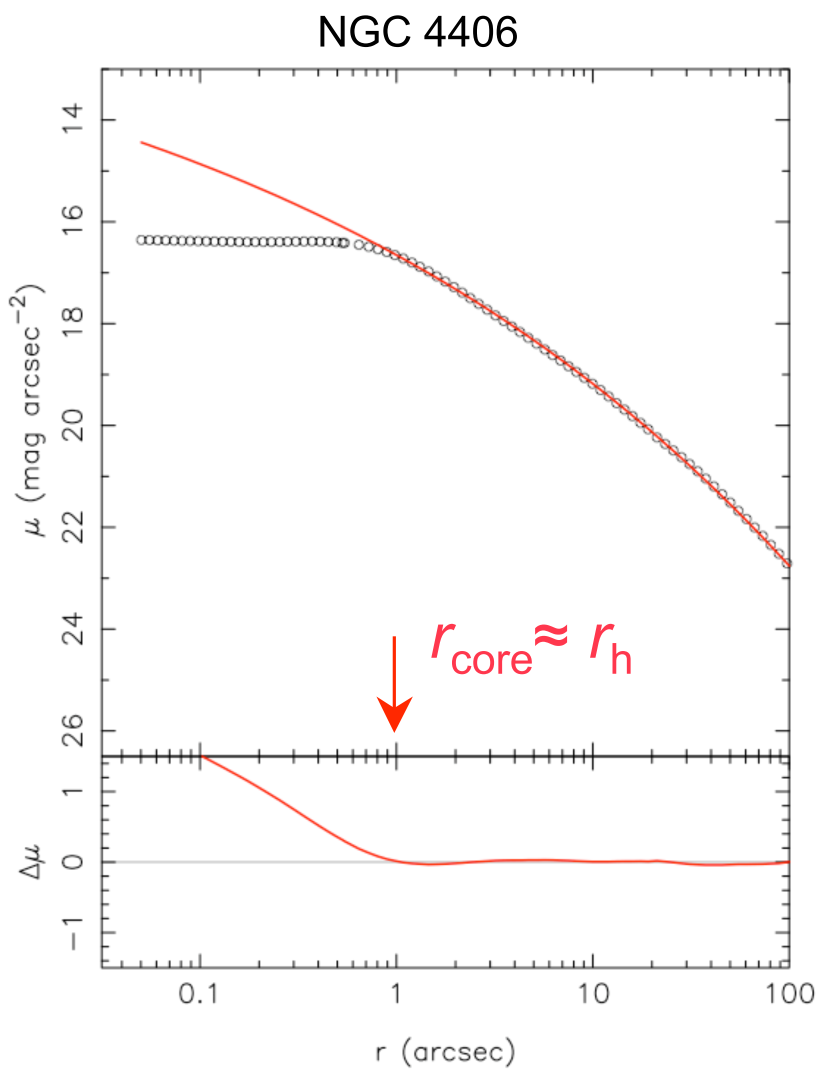
$$\frac{d\ln\Sigma}{d\ln R} = -\frac{b}{n}\left(\frac{R}{R_e}\right)^{1/n}$$

Einasto profile:

$$\frac{d\ln\rho}{d\ln r} = -\frac{b}{n}\left(\frac{r}{R_e}\right)^{1/n}$$

An Einasto profile in the space density looks similar to a Sersic profile in the projected density.

Structure of Spheroids



Bright* spheroids exhibit **mass deficits**, or *cores*.

The core radius r_{core} is roughly the SBH influence radius r_h.

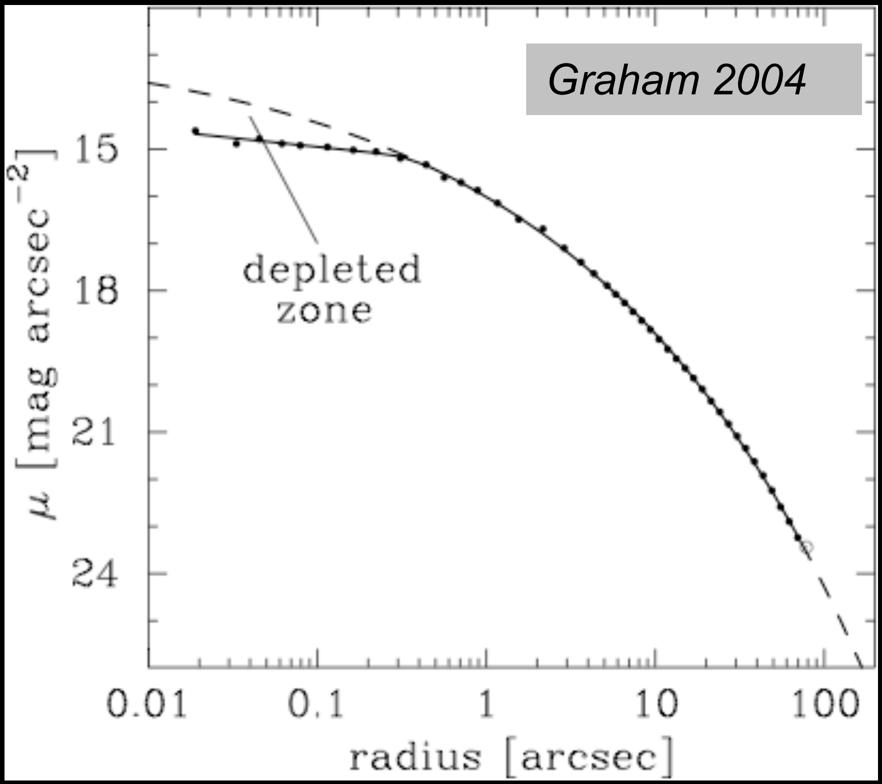
The core mass M_{def} is ~ the SBH mass $M_\bullet$.

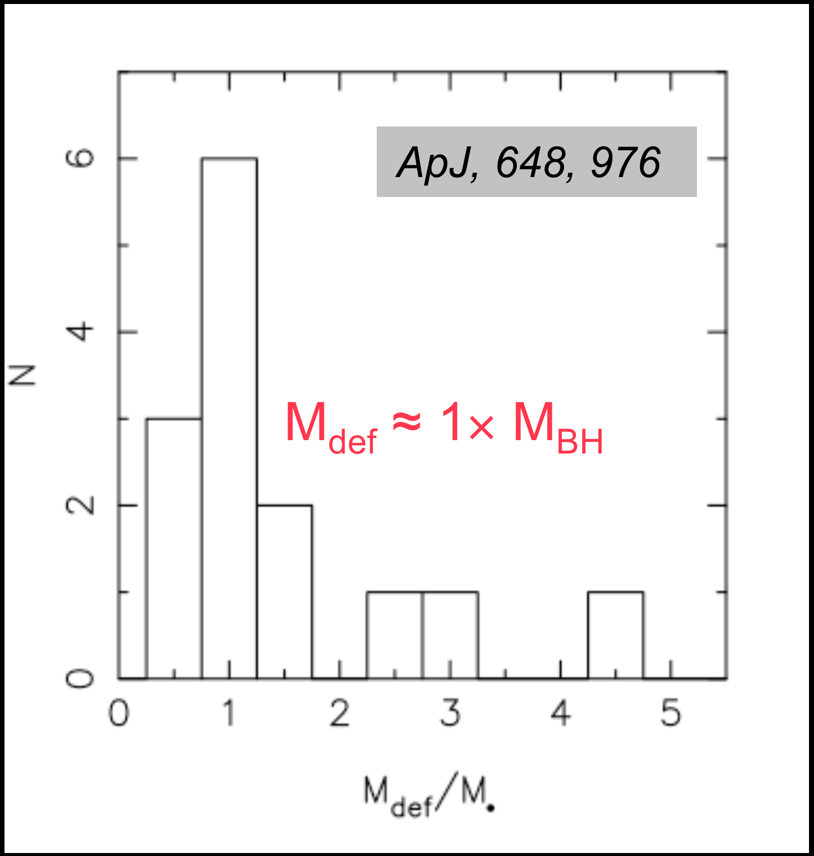
Influence radius:

$$r_h = G\, M_\bullet / \sigma^2$$

*$M_V < -21.5$

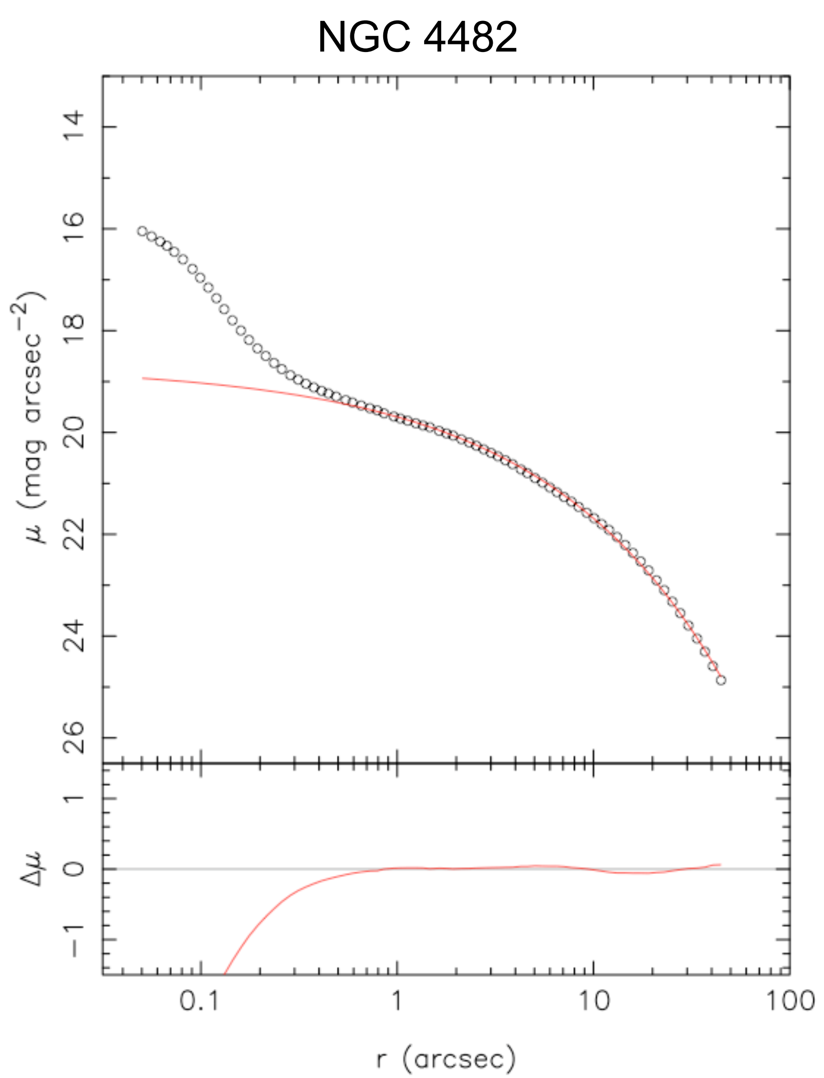
Mass Deficits





Milosavjlevic et al. 2002
Ravindranath et al. 2002

Structure of Spheroids



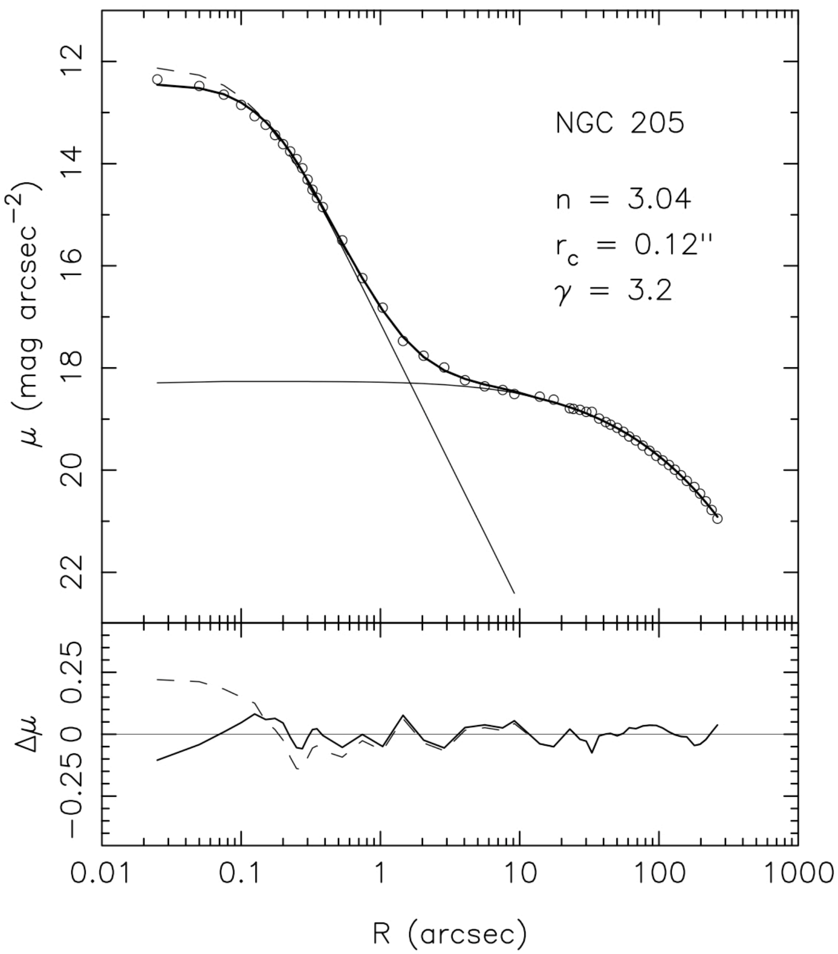
Faint* spheroids exhibit central excesses, or *nuclei*.

