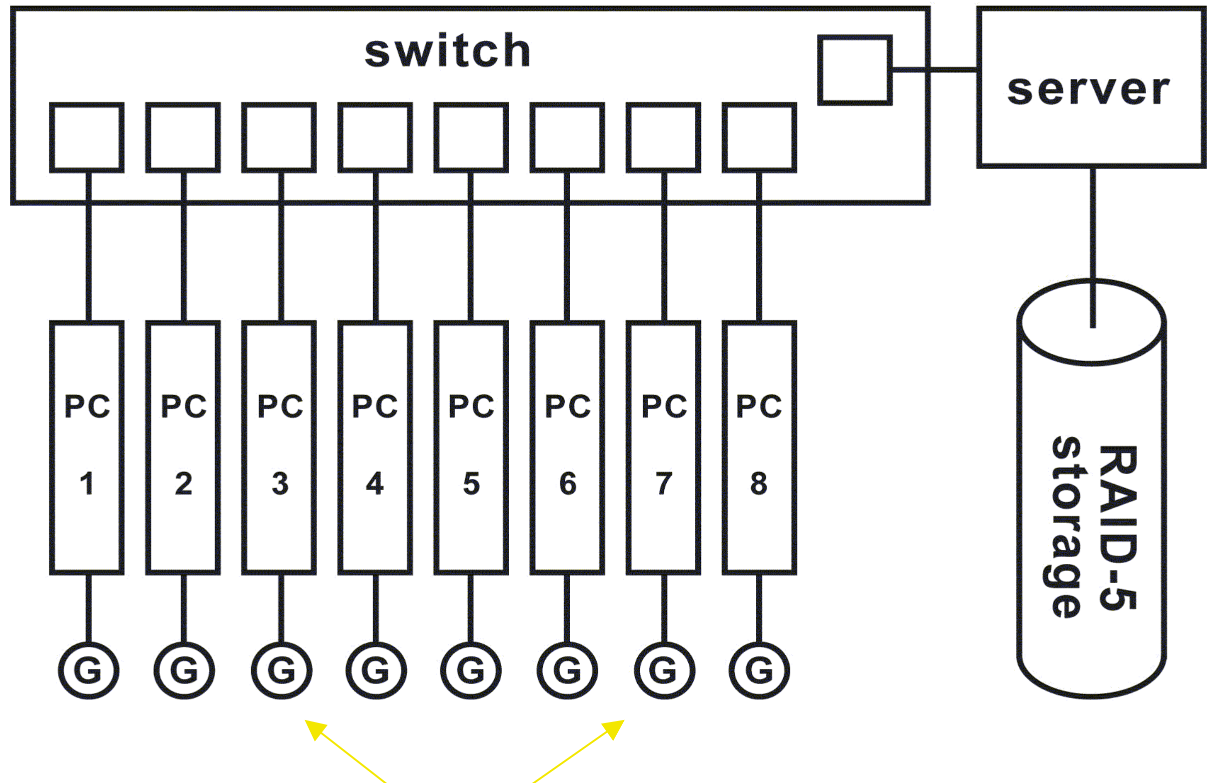
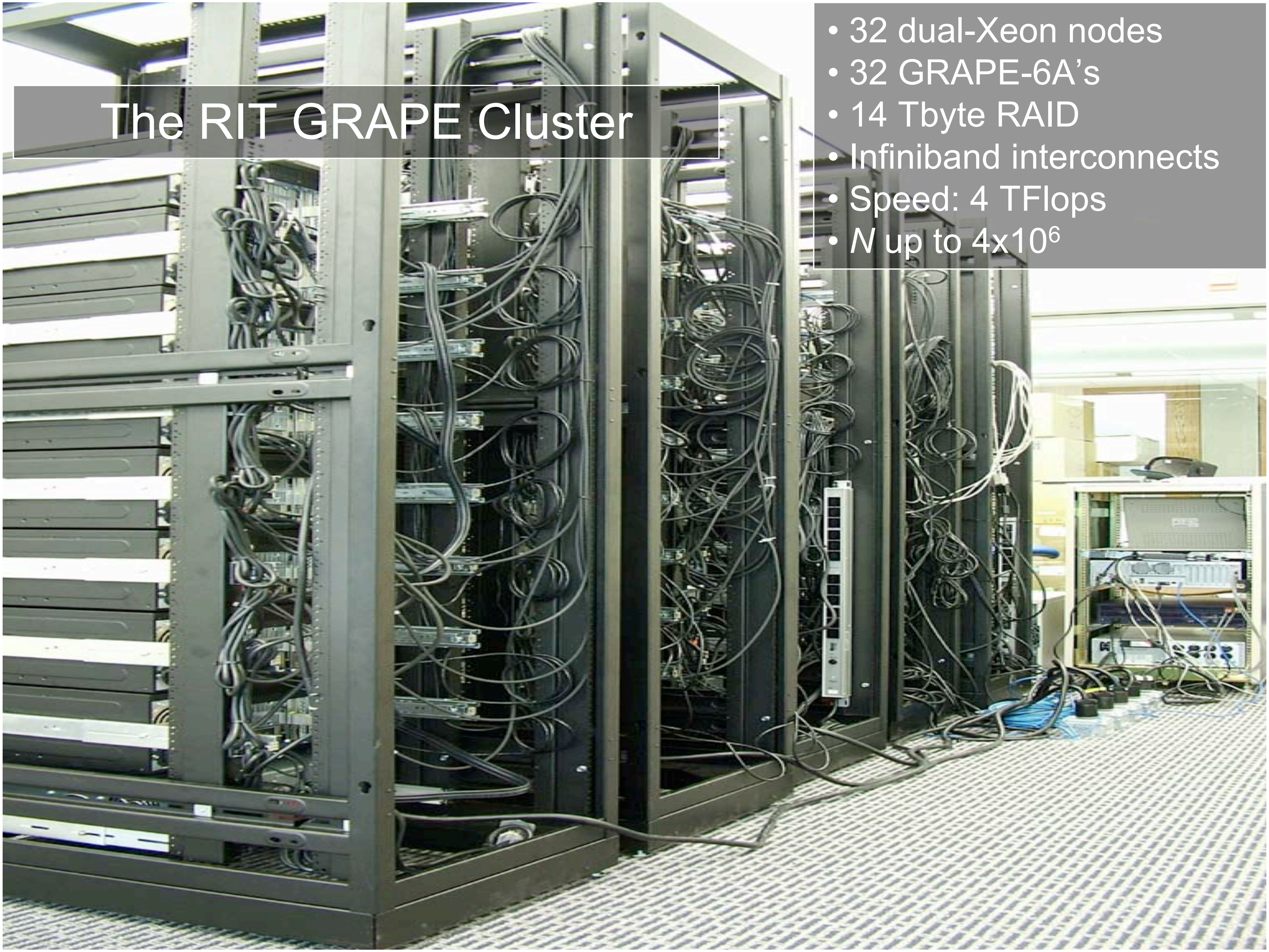
Solution (ca. 2004): The GRAPE Cluster



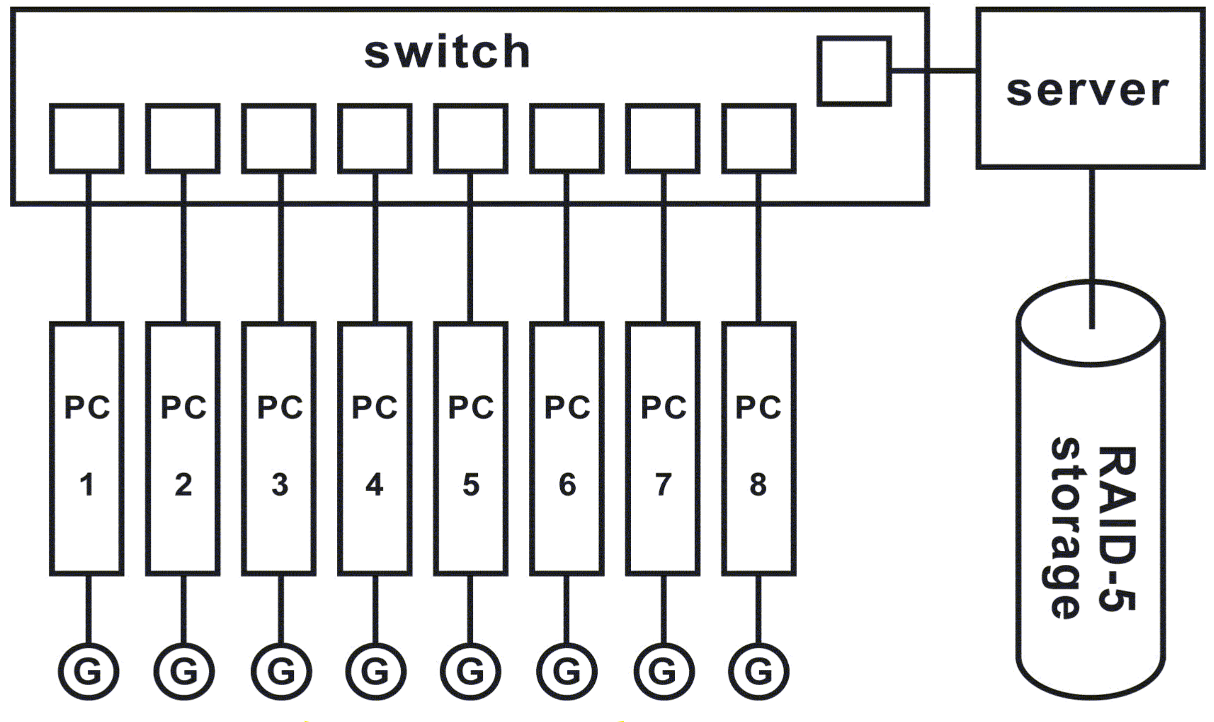
mini-GRAPEs
(GRAPE-6A)



The RIT GRAPE Cluster

- 32 dual-Xeon nodes
- 32 GRAPE-6A’s
- 14 Tbyte RAID
- Infiniband interconnects
- Speed: 4 TFlops
- *N* up to $4x10^6$

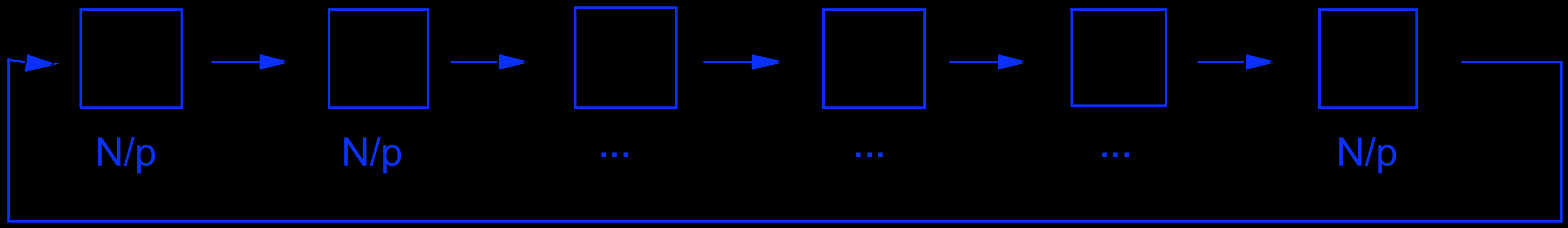
Solution (ca. 2008): The GPU Cluster



GPUs
(NVIDIA)

Parallel Algorithms

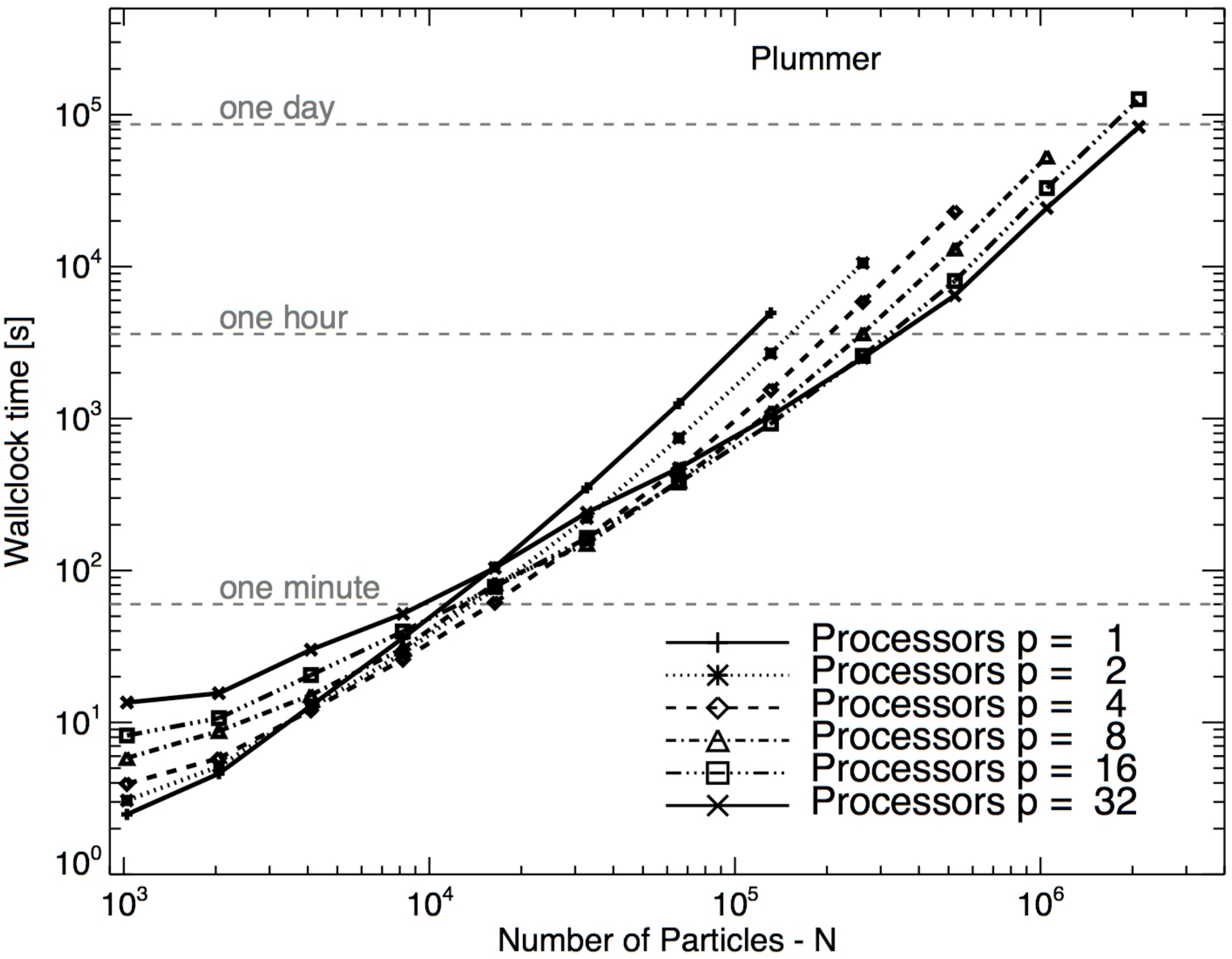
- Spread the *N* particles evenly among *p* processors
- Shift particles cyclically; compute and store partial forces ("systolic" algorithm)
- Repeat *p* times
- Collect and sum forces



Dorband, Hemsendorf & Merritt 2003
Makino 2003
Harfst et al. 2007

Performance of “φGRAPE”

(Harfst et al. 2007)



Schemes for “regularizing” close interactions with a BH (or BHs):

- Kustaanheimo-Stiefel (KS) regularization

  Coordinate transformation that effectively converts the Kepler problem to the simple harmonic oscillator. Often combined with a chain routine for multiple particles.

- Wheel-spoke regularization (WS)

  An alternative to the chain when there is a single dominant body. Treats every interaction with the BH via KS regularization while other interactions use a small softening.