The nuclear luminosity is ~$10^{-3.5}$ times the total luminosity.

The nucleus is typically unresolved.

*M_V > -18

NGC 205



Modelled with two components:

Galaxy: Einasto model:

$$j(r) = j_{gal} e^{-b\left[(r/r_{1/2})^{1/n} - 1\right]}$$

Nucleus: "Hubble" model:

$$j(r) = j_{nuc}\left(1 + r^2/r_c^{\,2}\right)^{-\gamma/2}$$

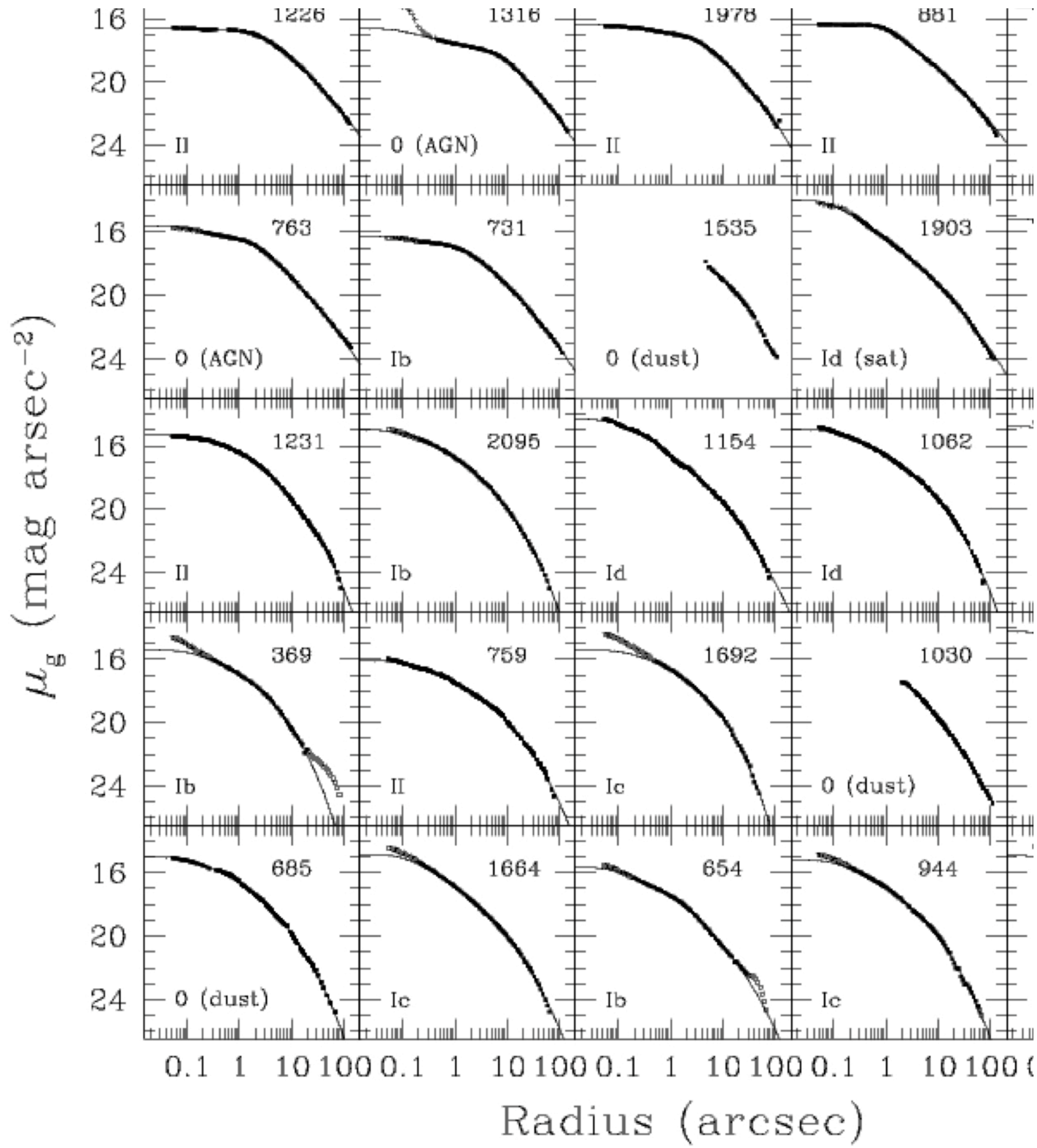
NB: $(M/L)_{nuc} \approx 0.3\,(M/L)_{gal}$

Properties of “Nuclear Star Clusters”

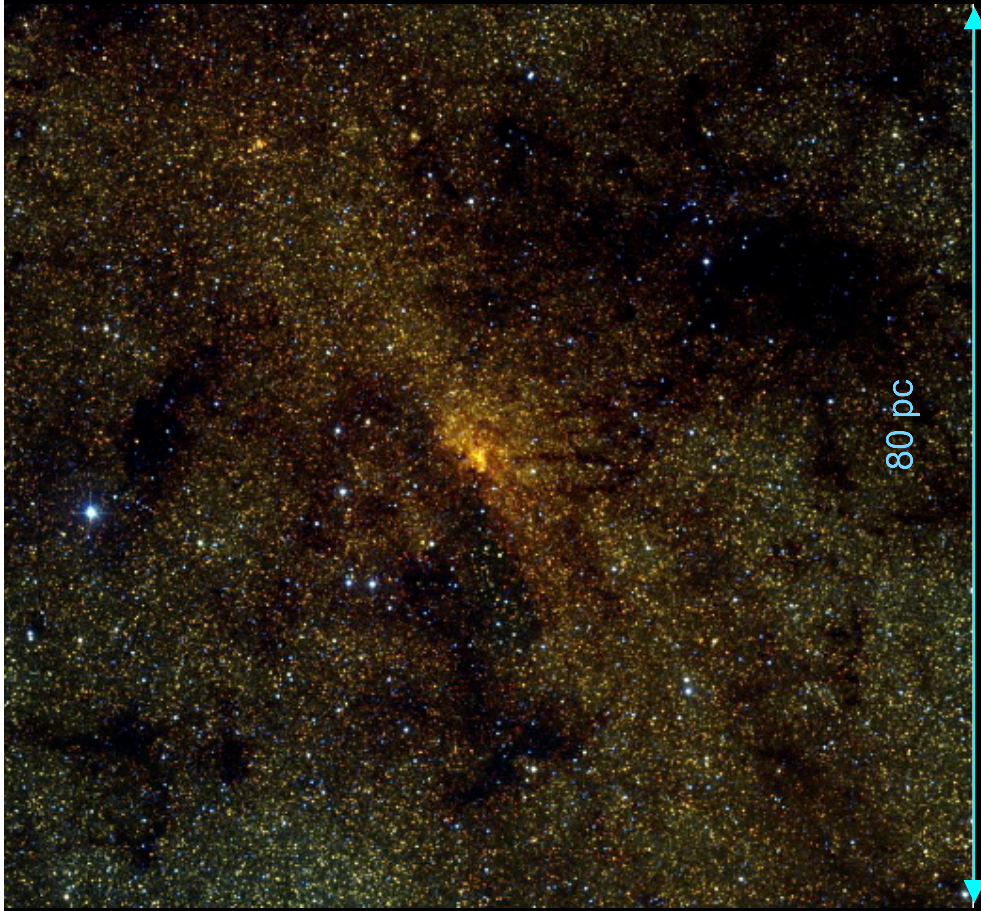
- Present in bulges of all Hubble types
- Frequency of nucleation is 50%-70%:
  -- Hard to see in bright (high-surface-brightness) galaxies
  -- Become rare at galaxy luminosities below $M_B \approx -12$
- 10-100 times brighter than globular clusters
- Sizes scale as $R \sim L^{0.5}$ (unlike GCs)
- Spectra reveal extended star formation histories:
  -- Mean stellar age correlates with Hubble type
  -- However, the dominant population is always old

Luminosity profiles of the **brightest** galaxies in the HST ACS Virgo cluster study.

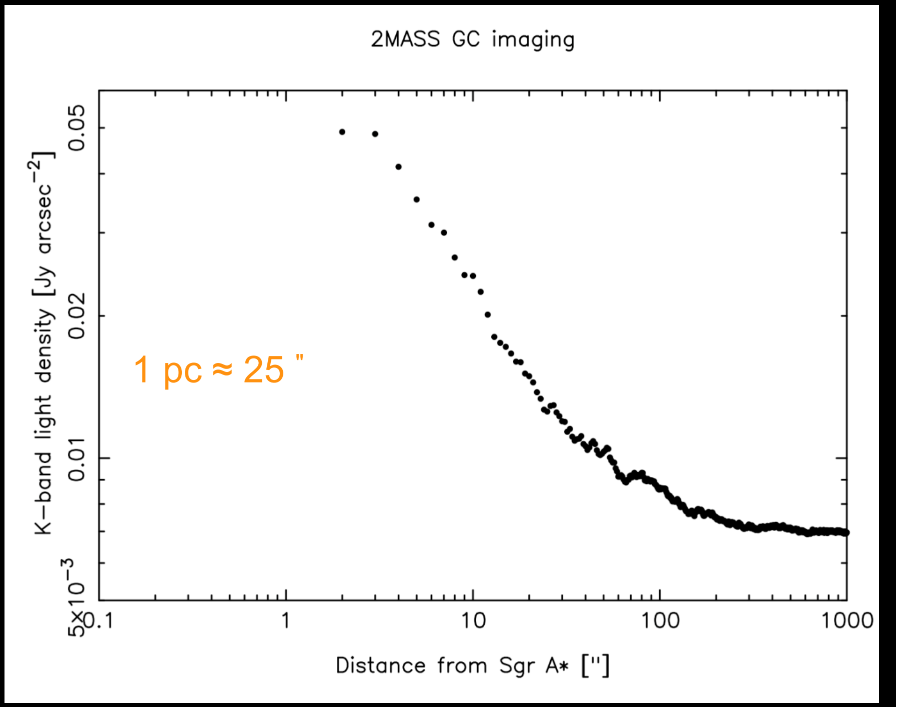
Cote et al. (2006)



Milky Way: Nuclear Star Cluster?