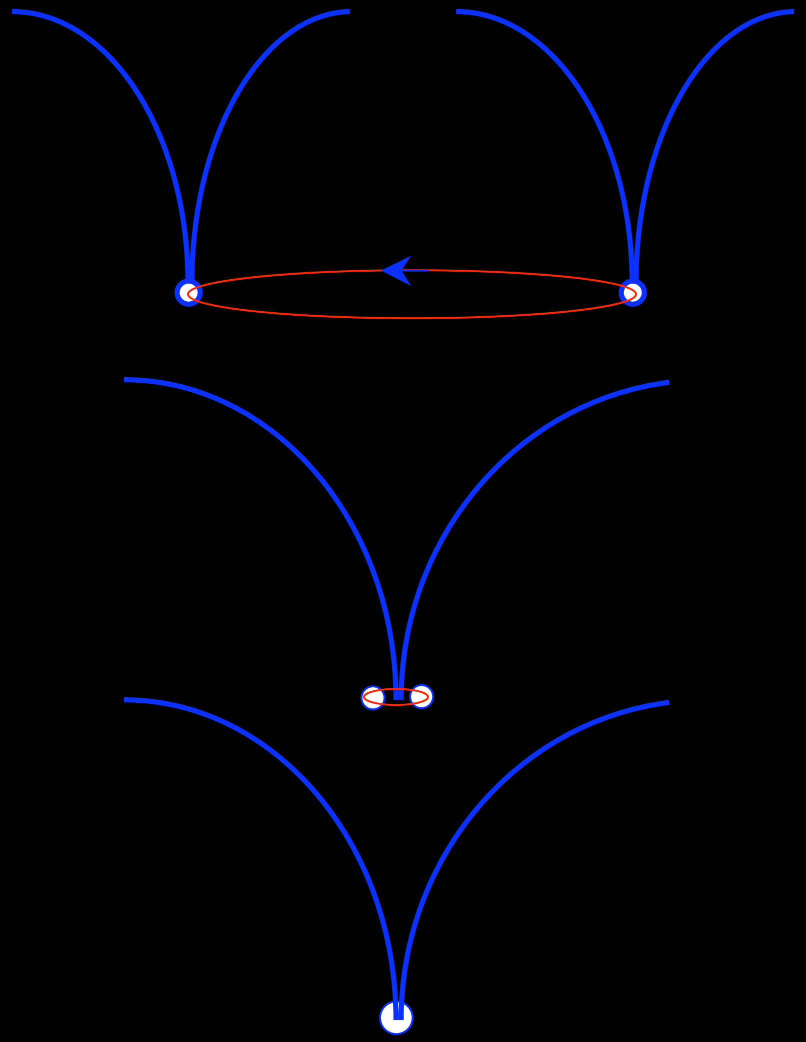
- Algorithmic regularization (AR)

  Removes singularities with a time transformation. Two versions:

  -- Logarithmic Hamiltonian (LogH)
  -- Time-transformed leapfrog (TTL)

Aarseth 2007

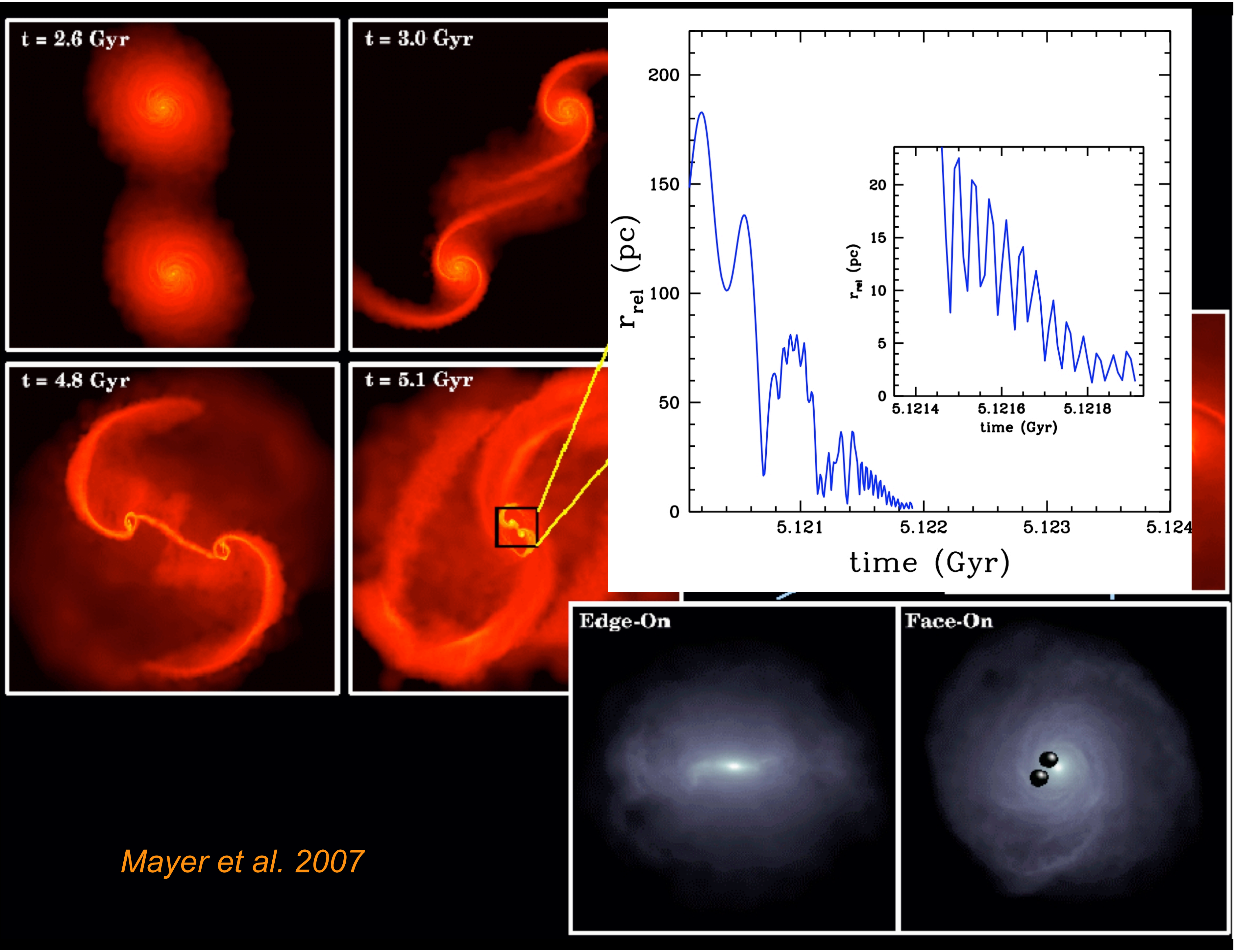
Bringing Them Together



Galaxies merge

Binary forms

Binary decays (?), via:
-- ejection of stars
-- interaction with gas



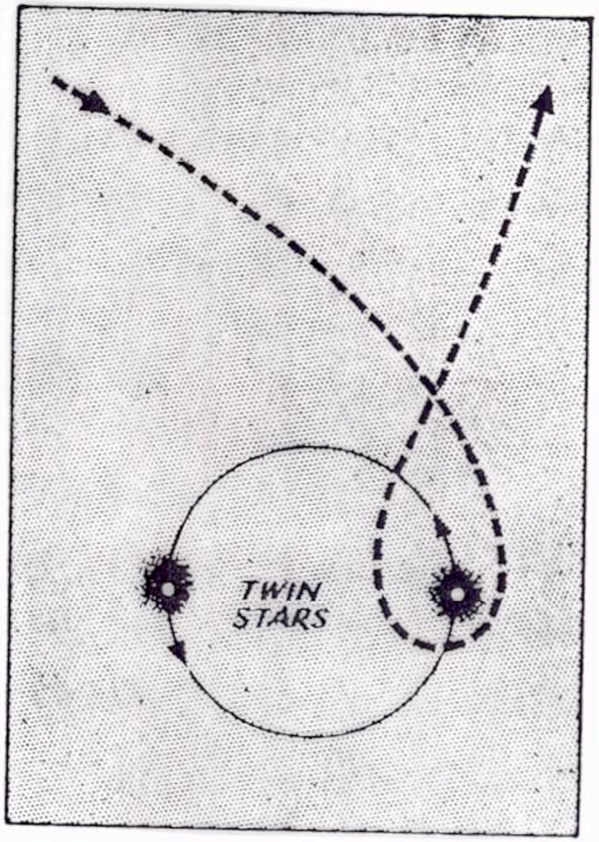
Mayer et al. 2007

Theory/Computation Challenges for LISA

(T. Prince, Dec. 14, 2003, CGWA)

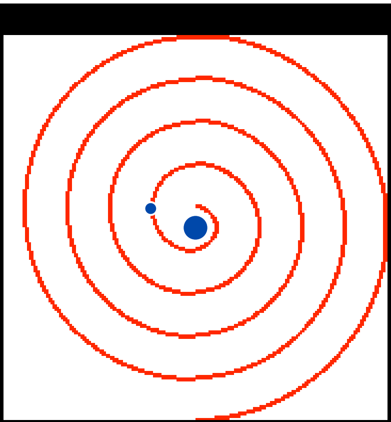
[As identified by the LISA International Science Team – March, 2002]

1) **Understanding the formation and evolution of nuclear star clusters around supermassive black holes**
2) Prediction of waveforms of compact objects spiraling into supermassive black holes
3) Development of methods for separating thousands of simultaneous wavetrains of diverse sources from a single time series
4) **Understanding the fate of merging supermassive black holes (the “final parsec problem”)**
5) Computing the emission from merging black holes
6) Predicting the stochastic primoridial background spectra due to inflation, phase transitions, brane worlds and other sources involving new physics
7) Understanding the astrophysics of tides and mass transfer in white-dwarf binaries



In Dyson's "gravity machine" an object is fired toward twin stars so that it circles the approaching star and is thrown back by that star's gravity, having gained much additional energy.

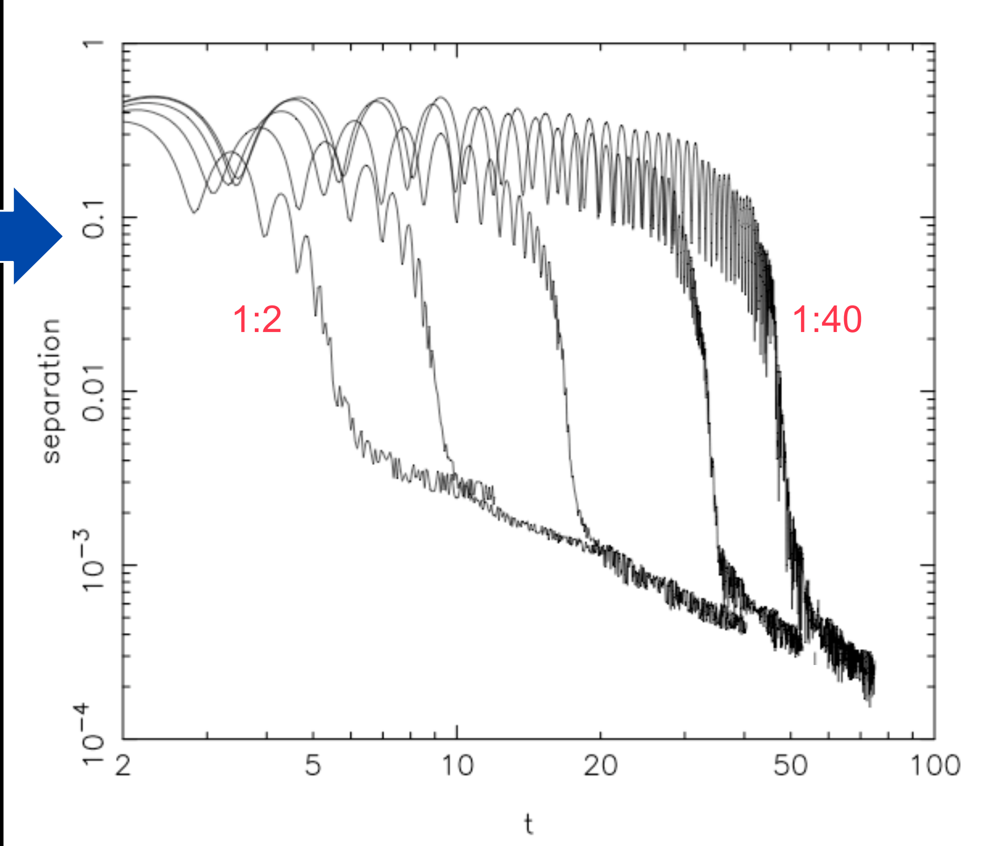
"Gravitational
Slingshot"

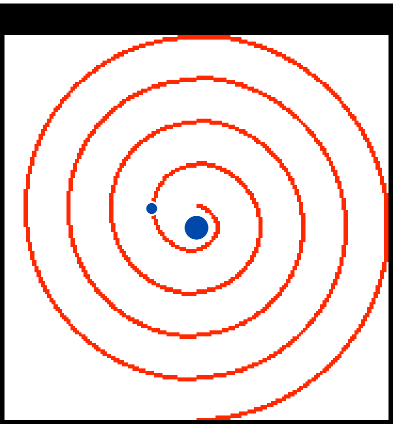


In-Spiralling Black Holes

$$r_h \approx \frac{GM_{12}}{\sigma^2}$$

$$\approx 1.54\ \mathrm{pc}\left(\frac{M_{12}}{3.5\times10^6\,M_{\mathrm{sun}}}\right)\left(\frac{\sigma}{100\ \mathrm{km\ s^{-1}}}\right)^{-2}$$



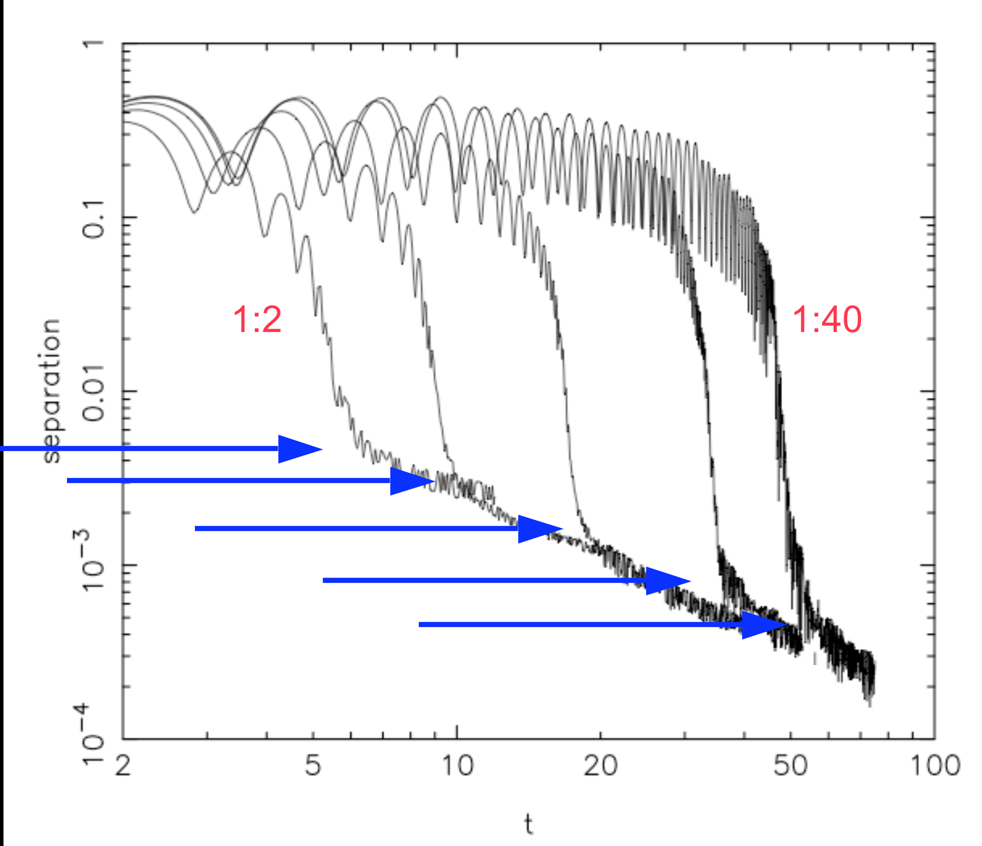


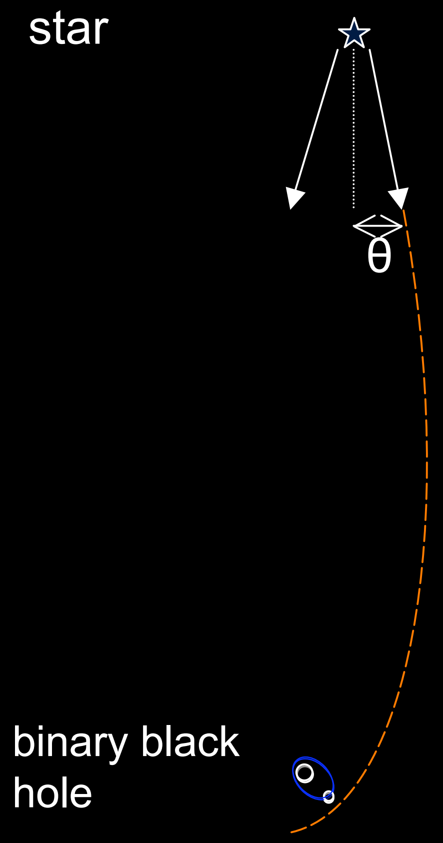
In-Spiralling Black Holes

$$a_h \approx \frac{GM_2}{4\sigma^2}$$

$$\approx 0.39\ \mathrm{pc}\ q \left(\frac{M_{12}}{3.5\times10^6 M_{\mathrm{sun}}}\right)\left(\frac{\sigma}{100\ \mathrm{km\ s^{-1}}}\right)^{-2}$$

$$q \equiv M_2 / M_1$$

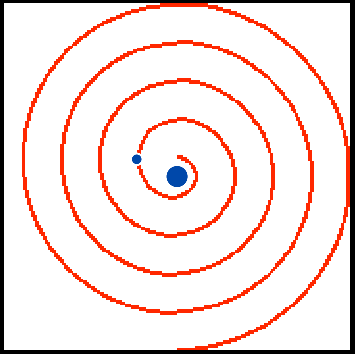




At the “hard binary” separation, the binary efficiently kicks out all stars on loss-cone orbits.

This requires a **smaller separation** for less-massive black holes.

Subsequent evolution (I.e. shrinking) requires new stars to be scattered onto these orbits.

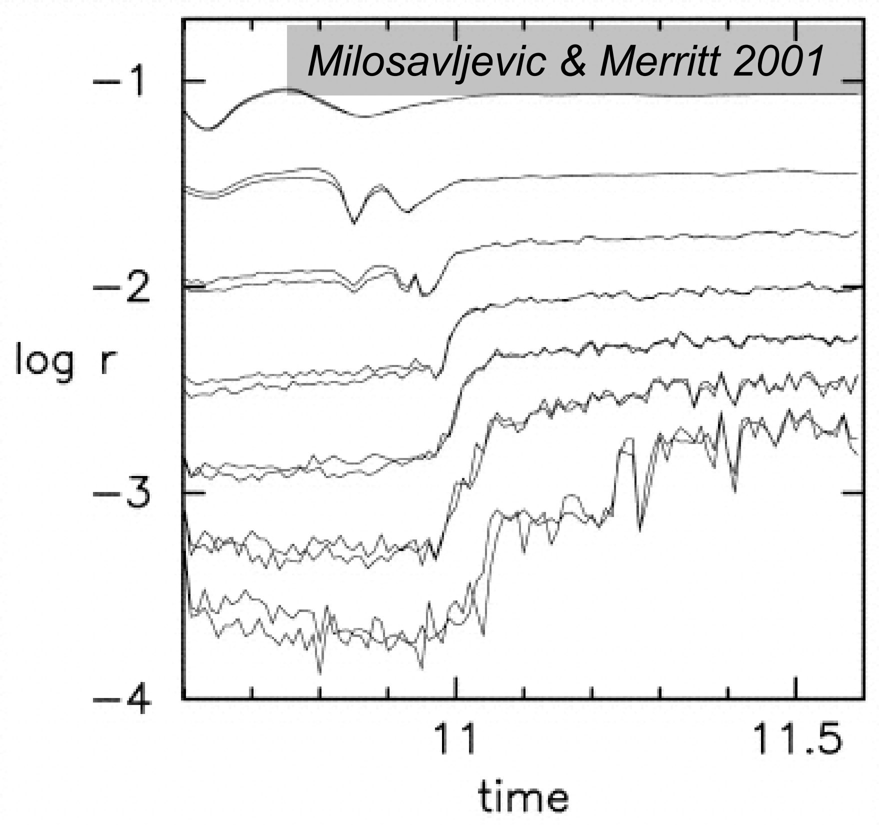


Binary SMBH forms by displacing stars.

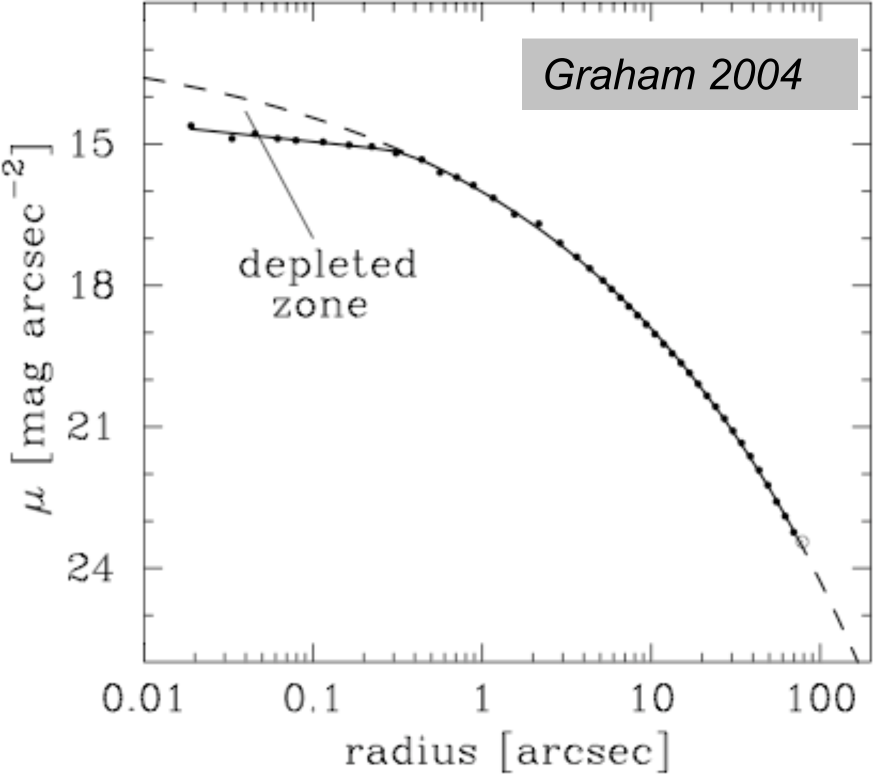
Energy released in reaching the “hard binary” separation, $a \approx a_h$, is:

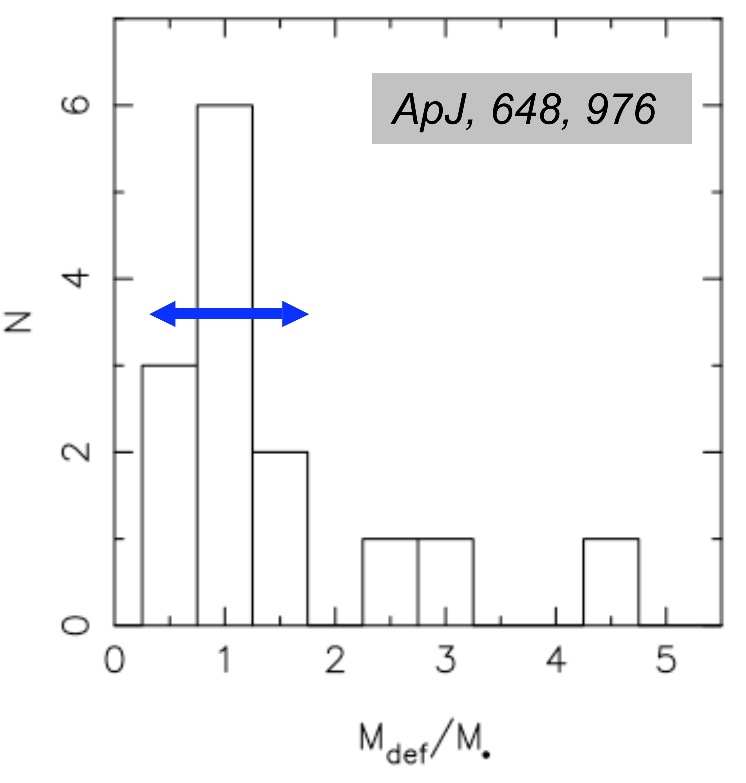
$$\begin{aligned} \Delta E &\approx -\frac{GM_1M_2}{2r_h} + \frac{GM_1M_2}{2a_h} \\ &\approx -\frac{1}{2}M_2\sigma^2 + 2M_{12}\sigma^2 \\ &\approx 2M_{12}\sigma^2 \end{aligned}$$

almost **independent** of the binary mass ratio M_1/M_2.



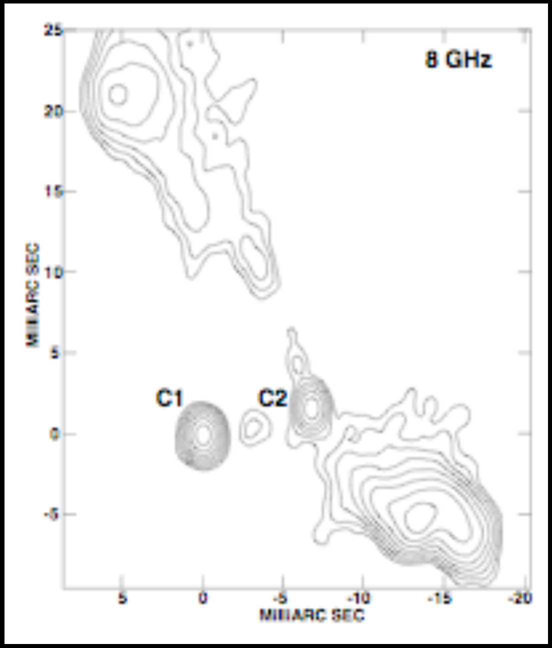
Mass Deficits





Milosavjlevic et al. 2002
Ravindranath et al. 2002

A Bona-Fide Binary Black Hole?

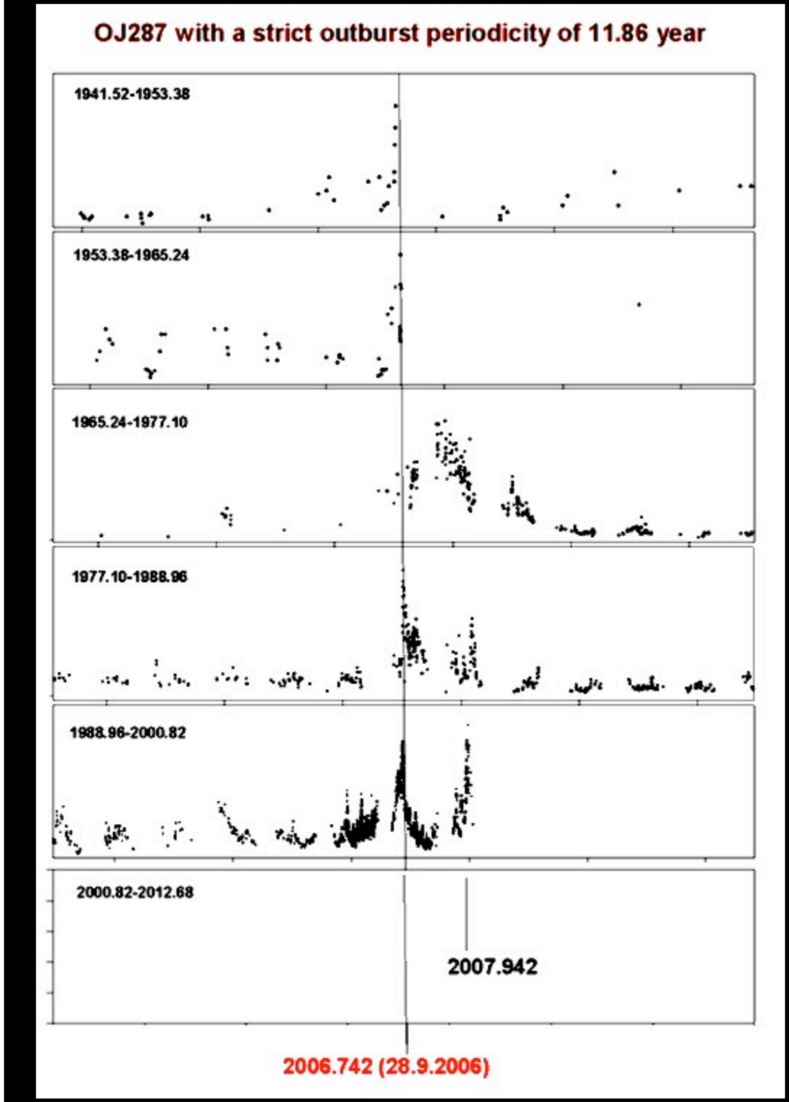


Rodriguez et al. 2006

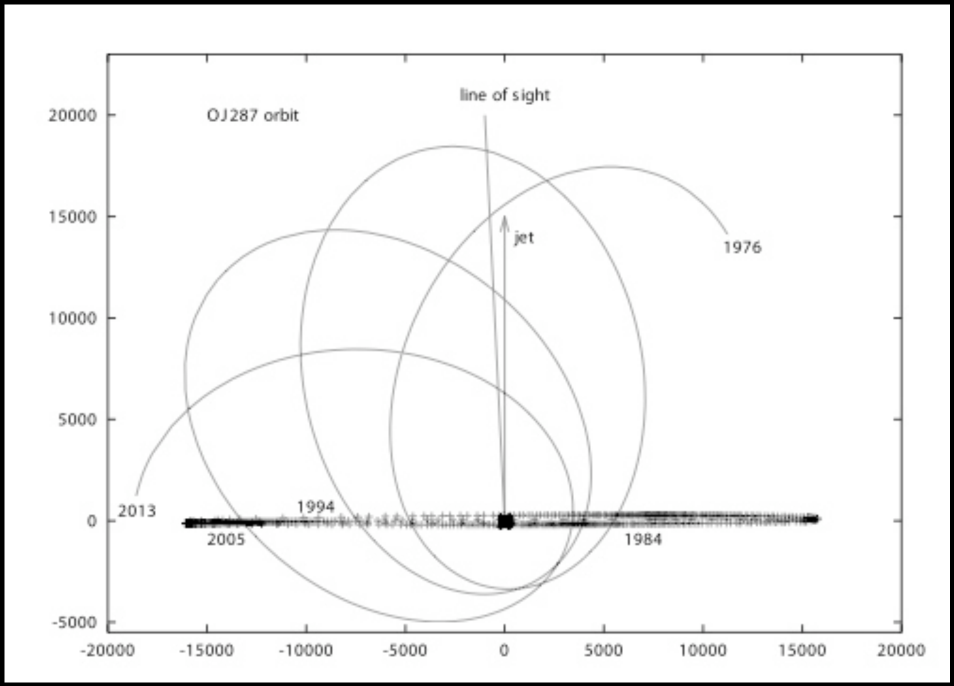
The observed (projected) separation of ~7 pc is the expected stalling radius for a $\sim 10^9 M_{sun}$ binary SMBH.