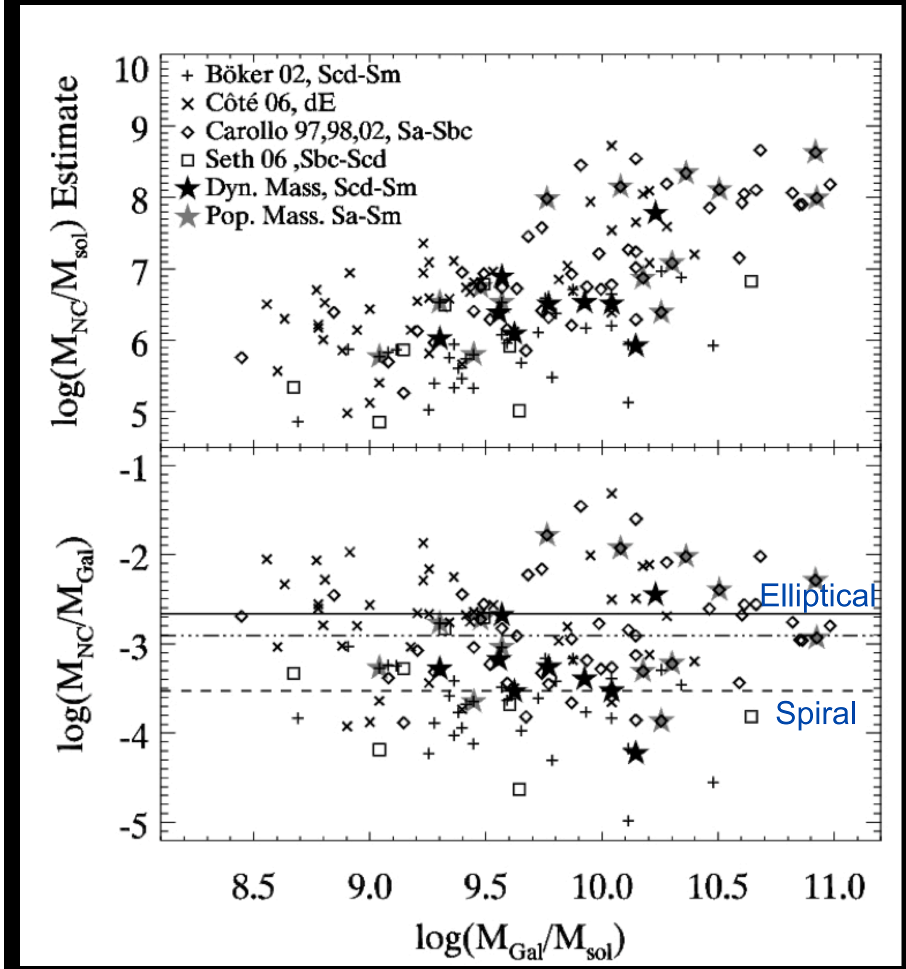
2MASS JHK Image



K-Band Light Density

(R. Schödel, unpub.)

Nuclear Star Clusters: Masses

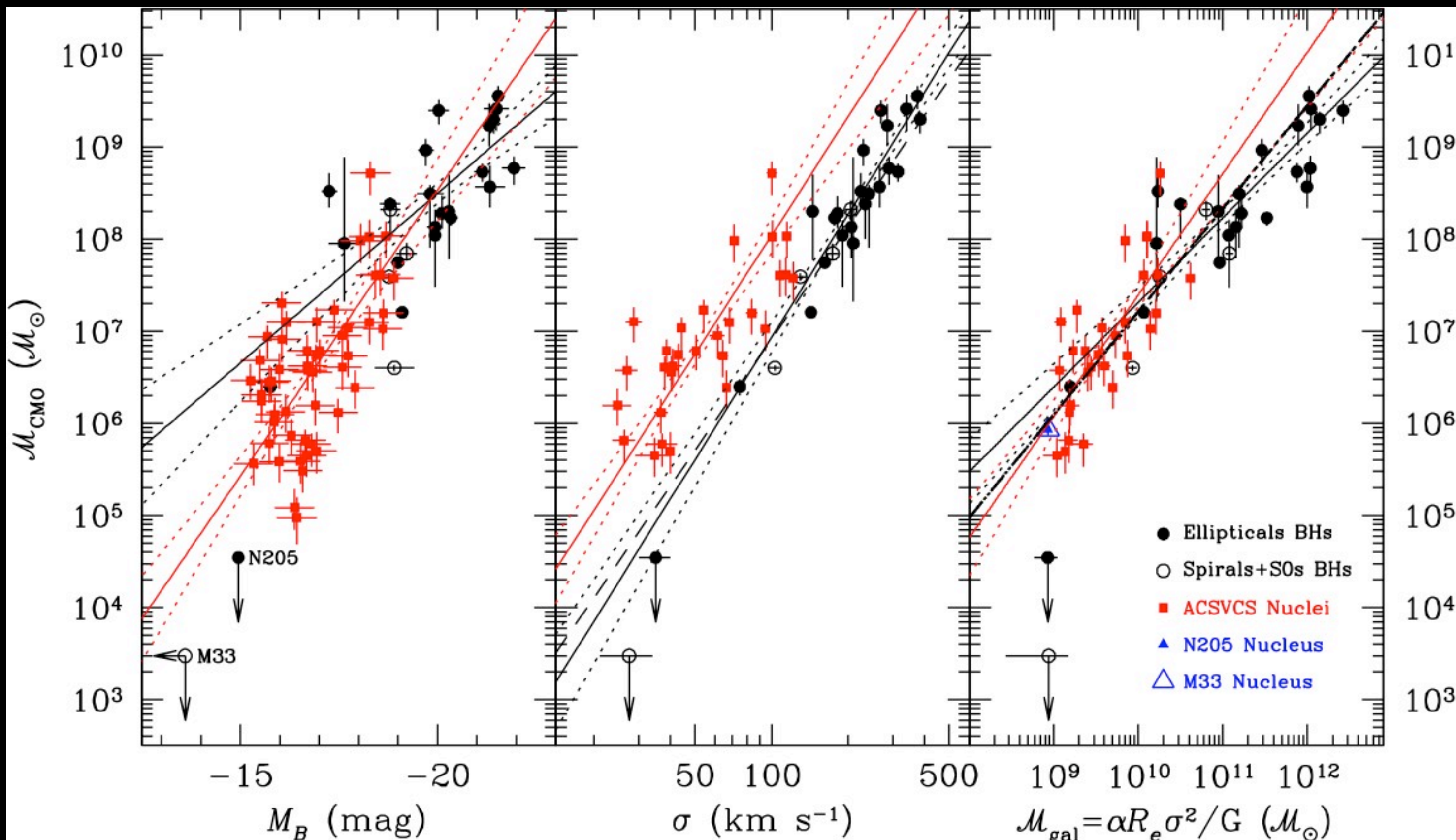


NSC mass vs. galaxy mass

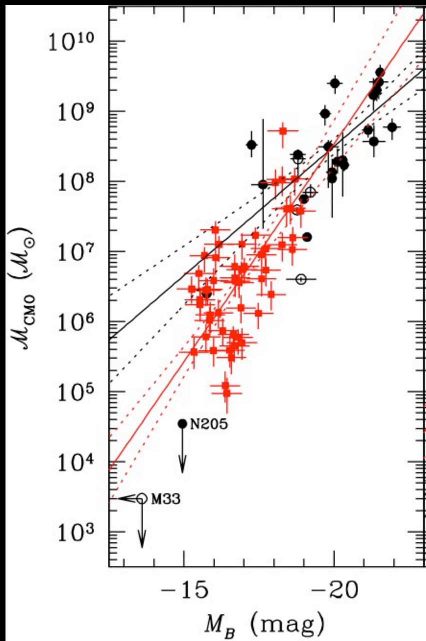
M_{nuc}/M_{gal} vs. galaxy mass

Seth et al. 2008

"Central Massive Objects"

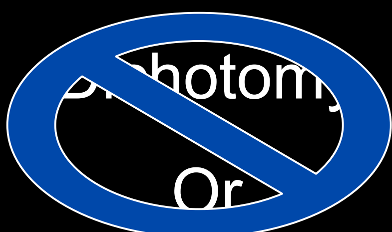


Ferrarese et al. 2006
Wehner & Harris 2006

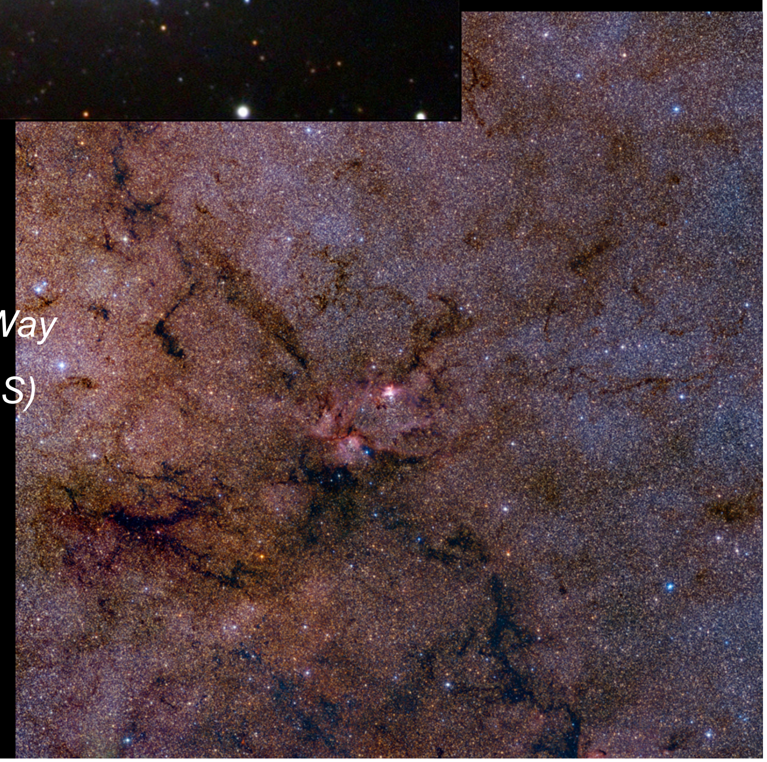


Ferrarese et al. 2006
Wehner & Harris 2006

NGC 4395



Co-Existence!

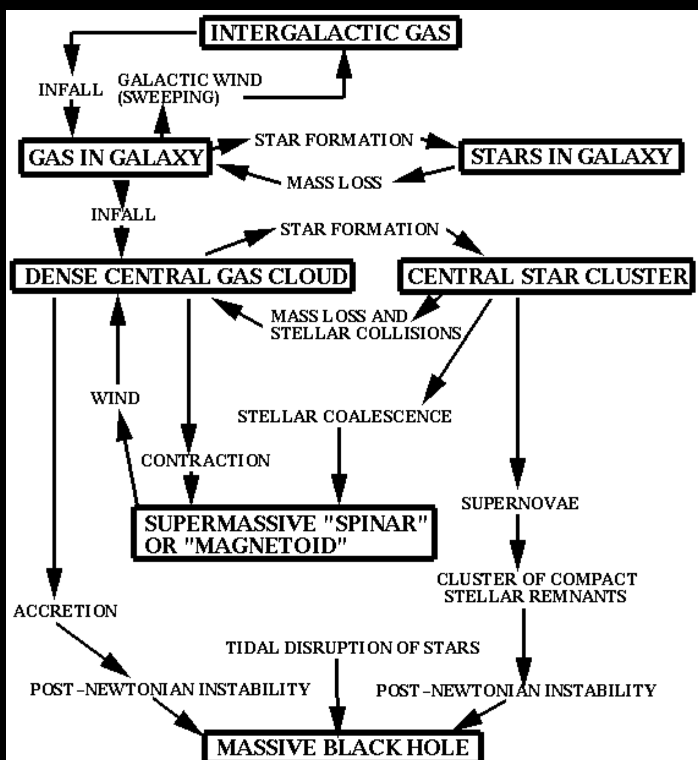


Milky Way

(2MASS)