OJ 287

Valtonen et al. 2006, 2008



Light Curve



Precessing Orbit Model

Overcoming the “Final-Parsec Problem”

I.e. how to bring binary separations from ~ 1 pc down to ~0.001 pc

1. Allow the BHs to interact with gas
2. Prolong BH-star interactions, by…

    -- Collisionless loss-cone refilling

    -- Collisional loss-cone refilling

3. Add additional BHs

Overcoming the “Final-Parsec Problem”

I.e. how to bring binary separations from ~ 1 pc down to ~0.001 pc

1. Allow the BHs to interact with gas
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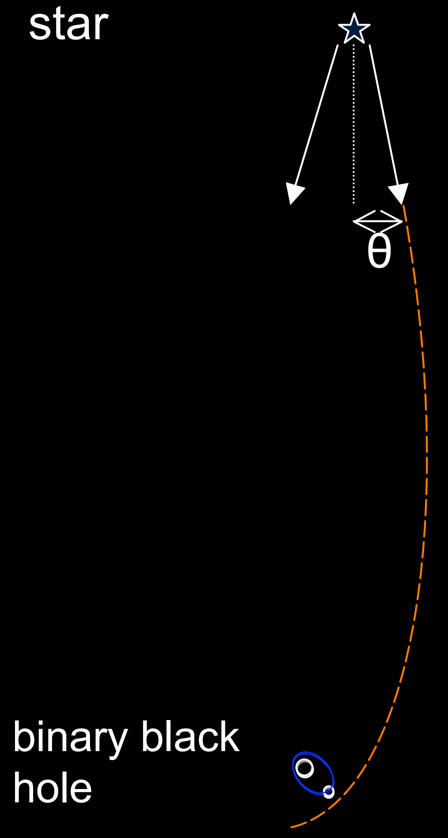
   - -- Collisionless loss-cone refilling
   - -- Collisional loss-cone refilling

3. Add additional BHs

Loss-cone around a binary black hole.

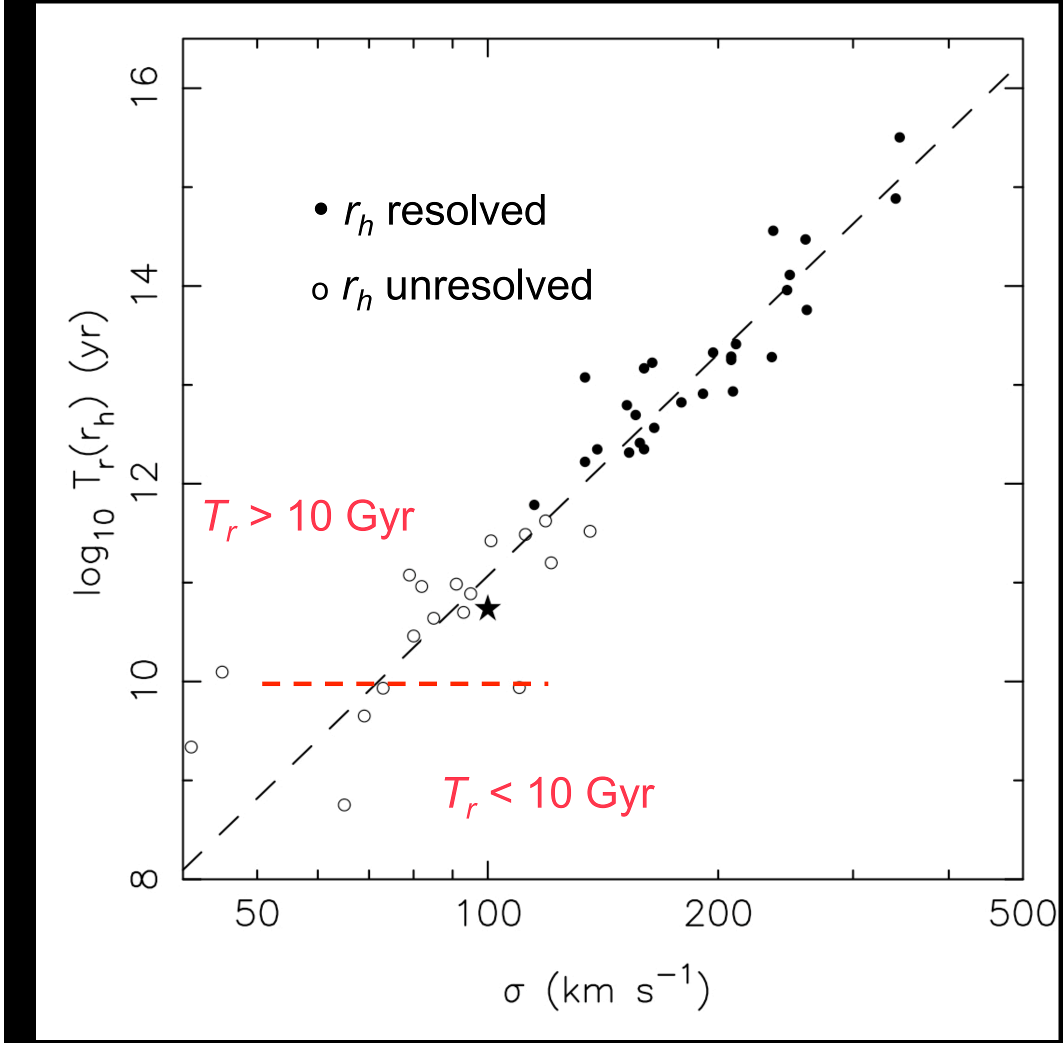


Stars are scattered into the binary, and ejected via the gravitational slingshot.
The binary responds by shrinking.

The shrinking rate *(d/dt)(1/a)* is limited by the rate of diffusion of stars into the loss cone.

$$\therefore \frac{d}{dt}\left(\frac{1}{a}\right) \propto T_r^{-1} \propto N^{-1}$$

Nuclear Relaxation Times

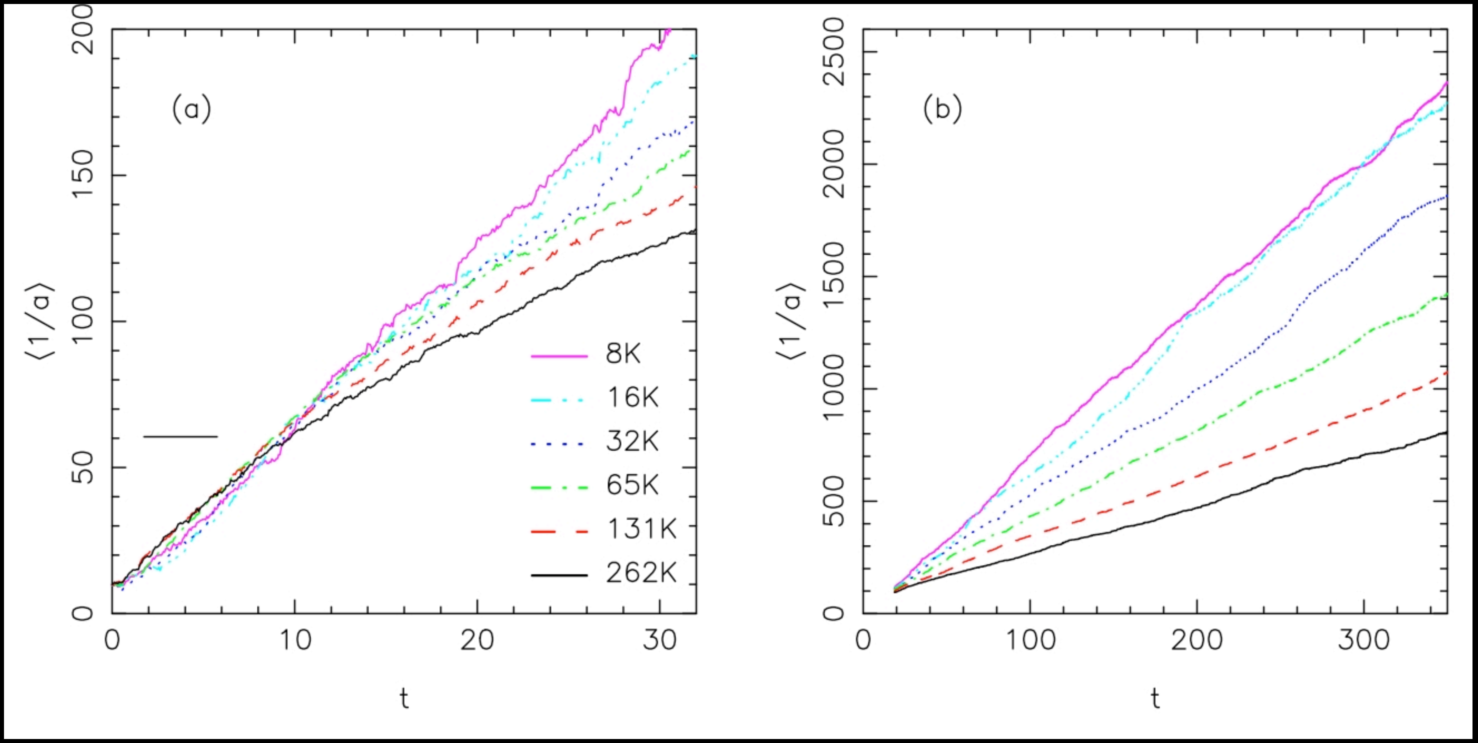


Stalling of the massive binary can be avoided if the nuclear relaxation time is shorter than ~a Hubble time.

This requires:

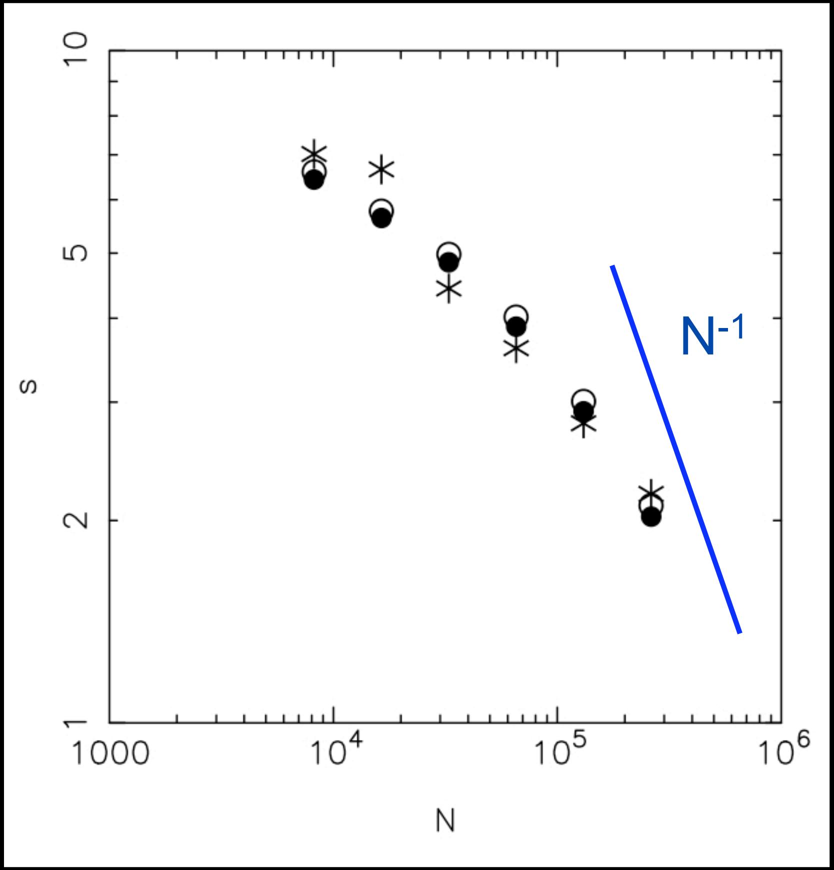
$\sigma \lesssim 100$ km s^{-1}

N-Dependence of Binary Evolution



Merritt, Mikkola & Szell 2007

N-Dependence of Binary Evolution

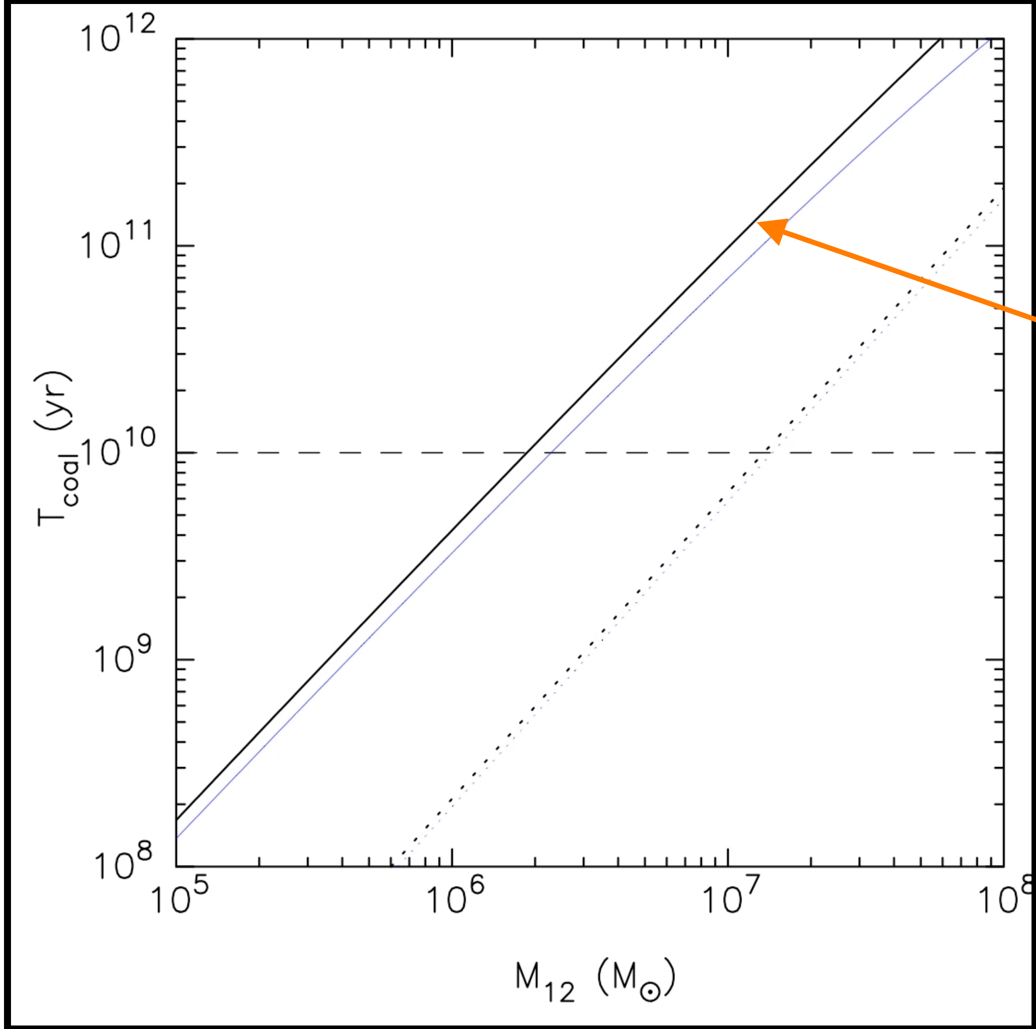


$$s \equiv \frac{d}{dt}\left(\frac{1}{a}\right)$$

- ✻ N-body
- ● Fokker-Planck
- ○ F.-P.+ “secondary slingshot”

Merritt, Mikkola & Szell 2007

Fokker-Planck!



Time to GW coalescence *vs*. binary mass.