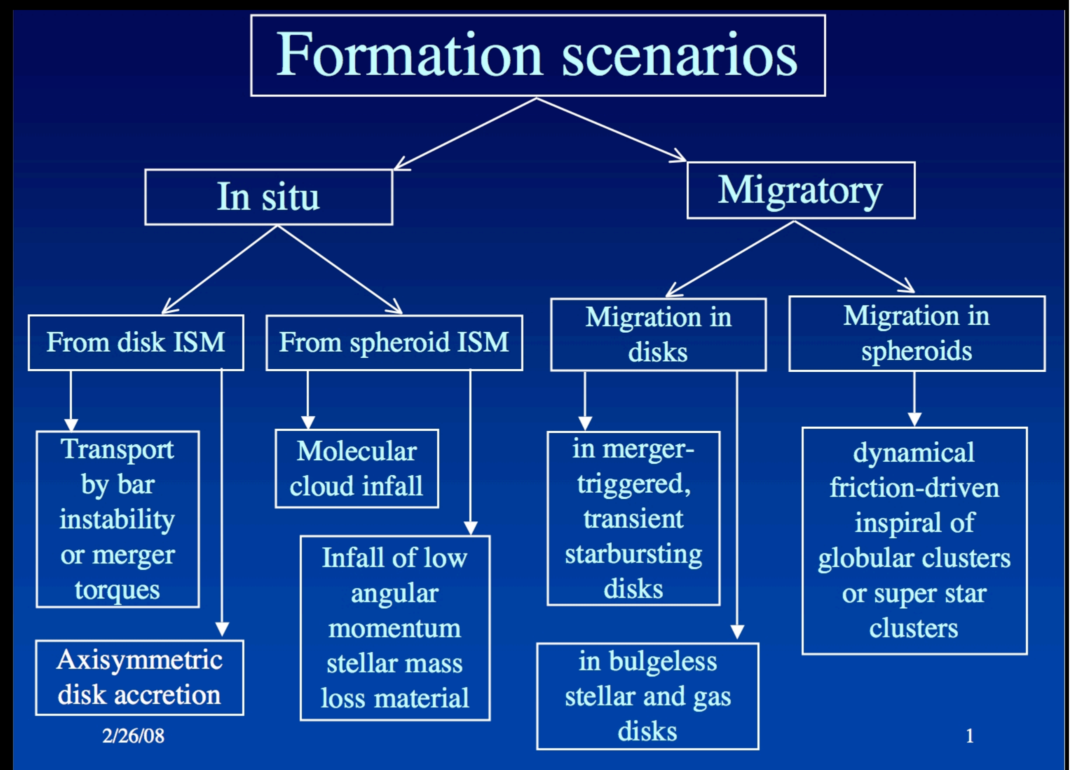
Where Did CMOs Come From?

Black Holes



Rees 1988

Nuclear Star Clusters

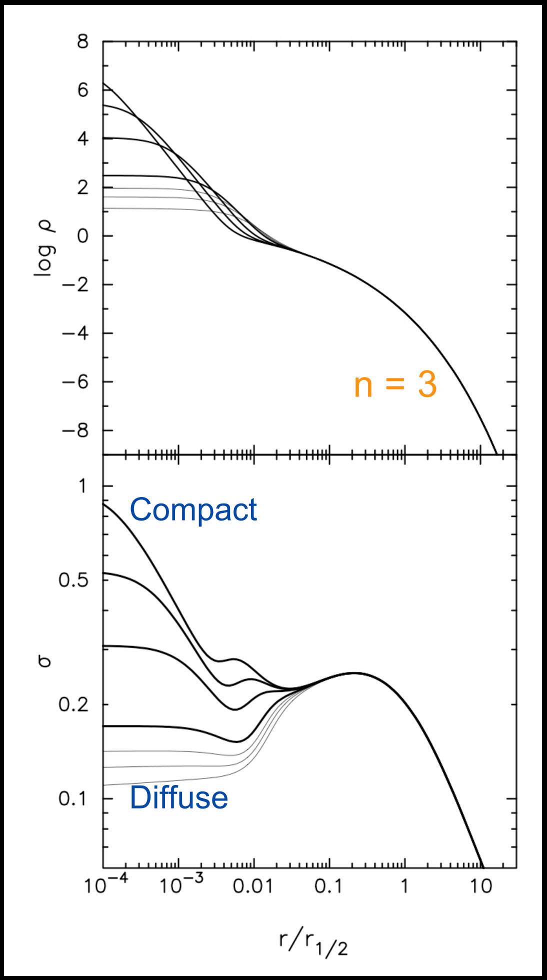


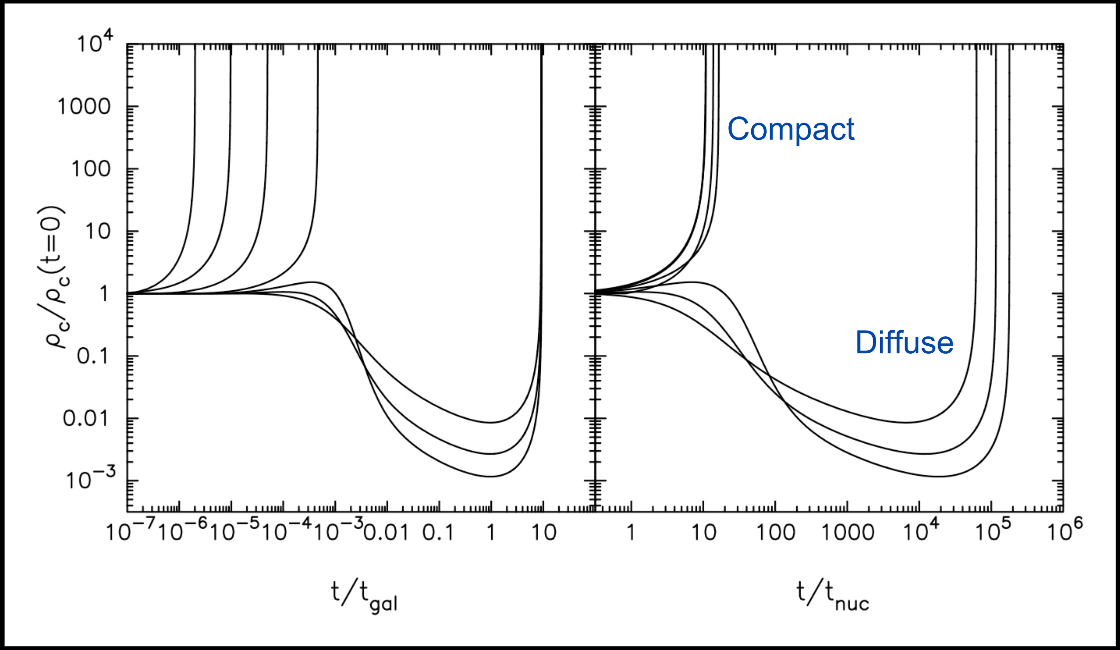
Milosavljevic 2008

Dynamical Modelling Methods: Comparison

•Fokker-Planck (direct or M.C.)	+ Efficient when modelling systems with high symmetry - Orbit-averaged form is a kludge - Complex to code and slow in the case of asymmetrical systems
•Fluid-Dynamical	+ Relatively efficient + Not restricted to symmetrical systems - Requires closure conditions
•N-Body	+ Exact! + Symmetry of problem irrelevant - Very compute-intensive

Nuclear Core Collapse
(no black holes!)

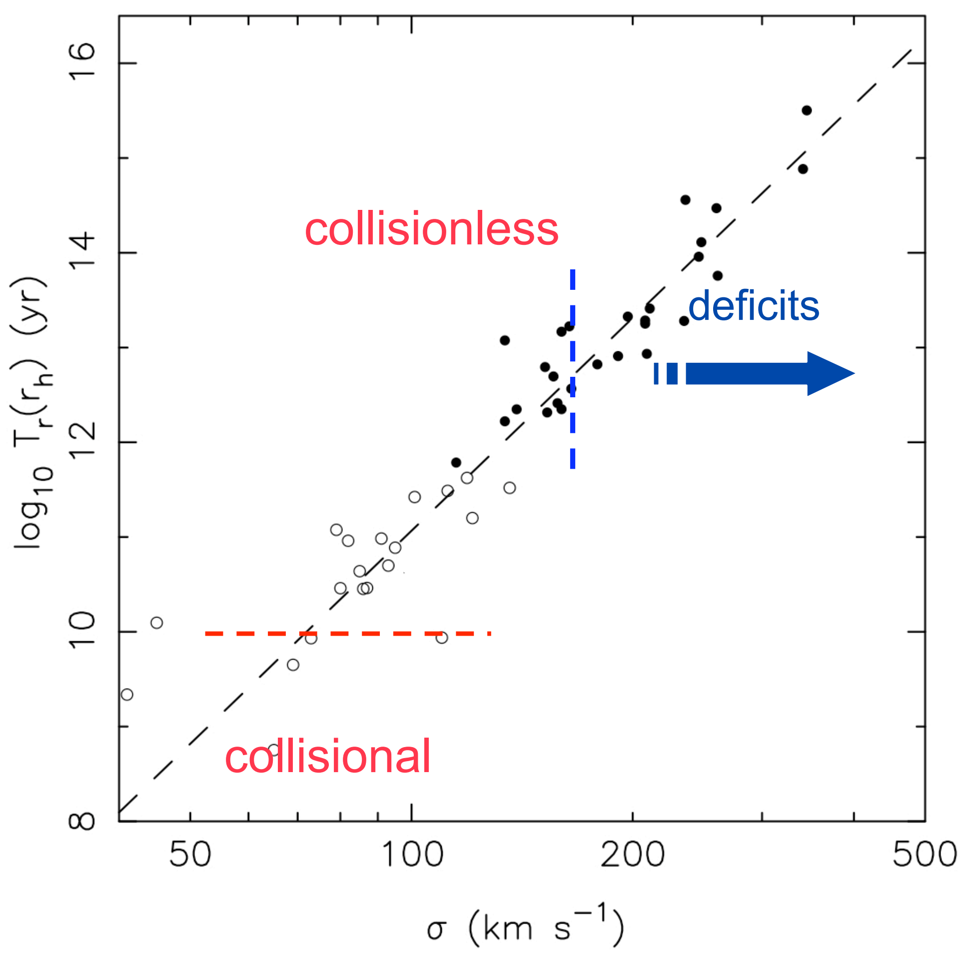




Evolution of the central density, for compact and diffuse nuclei.

(Isotropic, orbit-averaged, Fokker-Planck integration)

Nuclear relaxation times again (black holes are back in…)



Relaxation times in bright galaxies are **very long.**

Bright spheroids: “collisionless”

Faint spheroids: “collisional”

Bahcall-Wolf Solution

Two-body encounters lead to a redistribution of stars in energy space:

$$\frac{\partial f}{\partial t} = -\frac{\partial F_E}{\partial E},$$

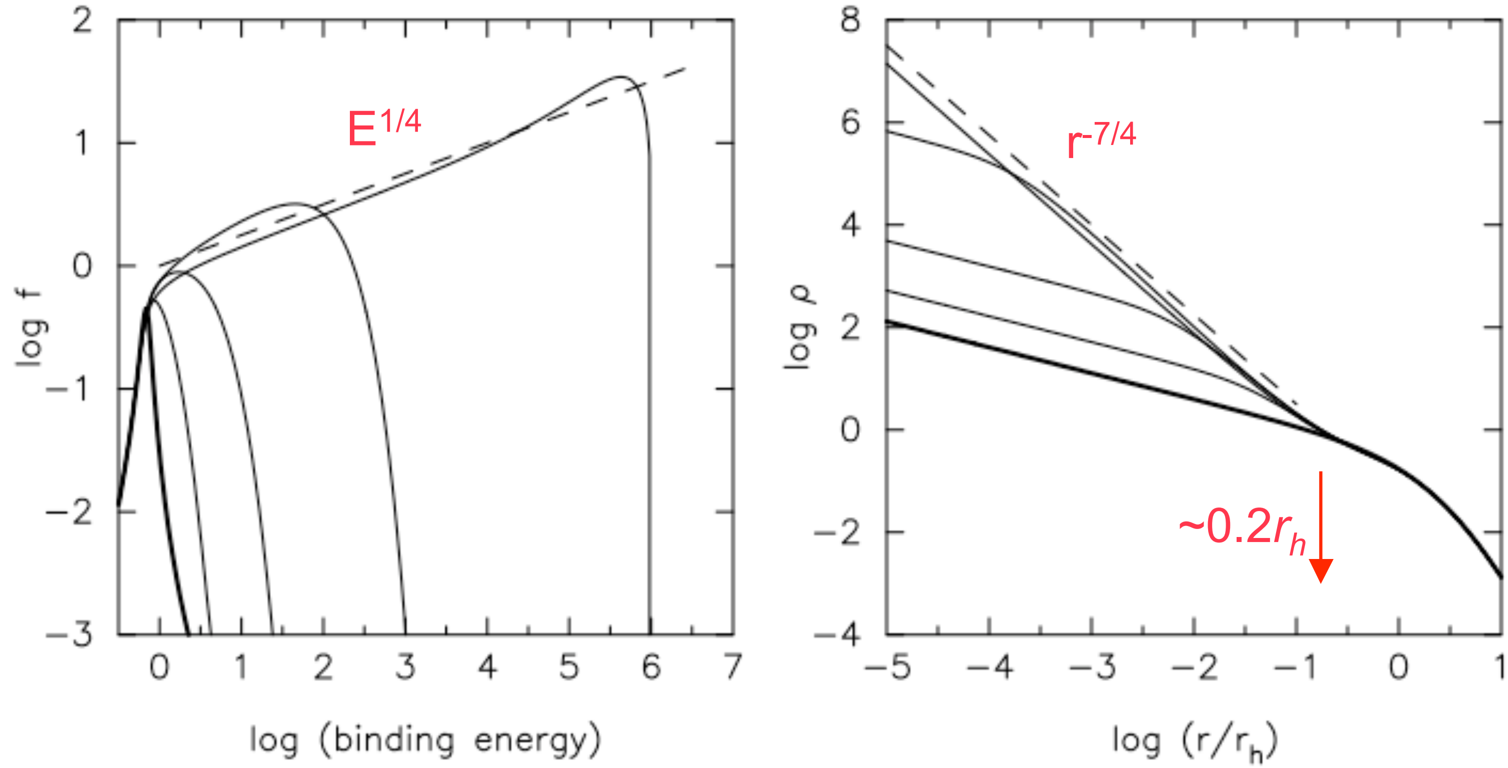
$$F_E = -D_E f - D_{EE}\frac{\partial f}{\partial E}$$

The most relevant solution is F_E= 0 ("zero flux"), which implies, in the potential of the BH:

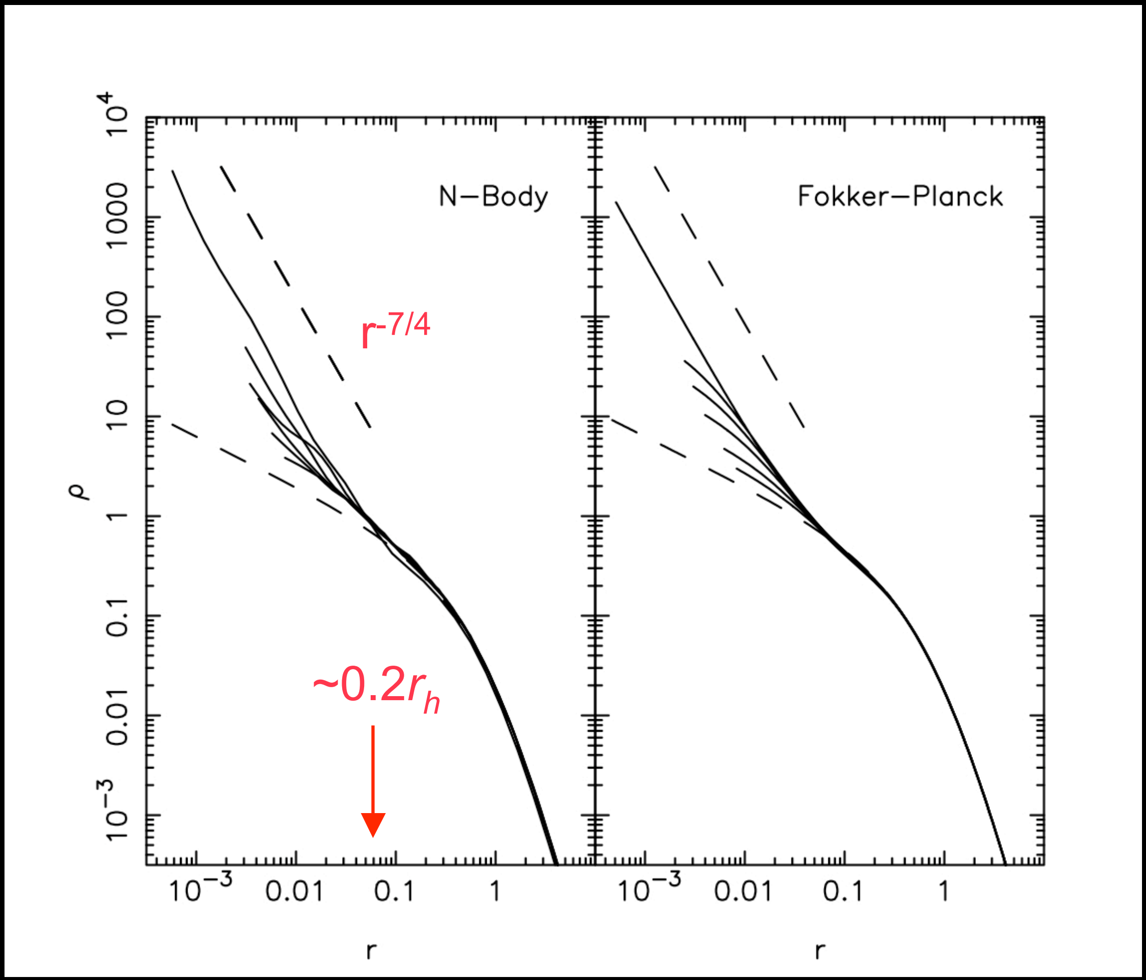
$$f \approx f_0 |E|^{1/4},$$

$$\rho \approx r^{-7/4}$$

The exact solution has $F_E \approx 0$; the flux is limited by the rate at which stars diffuse into the black hole.



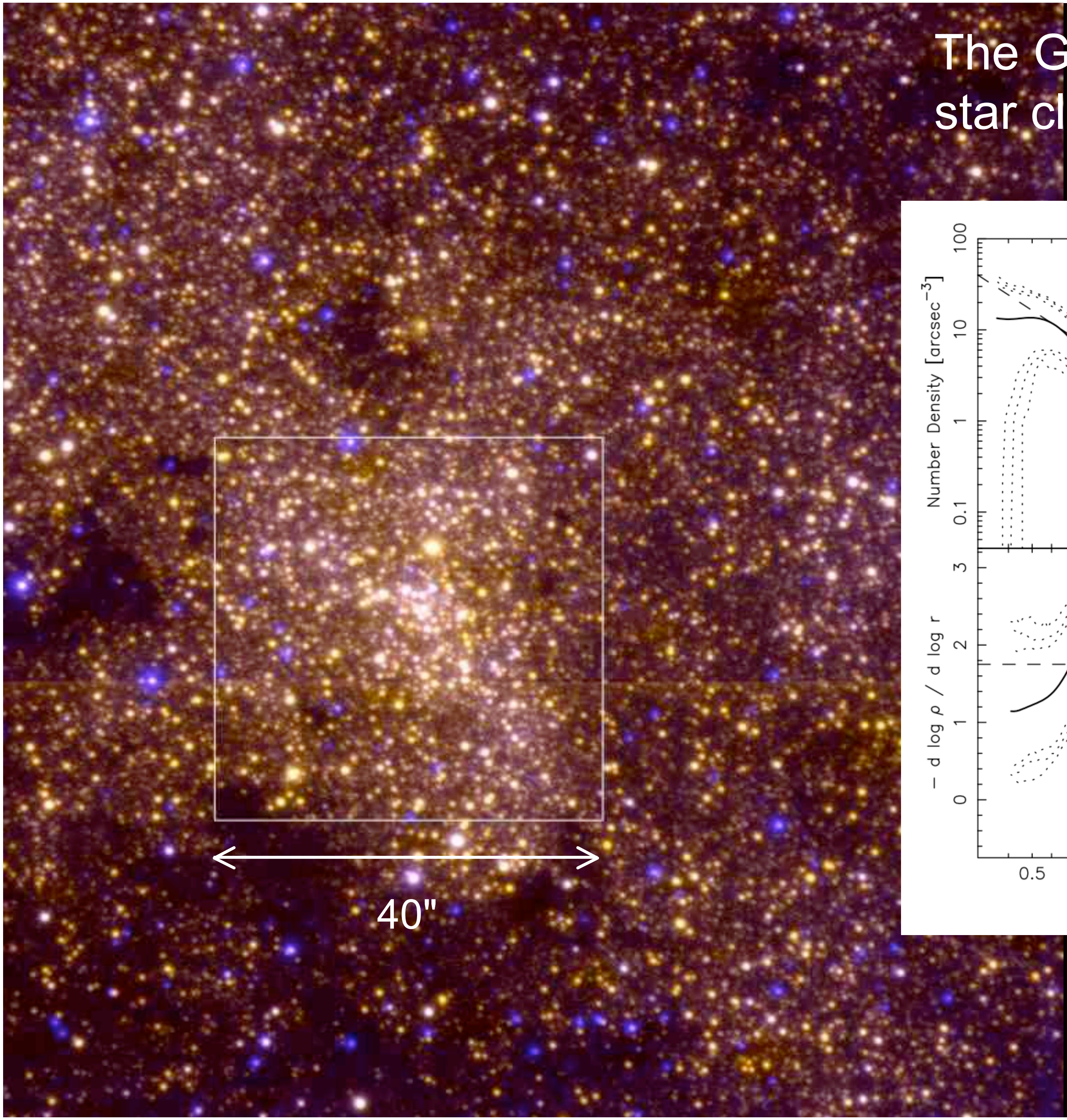
Radius of cusp ~ 0.2 r_h

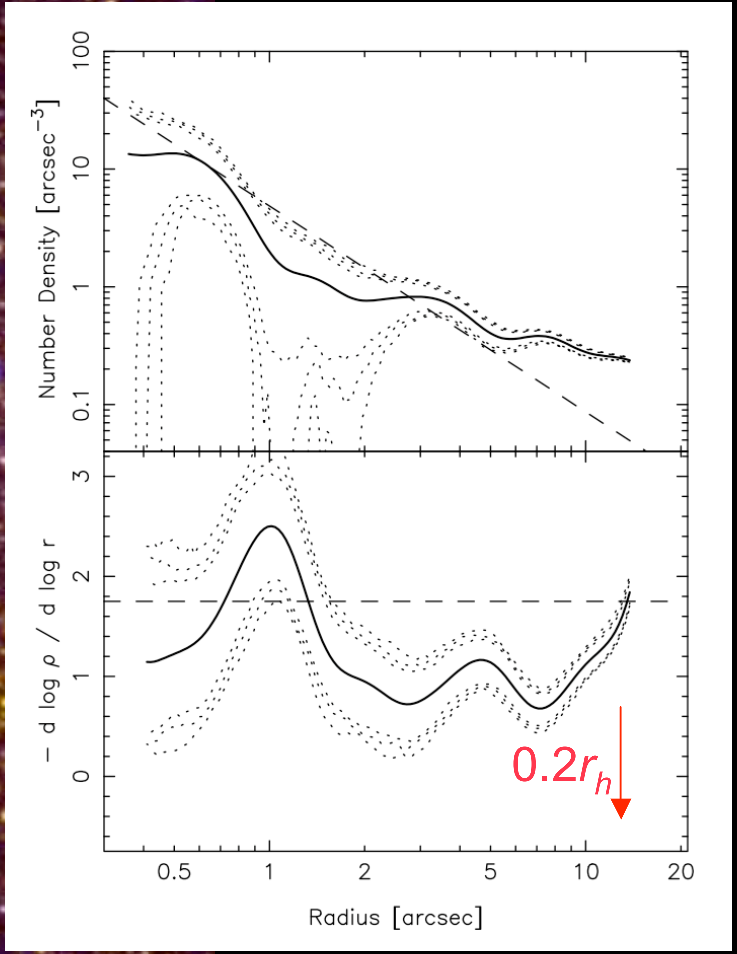


N-body growth of Bahcall-Wolf cusp.