Merritt, Mikkola & Szell 2007

Overcoming the “Final-Parsec Problem”

I.e. how to bring binary separations from ~ 1 pc
down to ~0.001 pc

1. Allow the BHs to interact with gas
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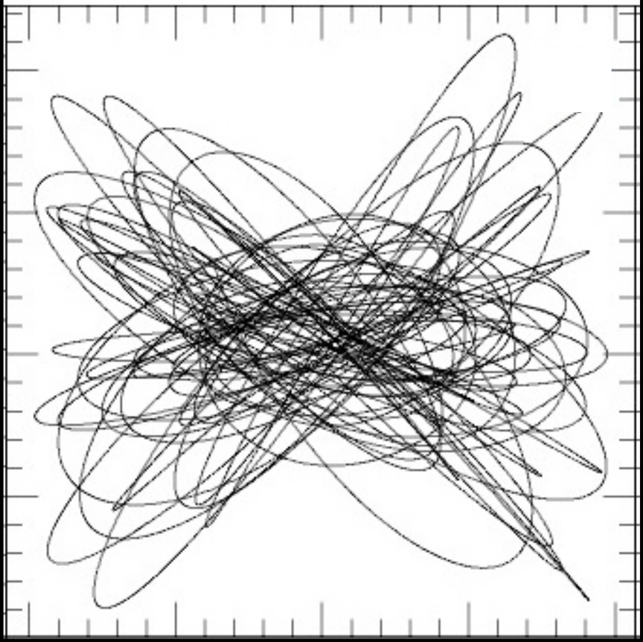
    - -- Collisionless loss-cone refilling
    - -- Collisional loss-cone refilling
3. Add additional BHs

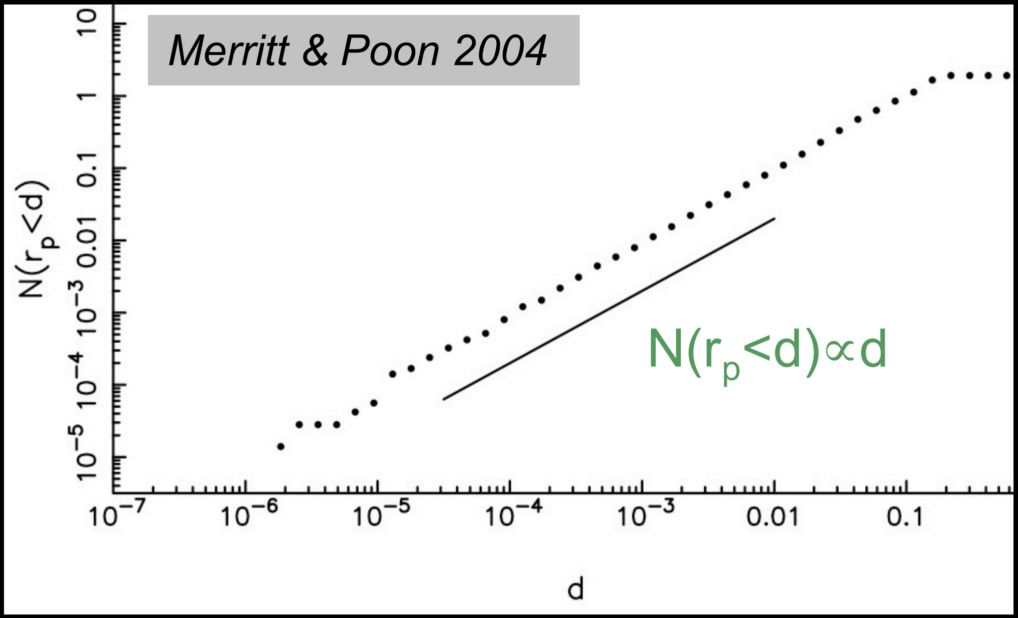
“Chaotic” (collisionless) Loss Cones

Box (chaotic) orbit



Holley-Bockelmann & Sigurdsson 2006
Merritt & Valluri 1999
…
Gerhard & Binney 1985
Norman & Silk 1983

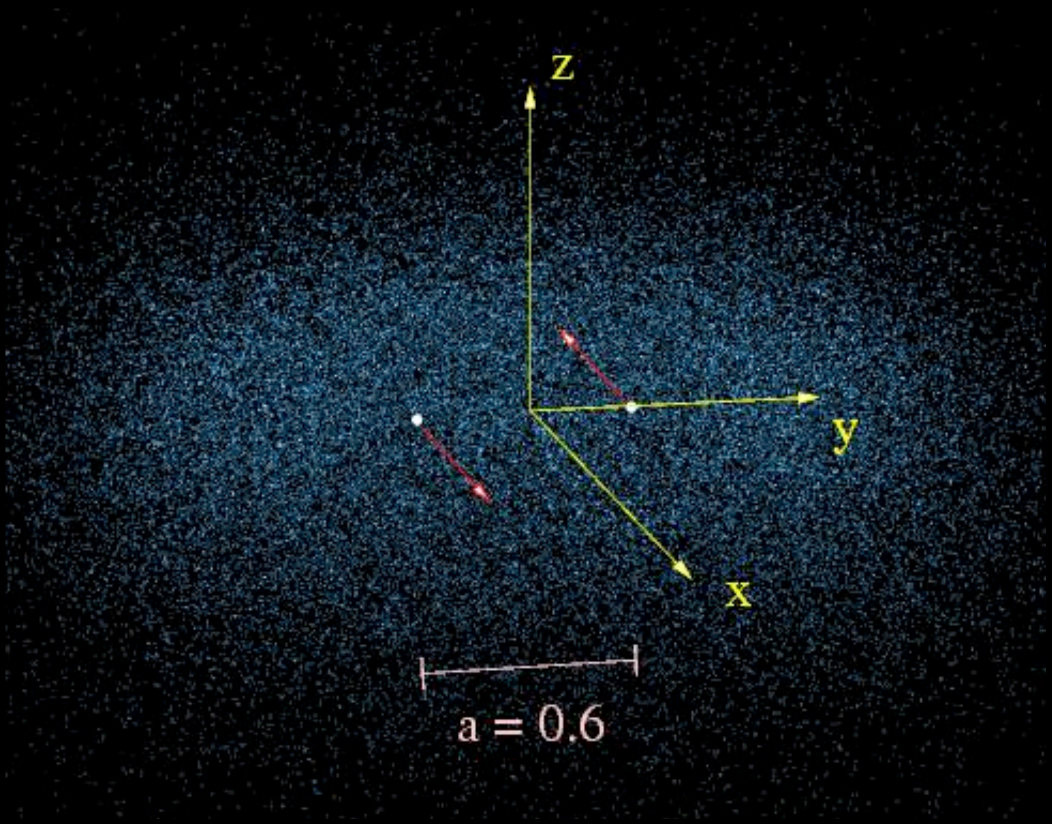
Distribution of pericenters



Implies feeding rate of

$$dM/dt \approx f_{box}\sigma^3/G$$

into a binary SMBH.

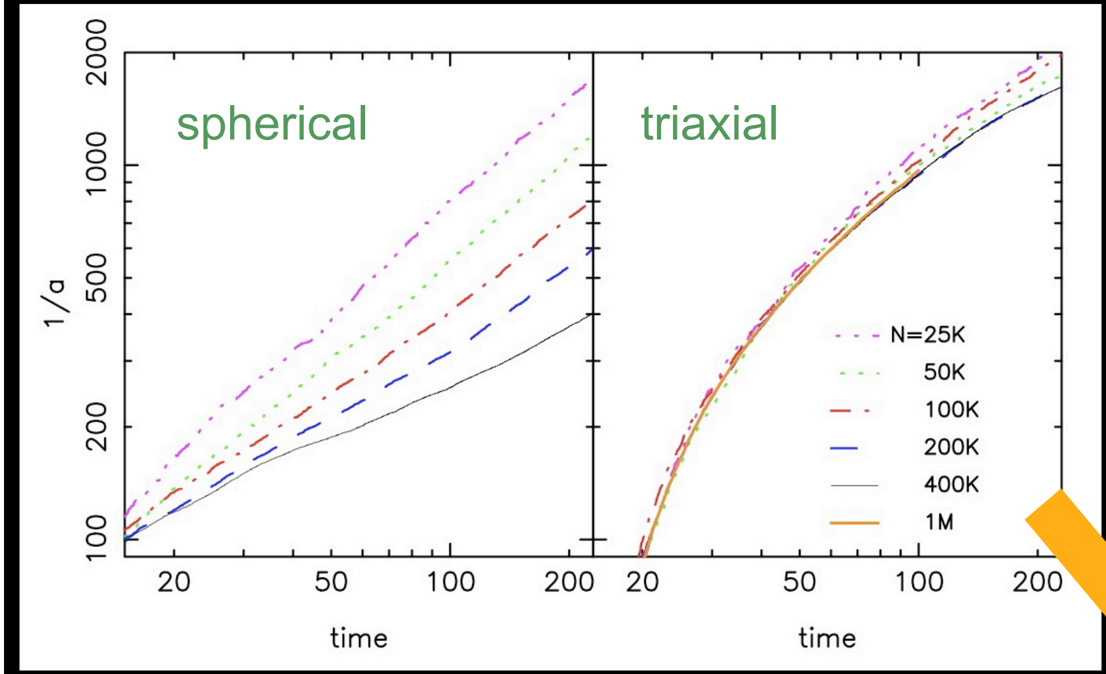


Initial conditions:

Rotating King model

Berczik et al. 2006

Berczik et al. 2006

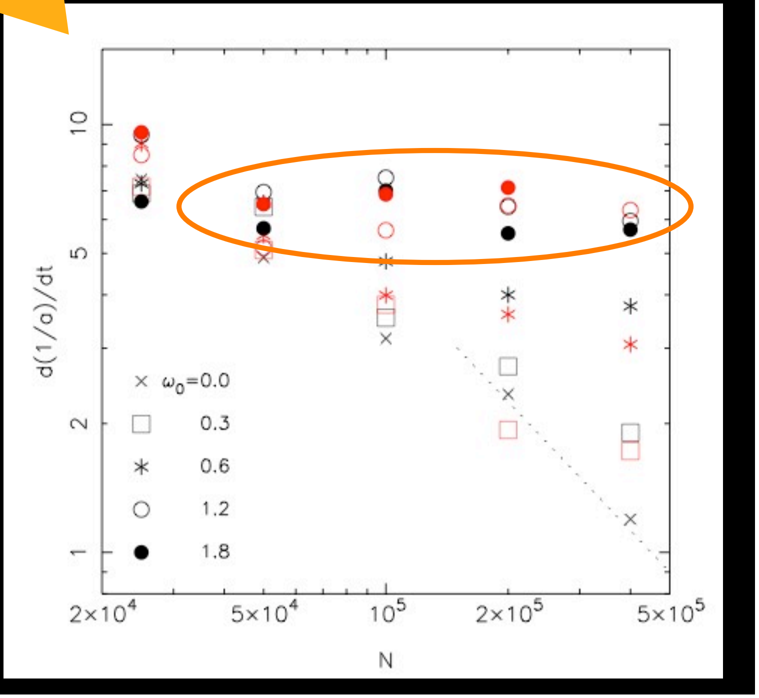


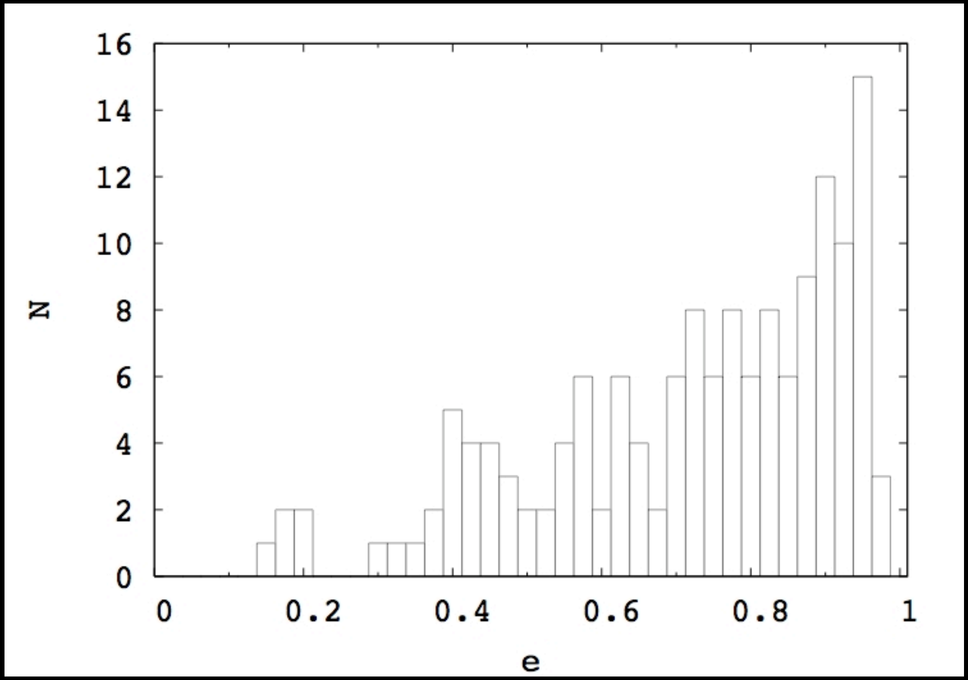
Evolution of semi-major axis

Berczik et al. 2006

Hardening rates *vs. N.*

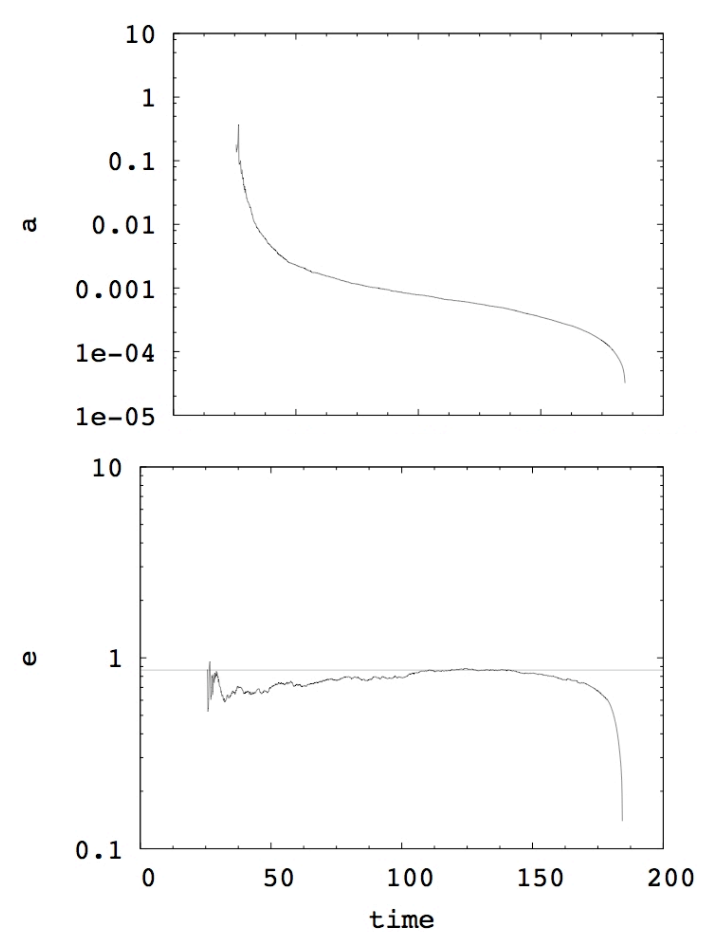
No N-dependence for triaxial models.





Eccentricity distribution at time of binary formation

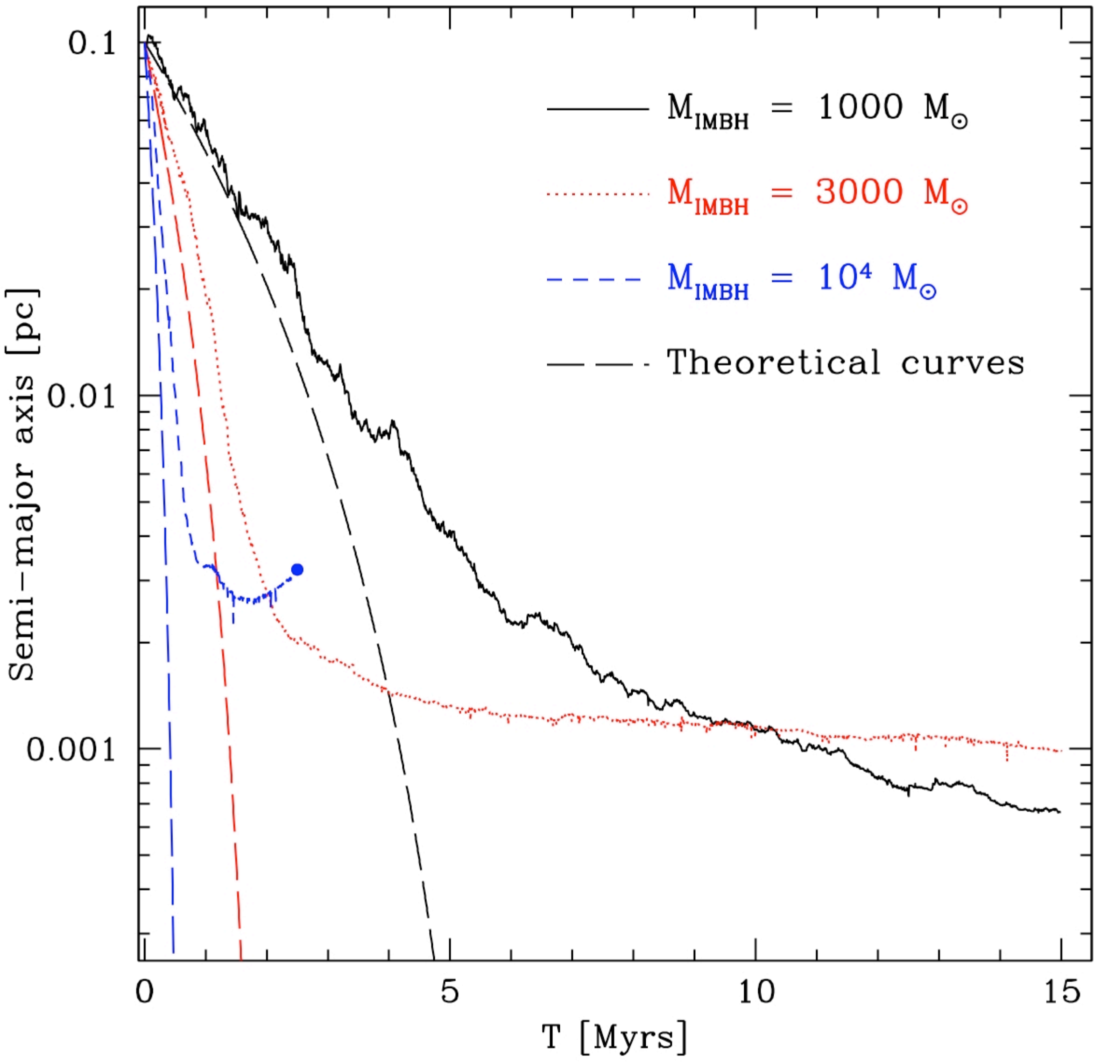
Berentzen et al. 2008



Binary evolution

(including terms up to PN2.5)

❑ Binary at the Galactic Center?

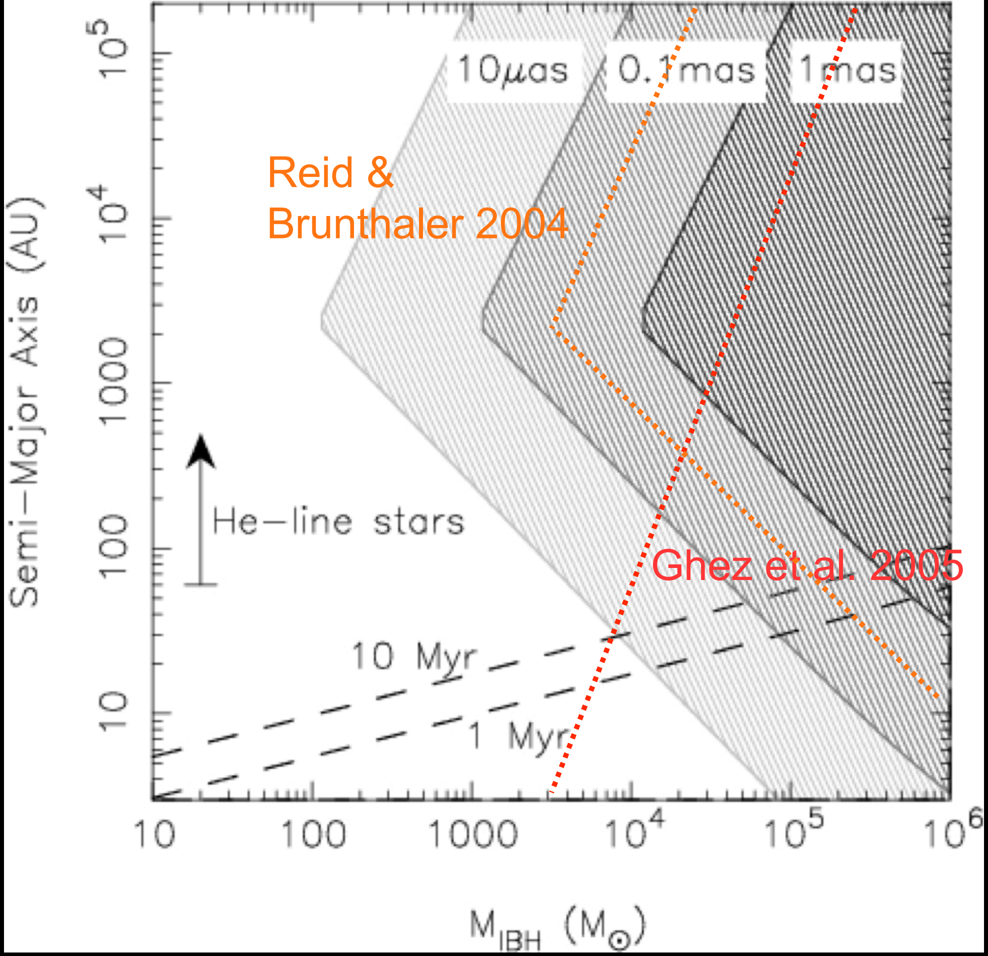


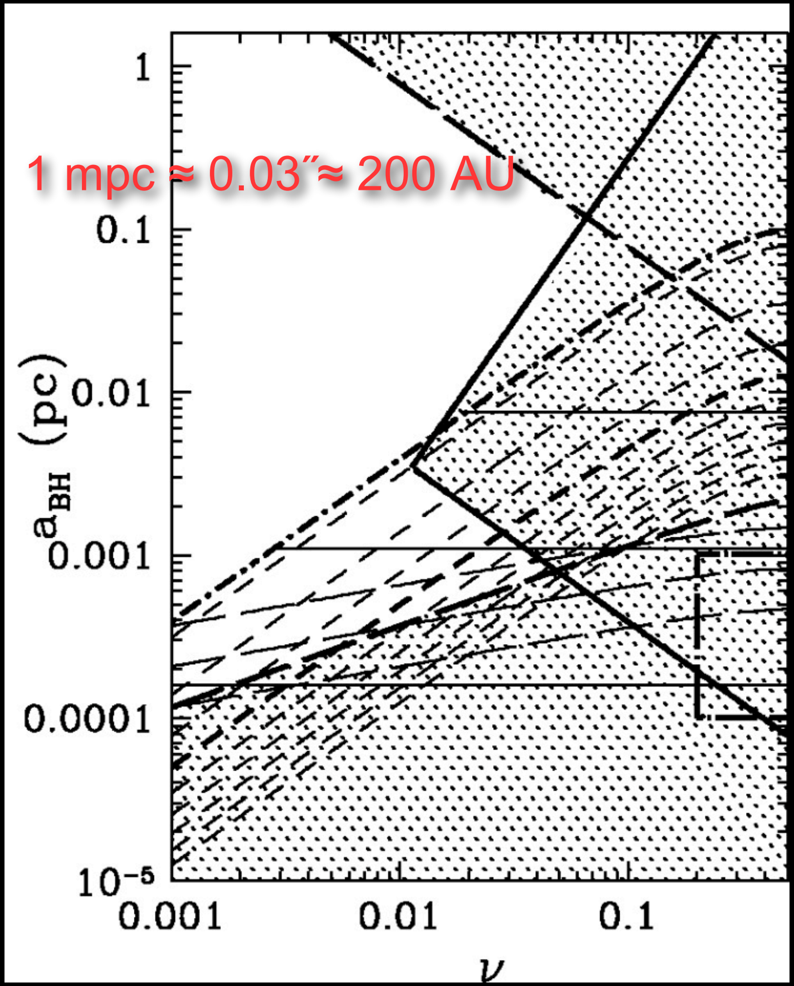
Evolution of the separation, for three values of M_{IMBH}.

Stalling radii are ~10^{-3} pc.

Baumgardt, Gualandris & Portegies Zwart (2005)

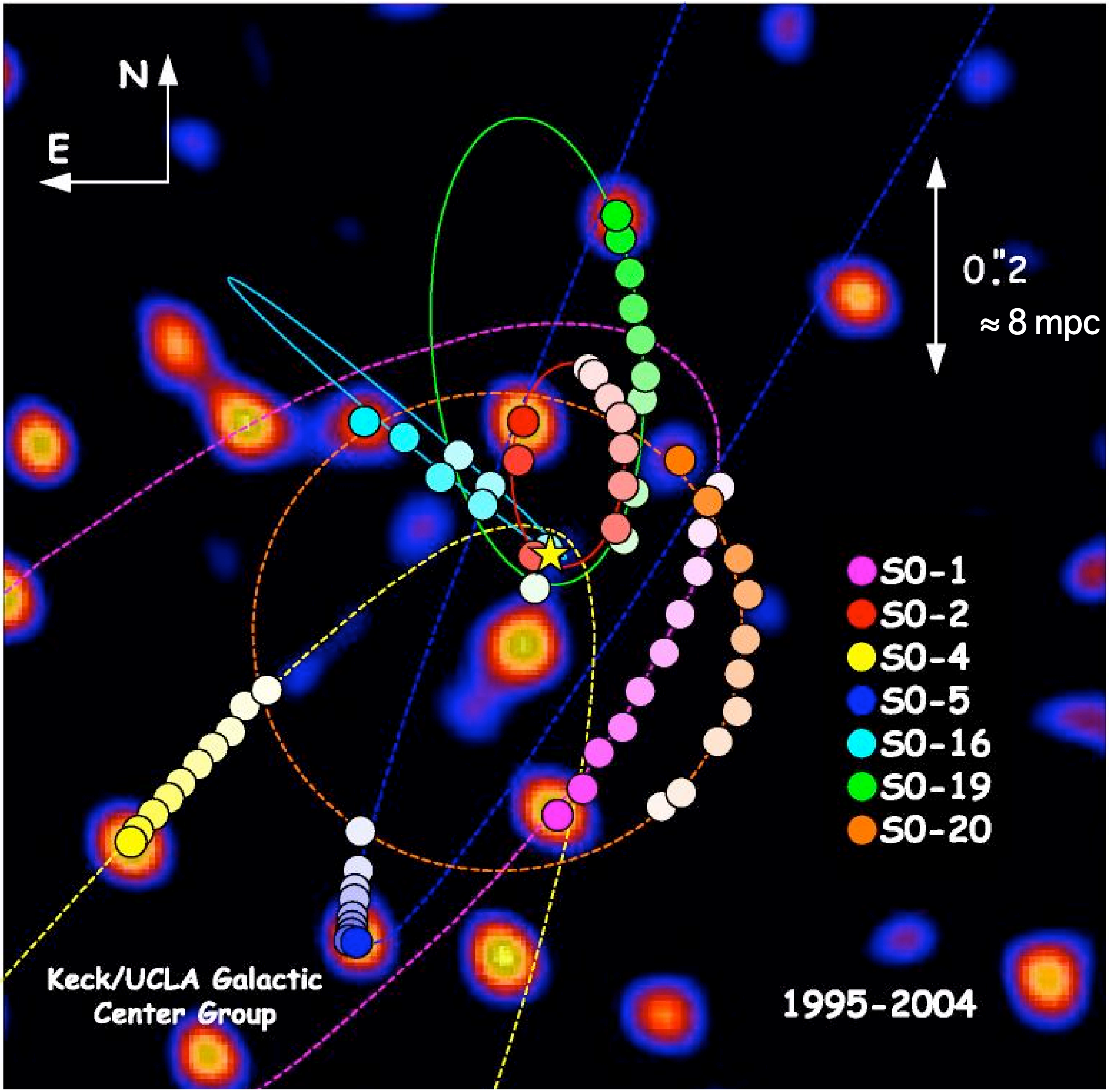
Constraints on IMBH at Galactic Center



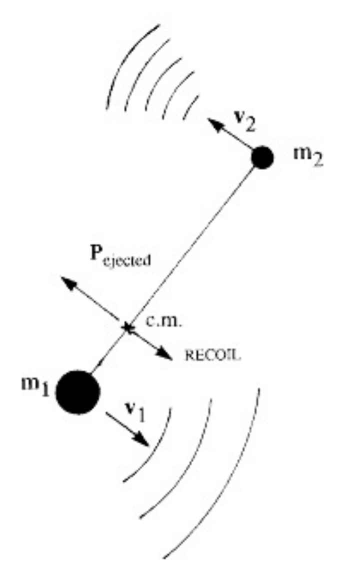


Hansen & Milosavljevic 2003

Yu & Tremaine 2003

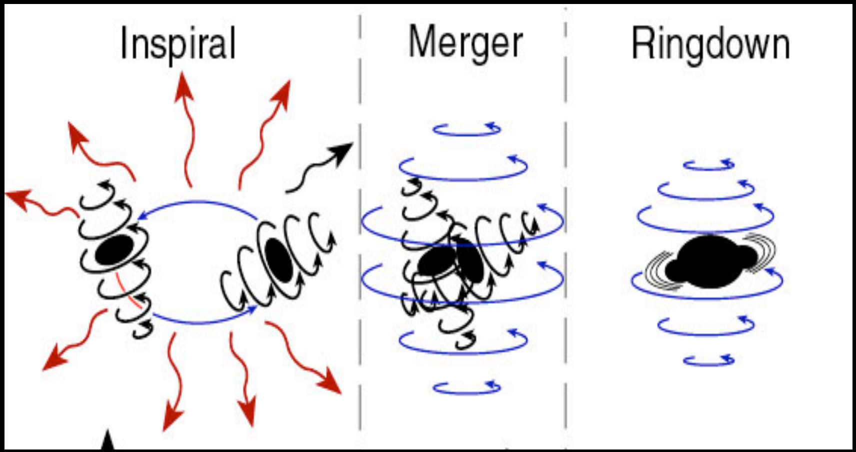

N
E
0.″2
≈ 8 mpc
S0-1
S0-2
S0-4
S0-5
S0-16
S0-19
S0-20
Keck/UCLA Galactic Center Group
1995-2004

❑ Kicking Them Out



Redmount & Rees (1989):

"…recoil speeds hundreds of times larger [than in the non-spinning case], hence larger than galactic escape velocities, might be obtained from the coalescence of rapidly rotating holes…This effect…might be largest for two holes of equal mass"



2005: Year of the Breakthrough(s)

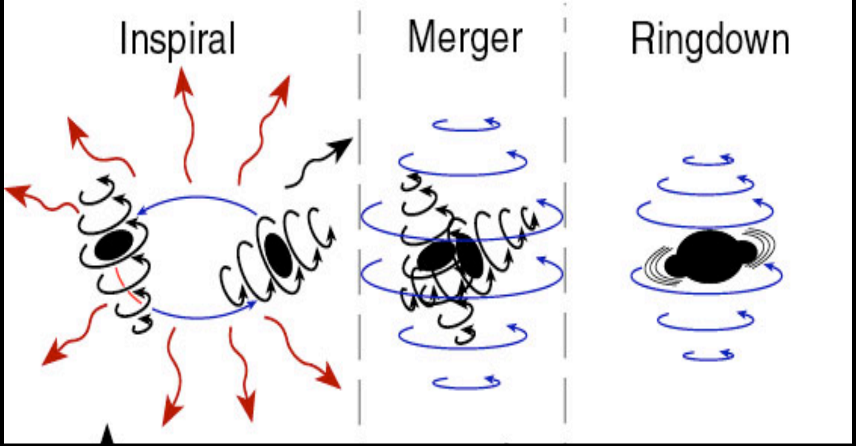
Generalized Harmonics

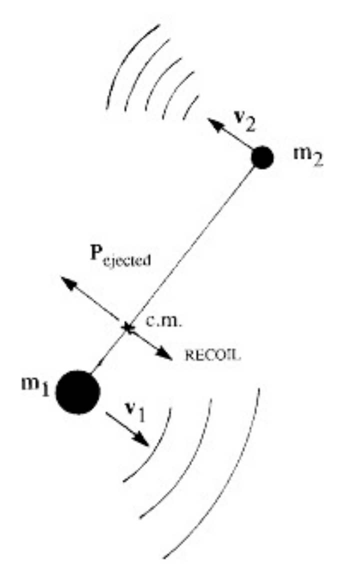
Pretorius, PRL, **95**, 121101 (2005)