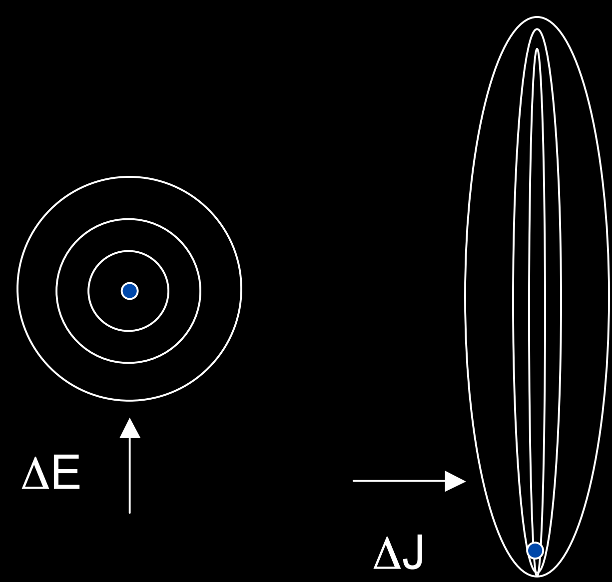
Preto et al. 2004

The Galactic center star cluster.





Schödel et al. 2007



In fact, loss of stars into the black hole is dominated by changes in J, not E.

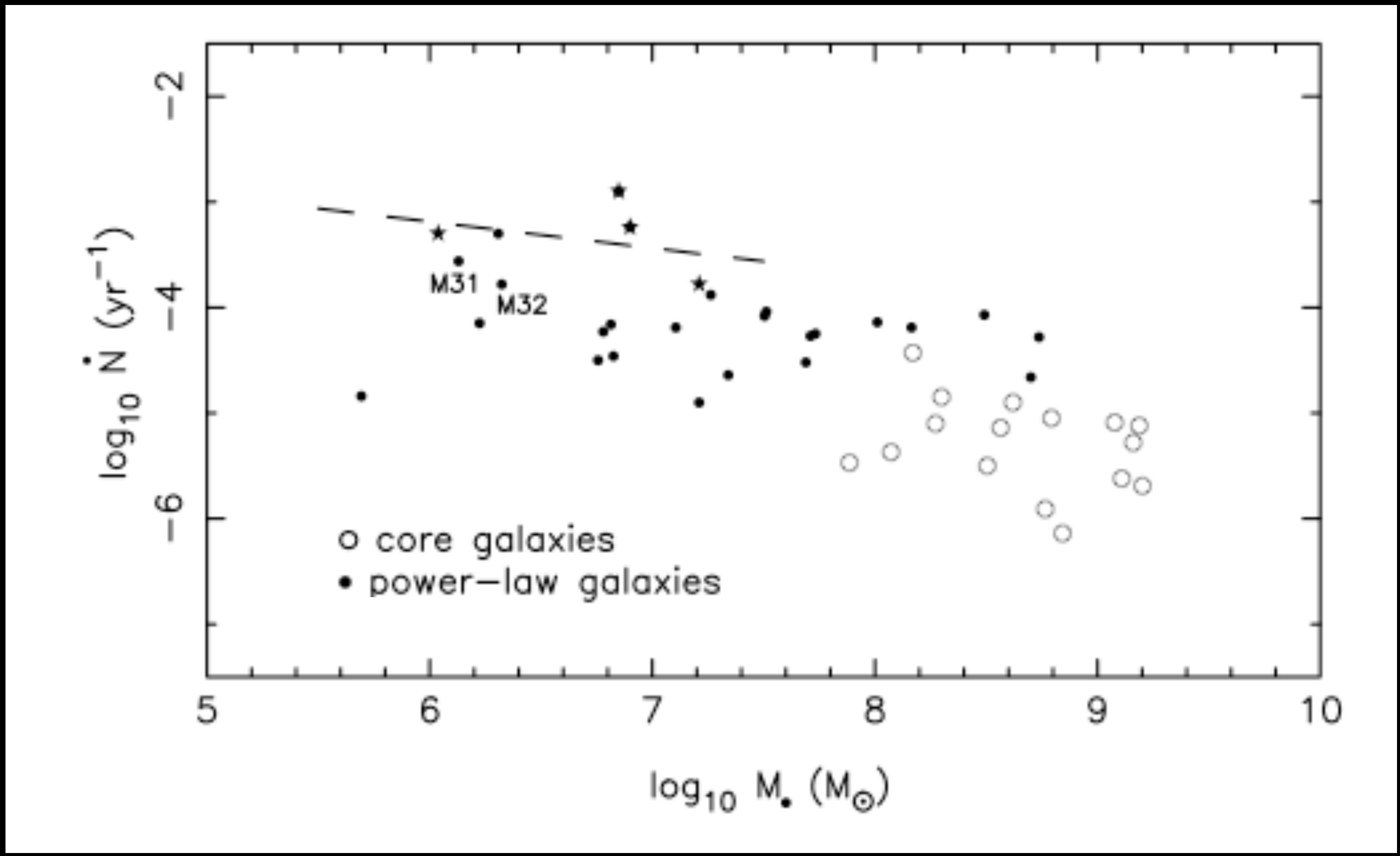
Write this loss term as $F_J(E)$. Then:

$$\frac{\partial f}{\partial t} \approx -\frac{\partial F_E}{\partial E} - \mathrm{F}_J(E)$$

$F_J(E)$ is “large”, in the sense that a mass $\sim M_{\mathrm{BH}}$ should be scattered into the black hole in a time $\sim T_R$:

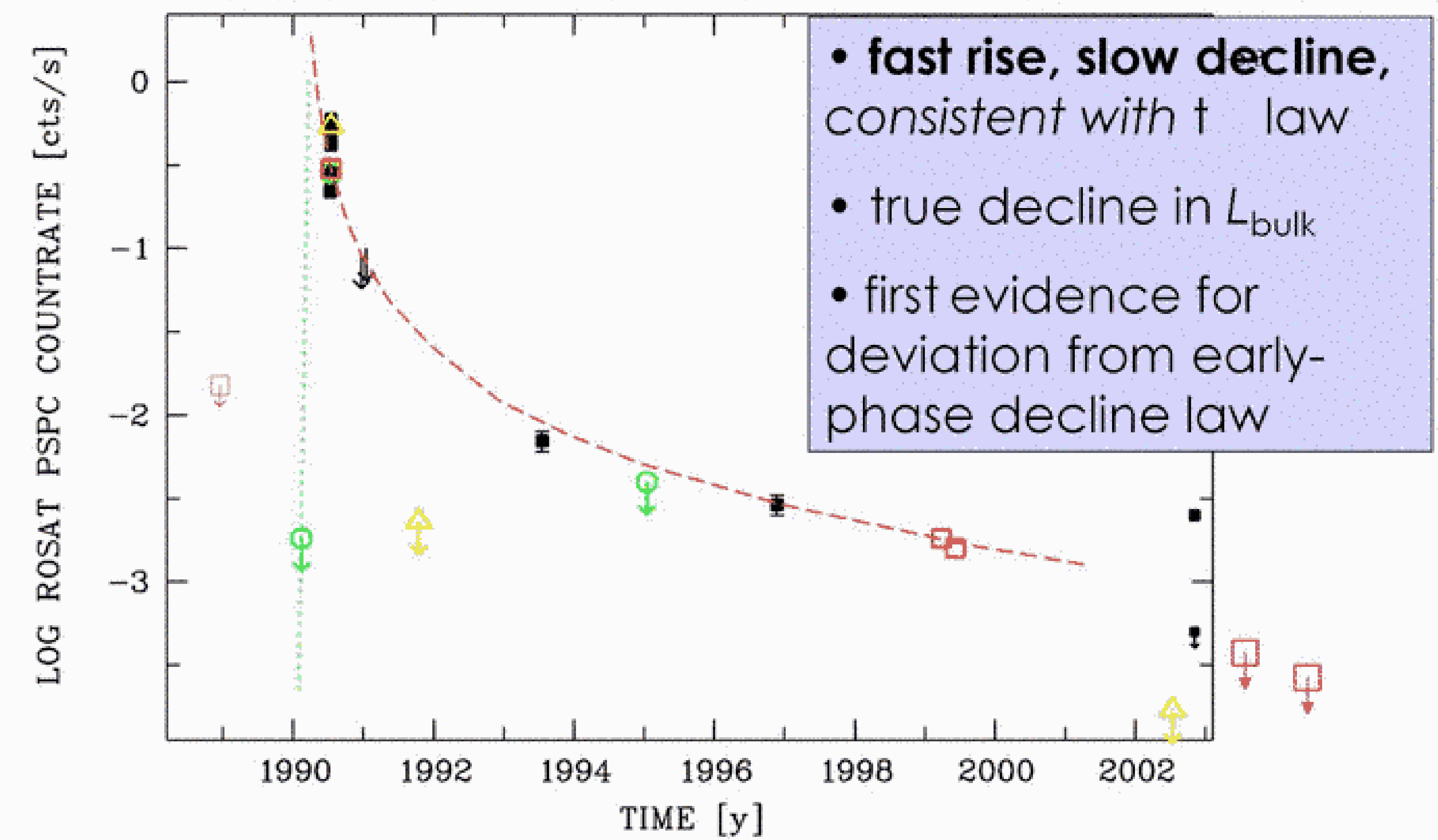
$N \approx M_{BH} / [T_R \ln (r_t/r_h)]$

Stellar Disruption Rates



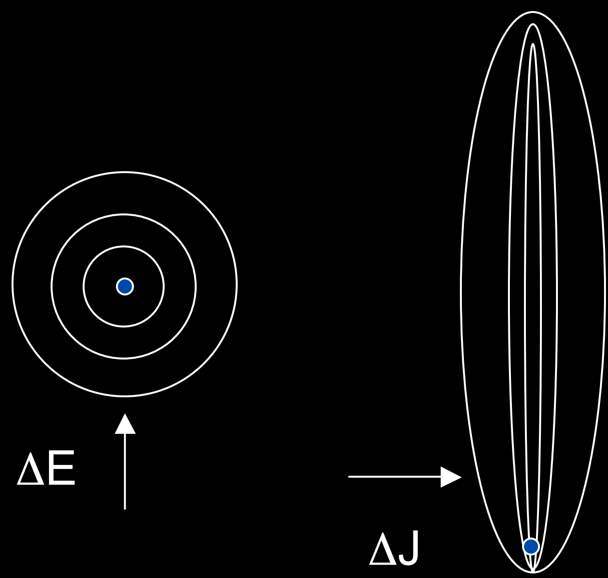
Wang & Merritt 2004

Tidal Disruptions Observed?



collective lightcurve, measured with *ROSAT* (<1997), *XMM* and *Chandra*, shifted to the same t_{max}. (dashed: $t^{-5/3}$ law, RXJ1242-11)

Komossa 2006



In fact, loss of stars into the black hole is dominated by changes in *J*, not *E*.