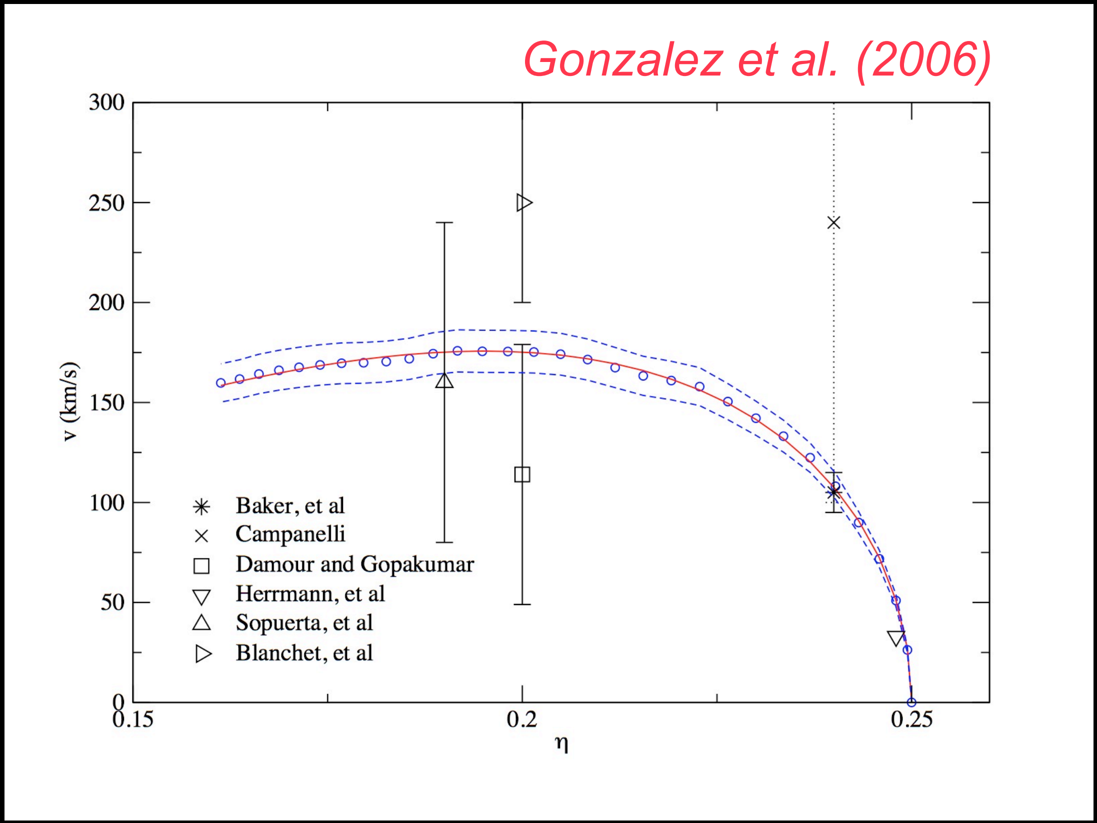
Followed by Caltech/Cornell/AEI

Moving Punctures

Campanelli et al., PRL, **96**, 111101 (2006)
Baker et al., PRL, **96**, 111102 (2006)
Followed by PSU/Jena/FAU/AEI/LSU/…






Gonzalez et al. (2006)

Rocket Effect

(no spins)

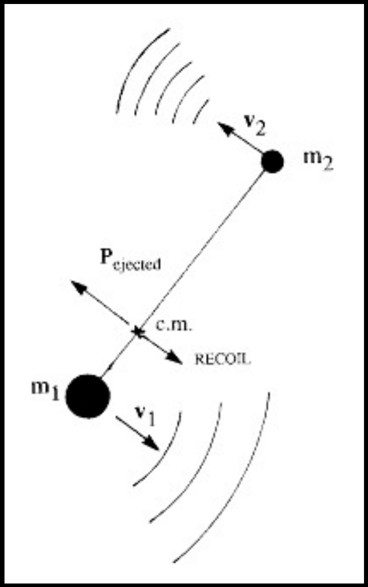
Kick maximized for:

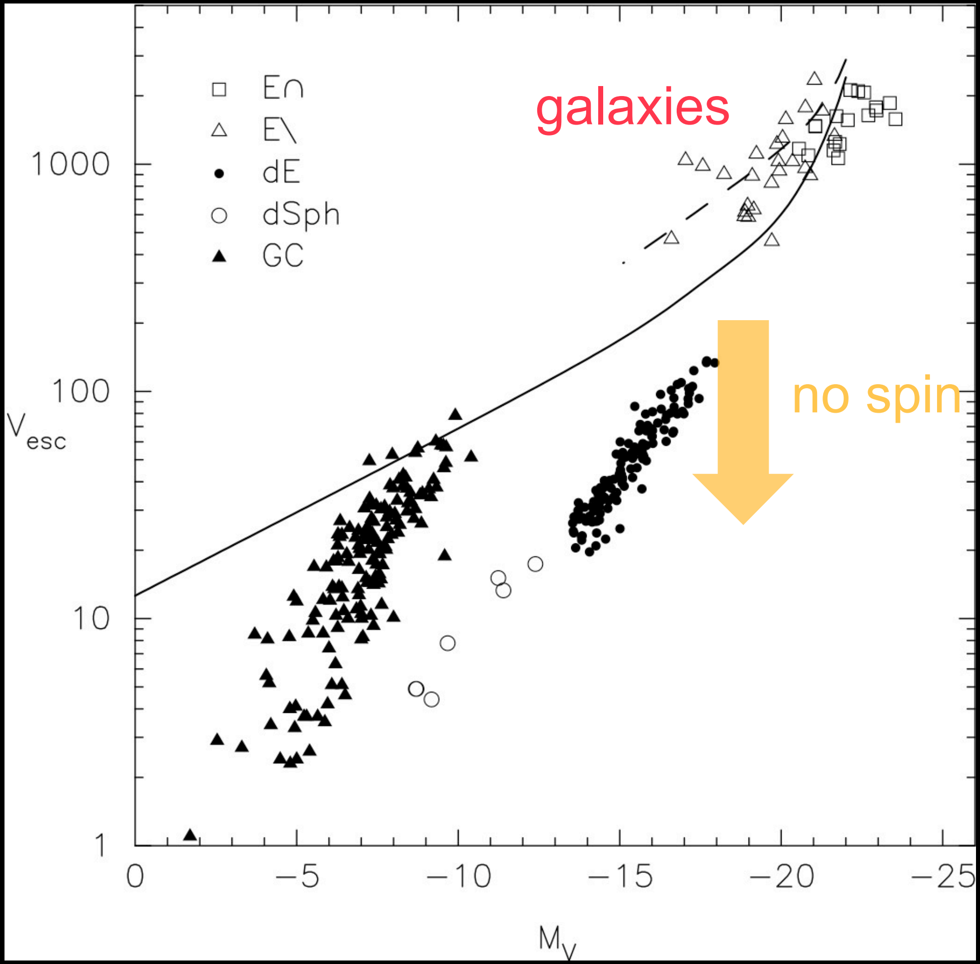
$\eta \approx 0.195$, i. e.

$M_2/M_1 \approx 0.36$

Baker et al. 2006
Sopuerta, Yunes & Laguna 2006
Herrmann, Shoemaker & Laguna 2006

Galaxy Escape Velocities





galaxies

no spin

Rocket Effect

(non-zero spins)

Koppitz et al. (2007):

$m_1=m_2 \;\; a_1=0.58 \;\; a_2/a_1 = -(0,1/4,1/2,3/4,1)$

$V = 128 \text{ km s}^{-1} (1-a_2/a_1)$ $\leq 256 \text{ km s}^{-1}$

Herrmann et al. (2007):

$m_1=m_2 \;\; a_1=-a_2 = (0.2,0.4,0.6,0.8)$

$V = 475\, a \text{ km s}^{-1}$ $\leq 392 \text{ km s}^{-1}$

Campanelli et al. (2007):

$m_1=2m_2 \;\; a_1=0.89 \; a_2 = 0$ $V = 454 \text{ km s}^{-1}$

$m_1=m_2 \;\; a_1=-a_2 = 0.5$ $V = 1830 \text{ km s}^{-1}$

Gonzalez et al. (2007):

$m_1=m_2 \;\; a_1=-a_2 = (0.73,0.80)$ $V = 2500 \text{ km s}^{-1}$

Tichy & Marronetti (2007):

$m_1=m_2 \;\; a_1=a_2 = 0.80$ $\leq 2500 \text{ km s}^{-1}$

Baker et al. (2007):

$m_1/m_2=2/3 \;\; a_1=a_2 = (0,\pm 0.2)$ $\leq 392 \text{ km s}^{-1}$

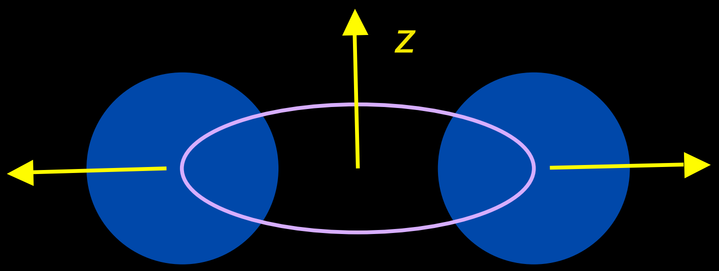
Rocket Effect

max. recoil when:

$M_1=M_2$,

$a_1=-a_2=1$,

a parallel to orbital plane

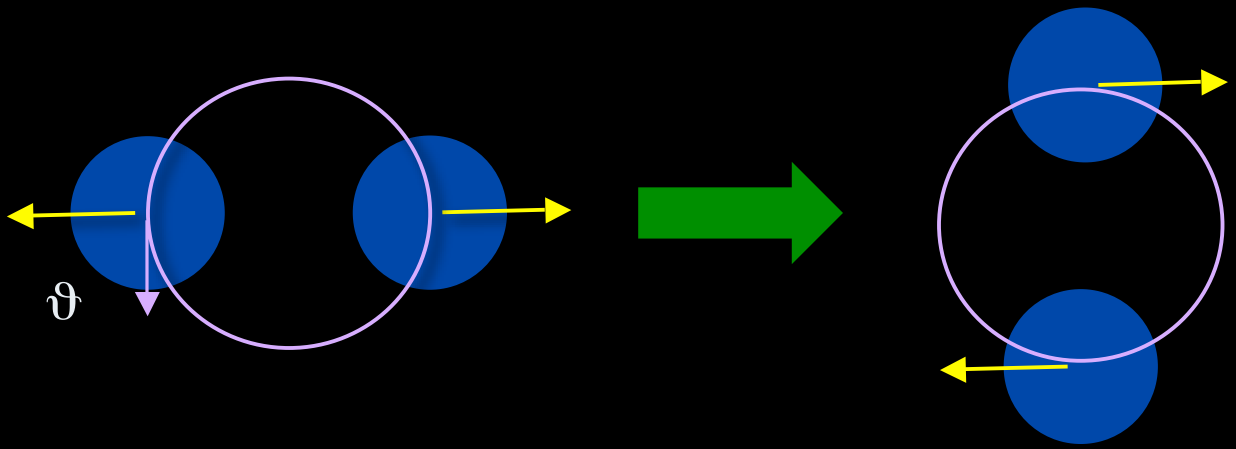


$$V_z \approx 6\times10^4\,\text{km s}^{-1}\,\frac{q^2}{(1+q)^4}$$

$$(q \equiv M_2/M_1)$$

Mass ratios as extreme as 5:1 can result in $V_{\text{kick}} > 1000$ km s^{-1}.

Recoil: Dependence on Orbital Phase

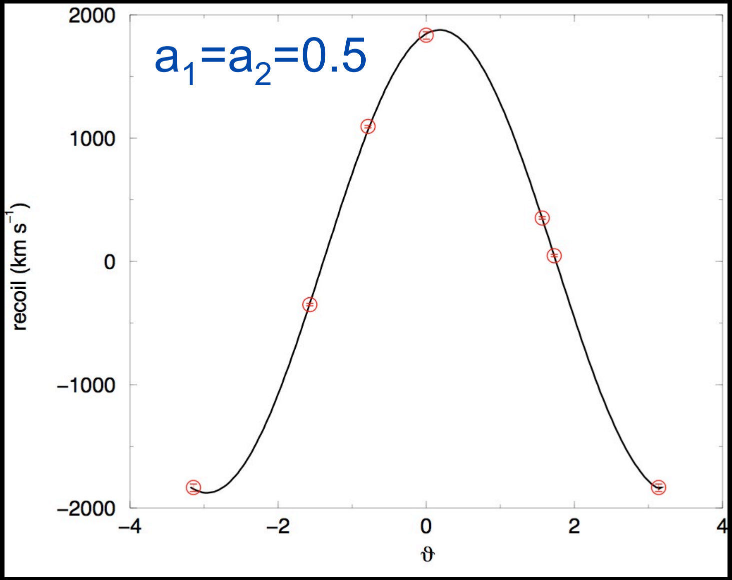


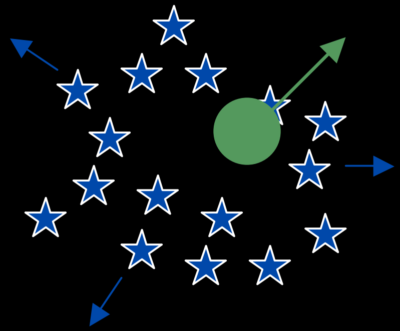
CLZM (2007):

Kick depends on *initial* orientation of BH spin *wrt initial* velocity vector.

$$V_z = 1875\ \mathrm{km\ s^{-1}}\cos(\vartheta-\vartheta_0)$$

⇒ 4000 km s^{-1} for $a_1=a_2=1$!