Write this loss term as $F_J(E)$. Then:

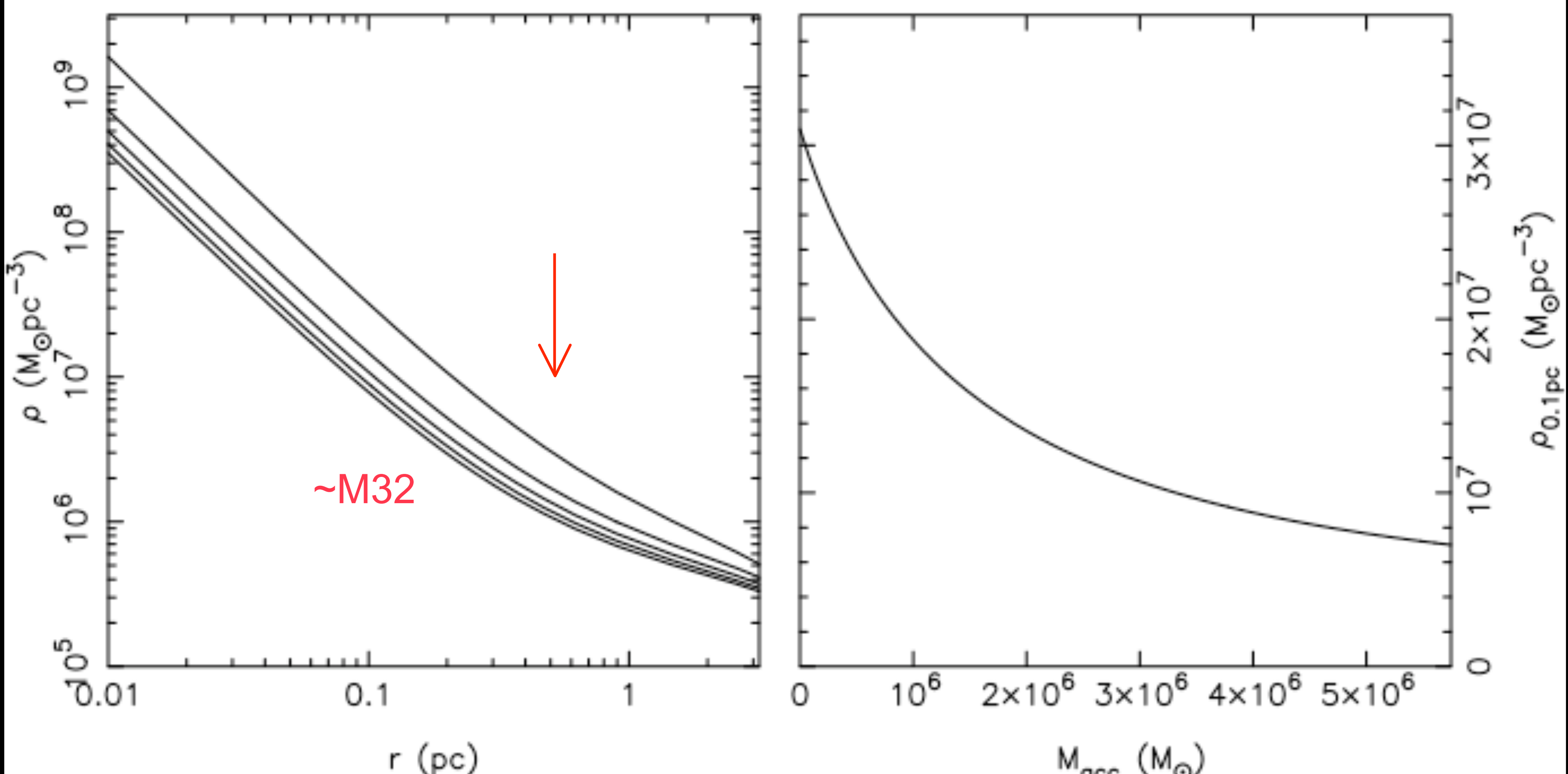
$$\frac{\partial f}{\partial t} \approx -\frac{\partial F_E}{\partial E} - \mathrm{F}_J(E)$$

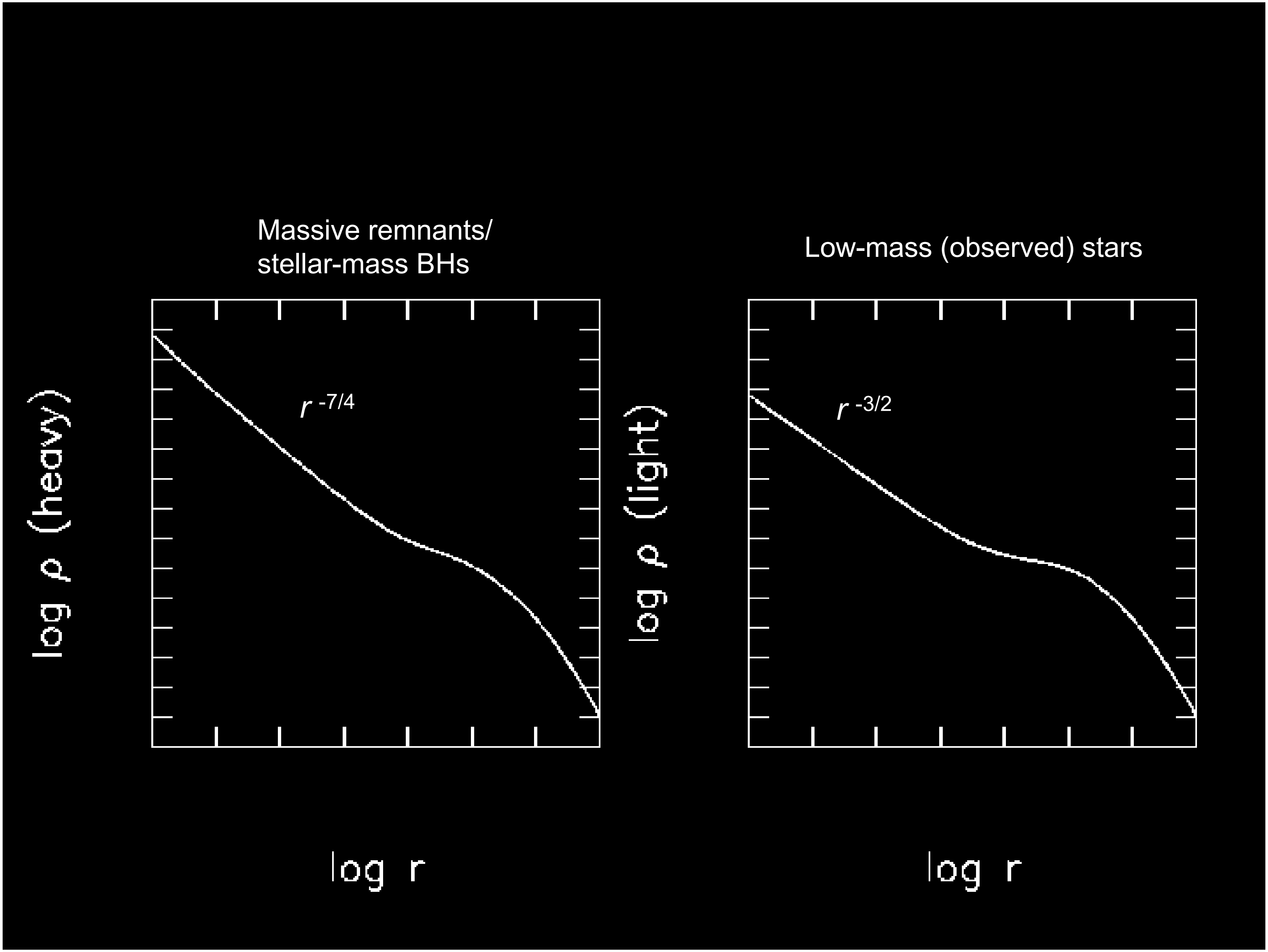
and a steady state requires:

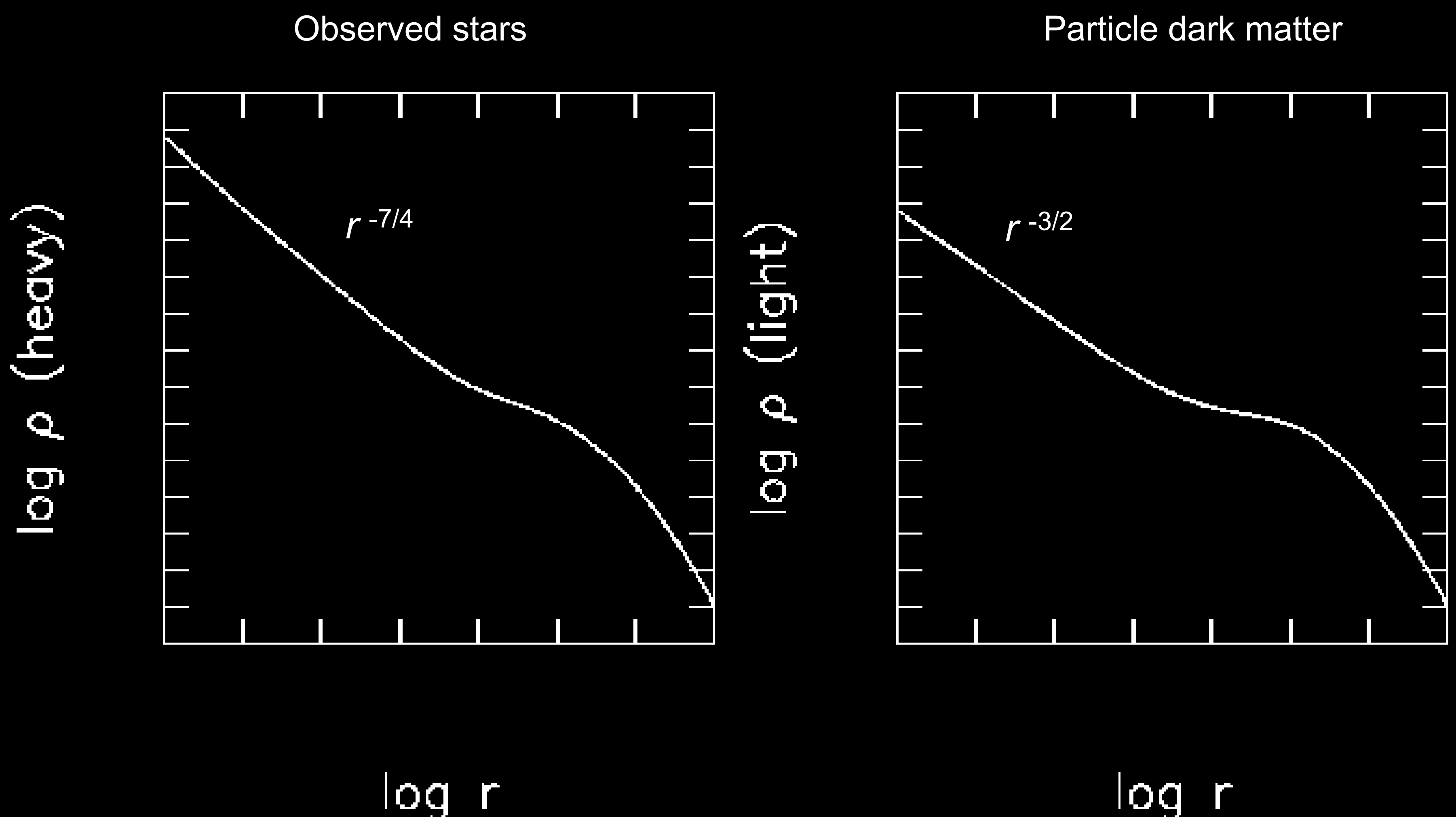
$$F_E \approx -\int \mathrm{F}_J dE,$$

i.e. the loss $\int F_J\, dE$ into the black hole must be balanced by “downward” diffusion in energy.

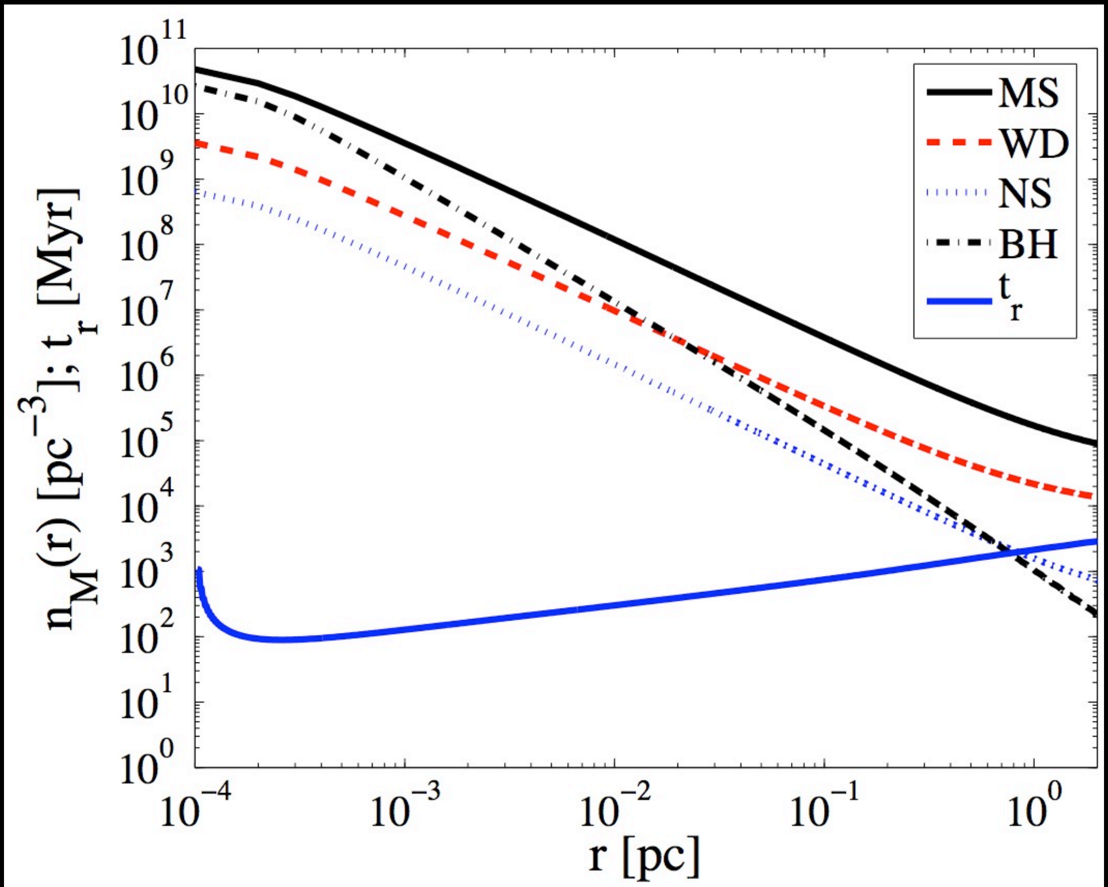
Nuclear Expansion due to a Black Hole




Massive remnants/
stellar-mass BHs
Low-mass (observed) stars
$r^{-7/4}$
$r^{-3/2}$
log ρ (heavy)
log ρ (light)
log r
log r


Observed stars
Particle dark matter
log ρ (heavy)
log ρ (light)
r -7/4
r -3/2
log r
log r

Galactic Center Mass Segregation



Density profiles of stars, stellar-mass BHs near the GC SMBH.

Hopman & Alexander 2006

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•Fluid-Dynamical	+ Relatively efficient + Not restricted to symmetrical systems - Requires closure conditions
•N-Body	+ Exact! + Symmetry of problem irrelevant - Very compute-intensive

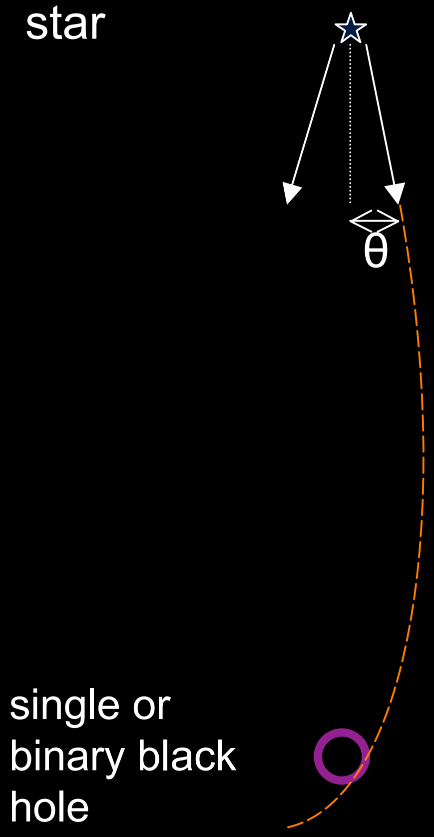
What Values of *N* are Required?

N fixes the ratio of **relaxation time** to **crossing time**:

N	T_{relax}/T_{cross}
10^2	2.2
10^3	14.5
10^4	109
10^5	870
10^6	7250
10^{11}	3.9×10^8

A physical scaling that depends on the separation of the two time scales, requires large *N*.

In loss-cone problems, this requirement is more severe.



Stars are scattered by other stars into the loss cone, where they can interact with the central object(s).

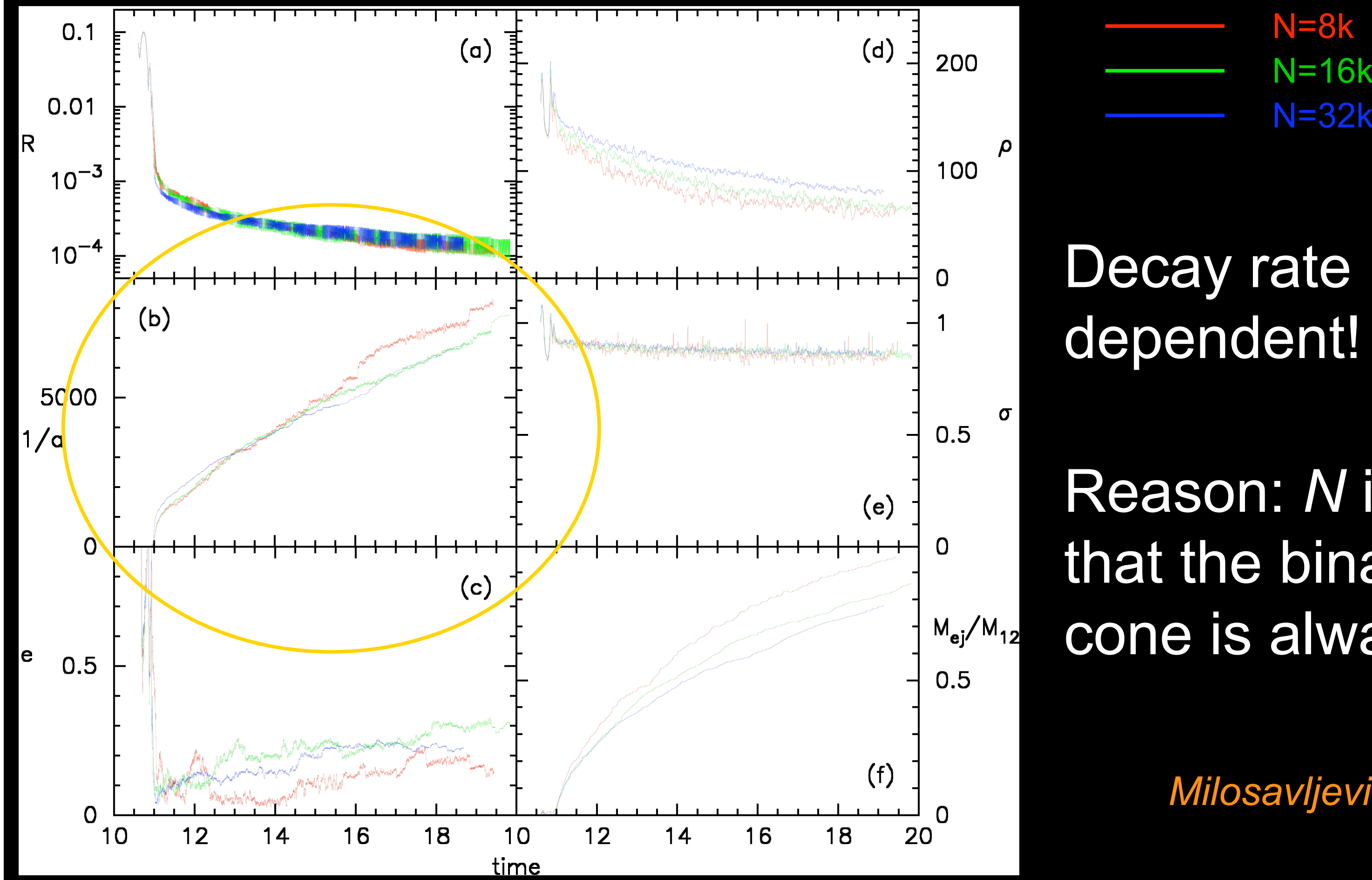
Scattering time is

$$\sim\theta^2 T_{relax} << T_{relax}$$

and separation of the two time scales requires

$$T_{relax} >> \theta^{-2} T_{cross}$$

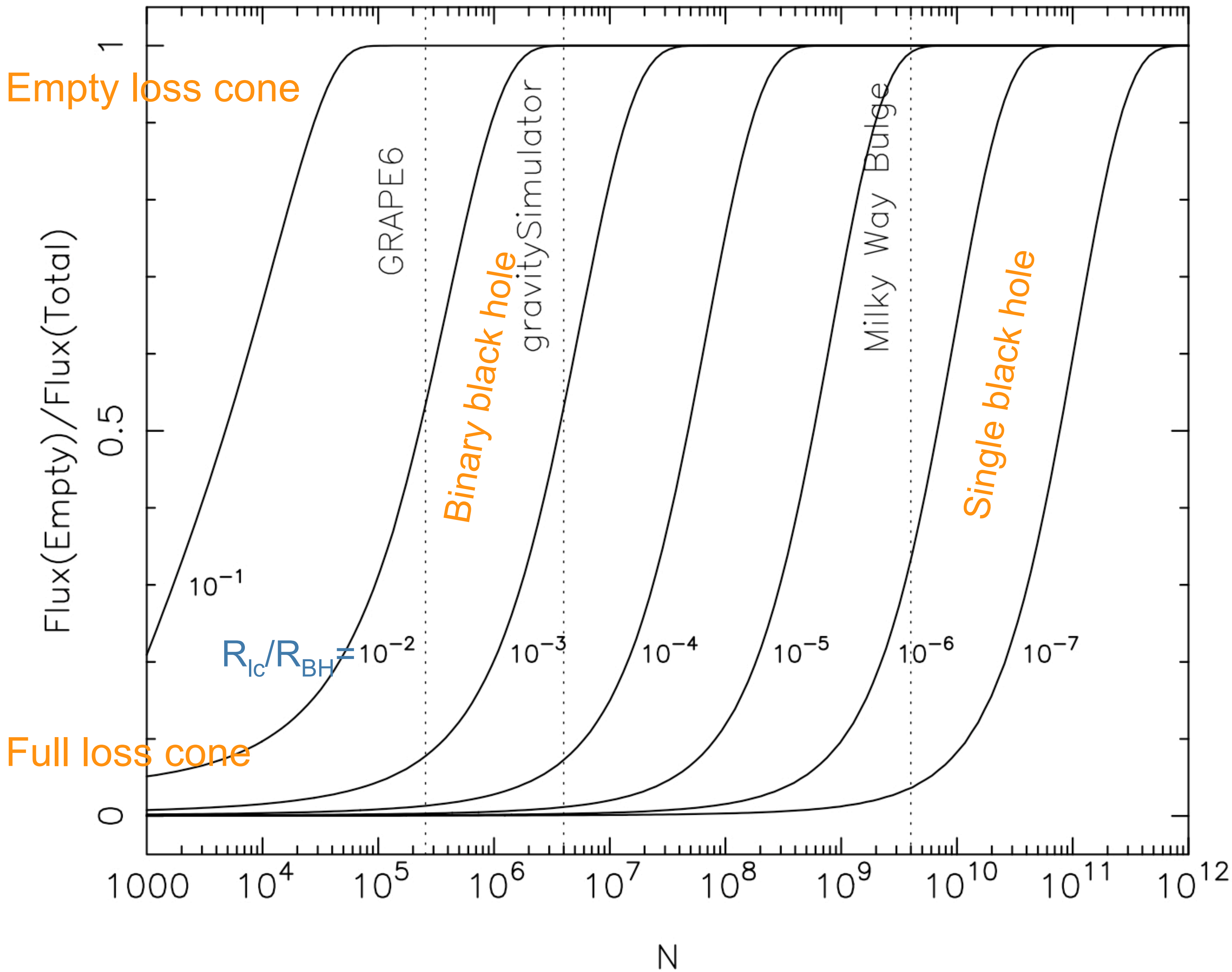
N-body Integration of Binary Black Hole



Decay rate is ***not*** *N*-dependent!

Reason: *N* is so small that the binary's loss cone is always full.

Milosavljevic & Merritt 2001



Empty loss cone
Full loss cone
Flux(Empty)/Flux(Total)
1
0.5
0
GRAPE6
gravitySimulator
Milky Way Bulge
Binary black hole
Single black hole
$R_{lc}/R_{BH}=10^{-2}$
10^{-1}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
1000
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
N