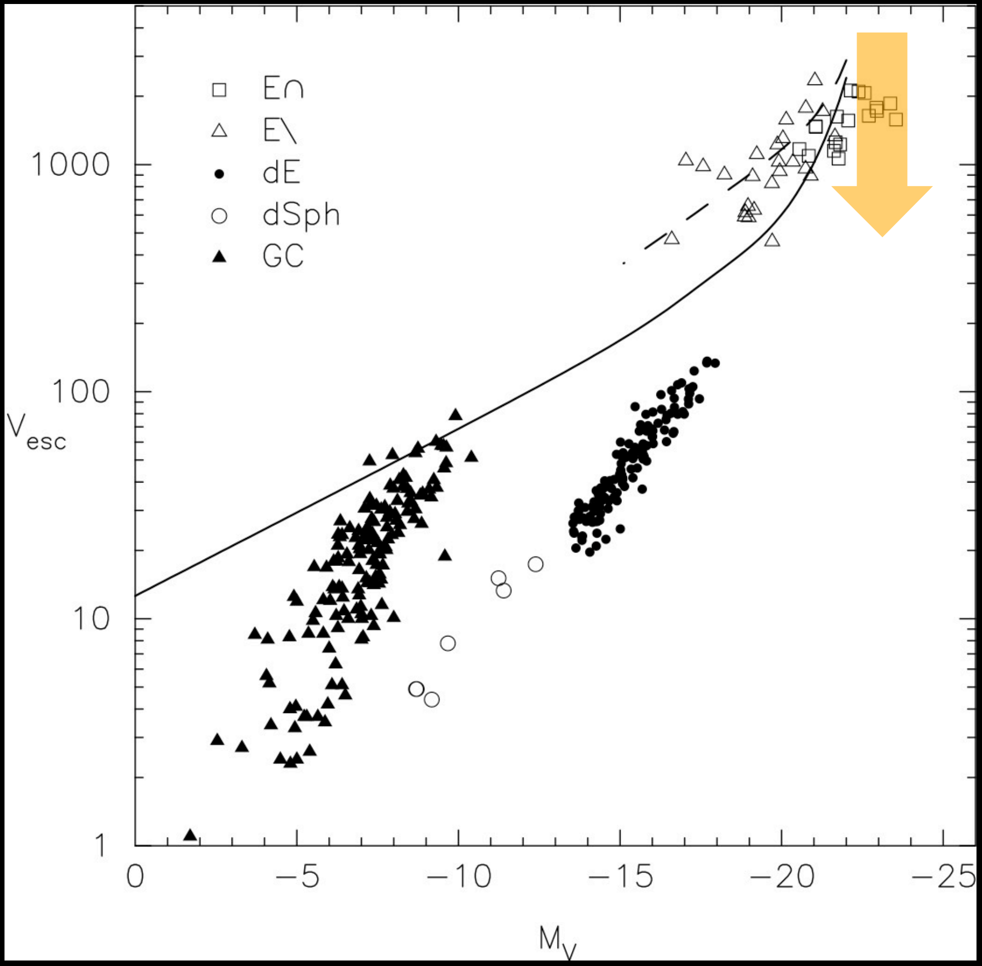


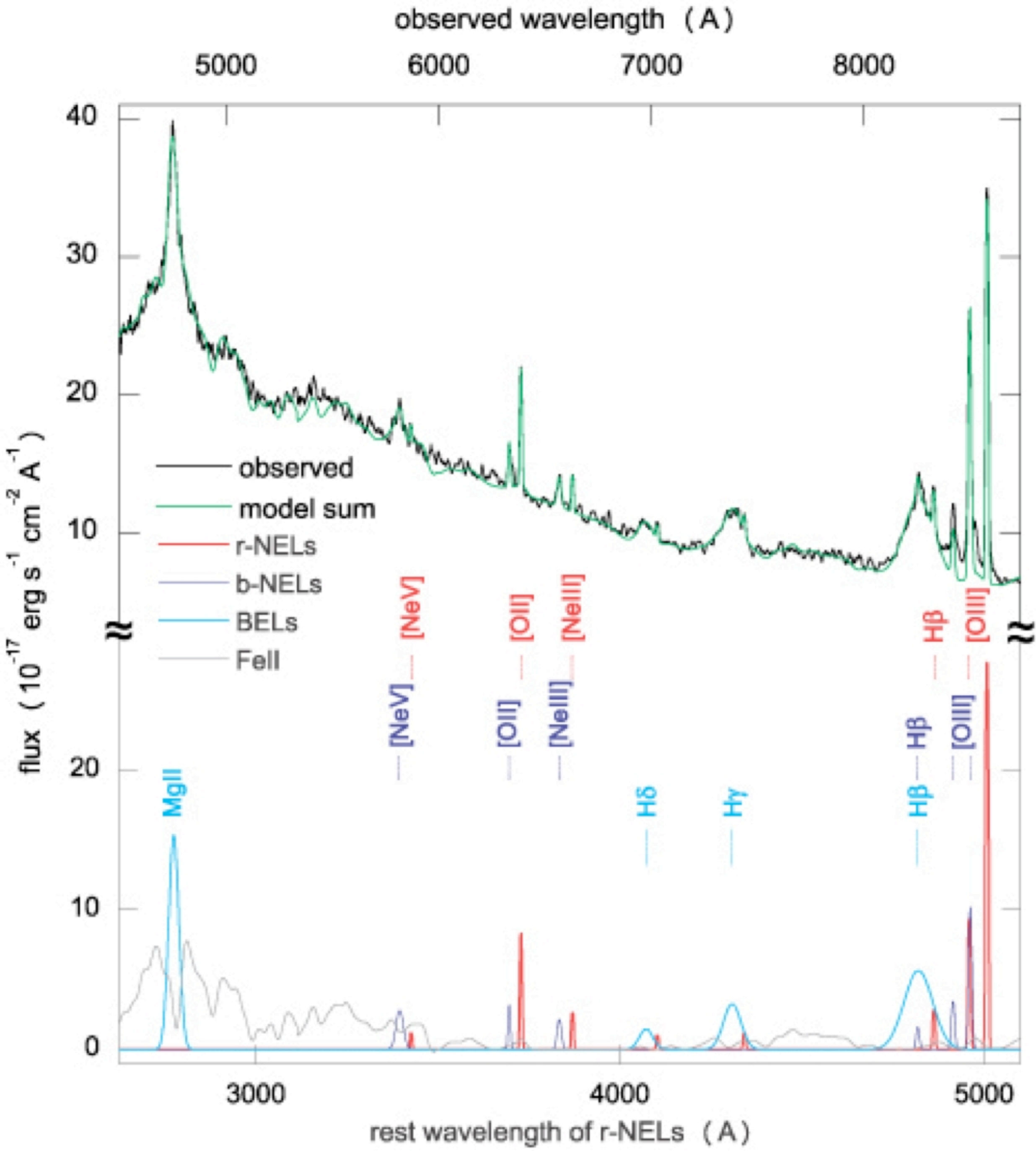


Kicks of ~4000 km/s are large enough to eject SMBHs even from the brightest galaxies!

Even ~400 km/s can substantially displace the BH from the center.

Volonteri 2007
Schnittman & Buonanno 2007
Bogdonavich et al. 2007





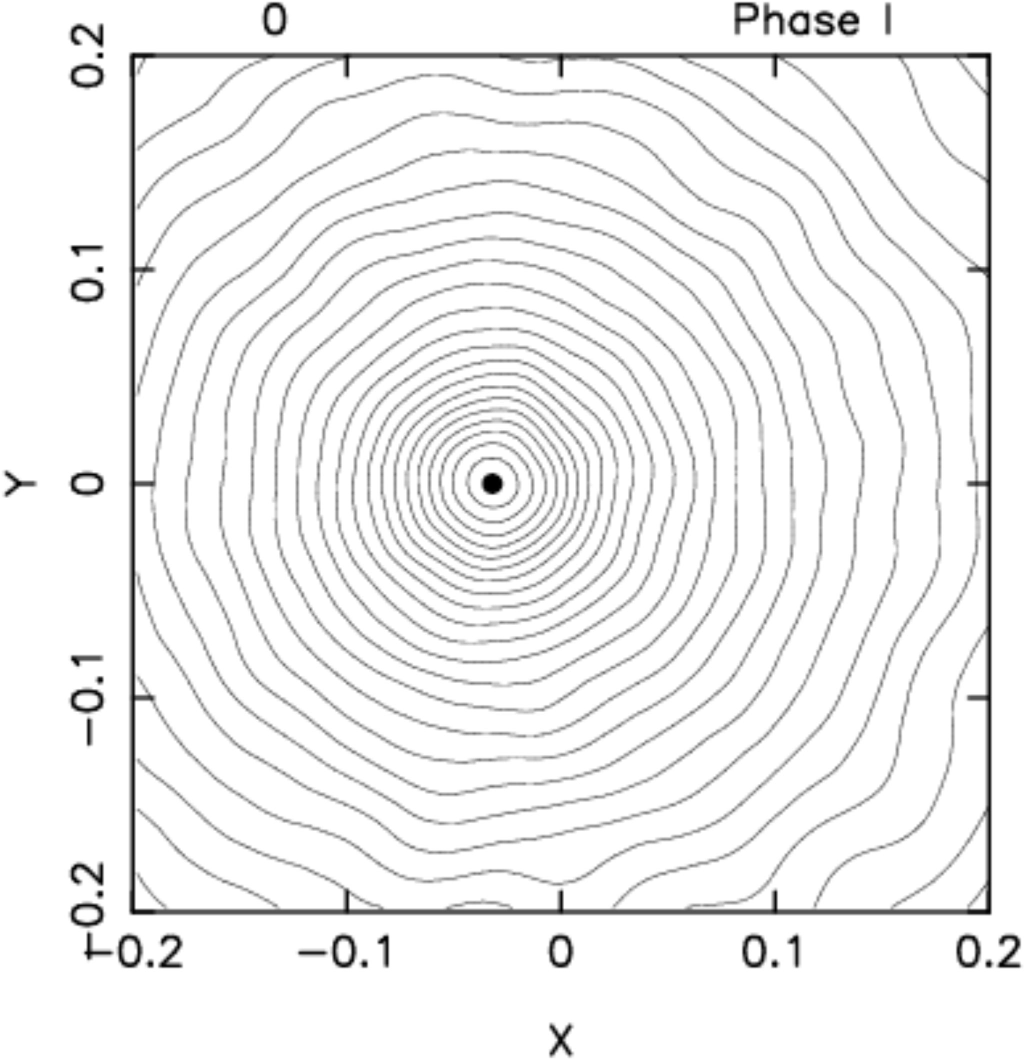
Komossa et al. (2008):

First compelling candidate for recoiling SMBH!

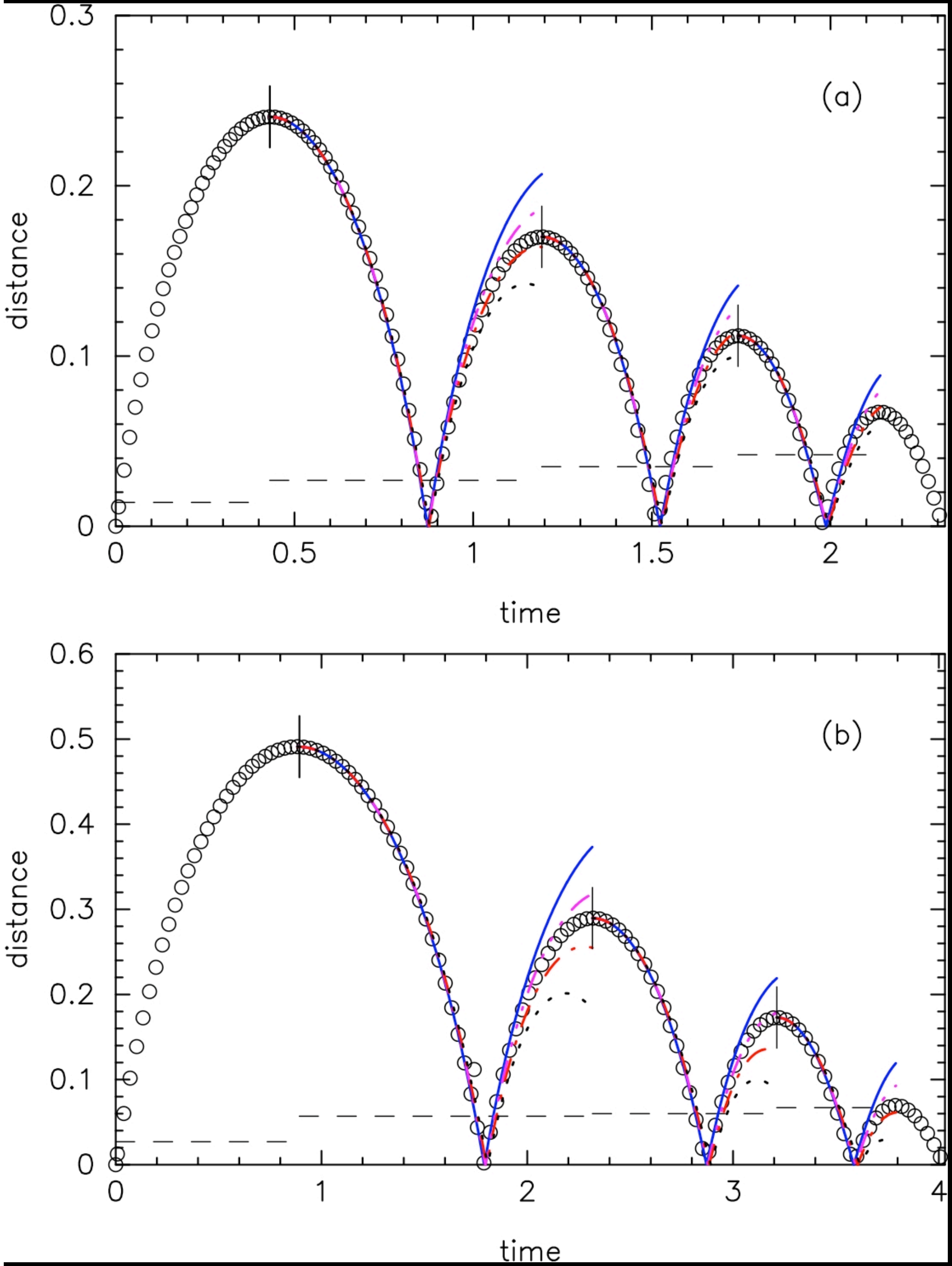
$\Delta V = 2650$ km s^{-1}

Kicked SMBH

$V_{kick} \approx (1/2)\, V_{escape}$



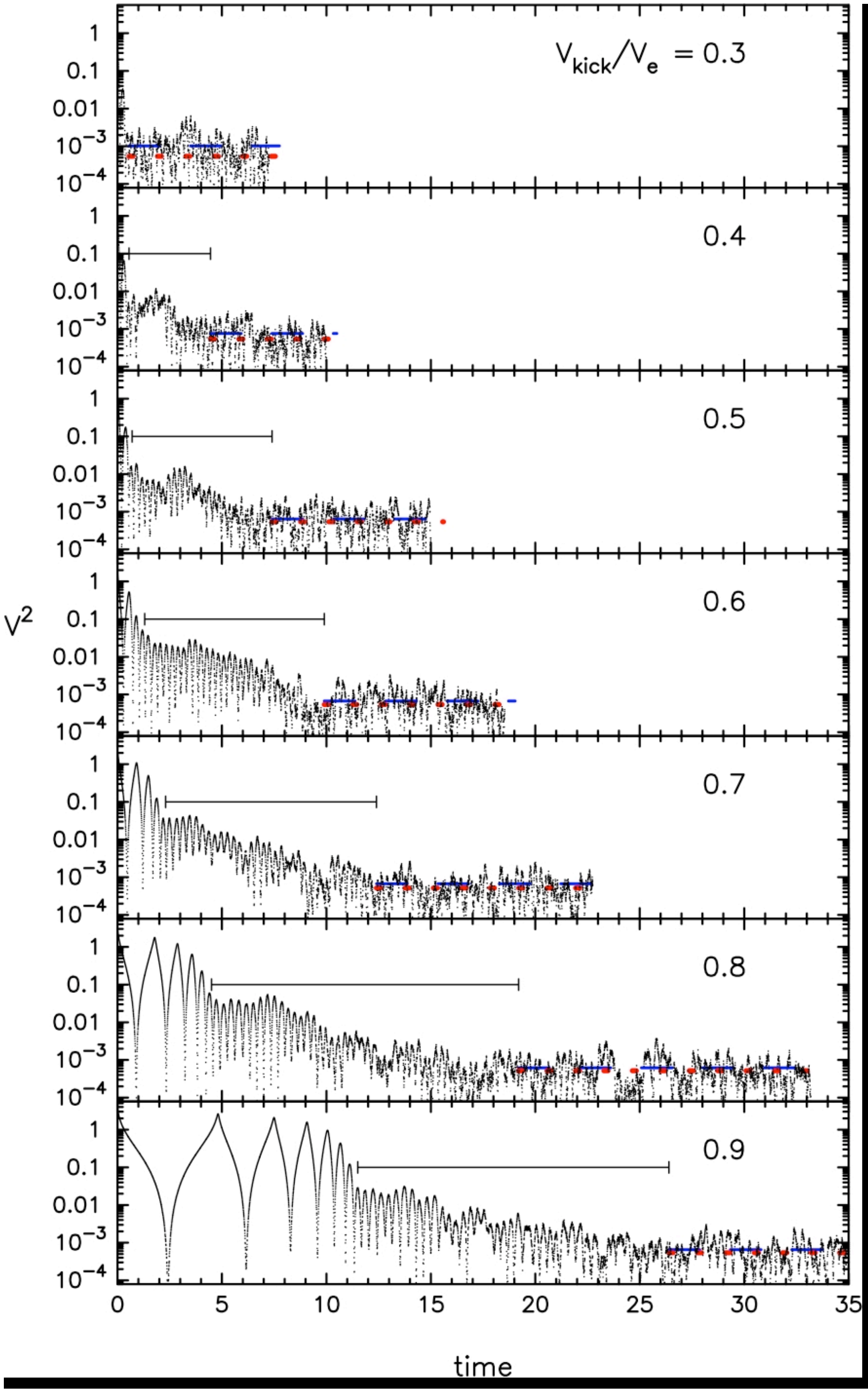
Gualandris & DM 2008



Early evolution of kicked BHs, for two values of V_{kick} (0.7, 0.8 x V_{esc}).

Curves show trajectories predicted by Chandrasekhar's formula, for ln Λ = (1,2,3,4).

Gualandris & Merritt 2008



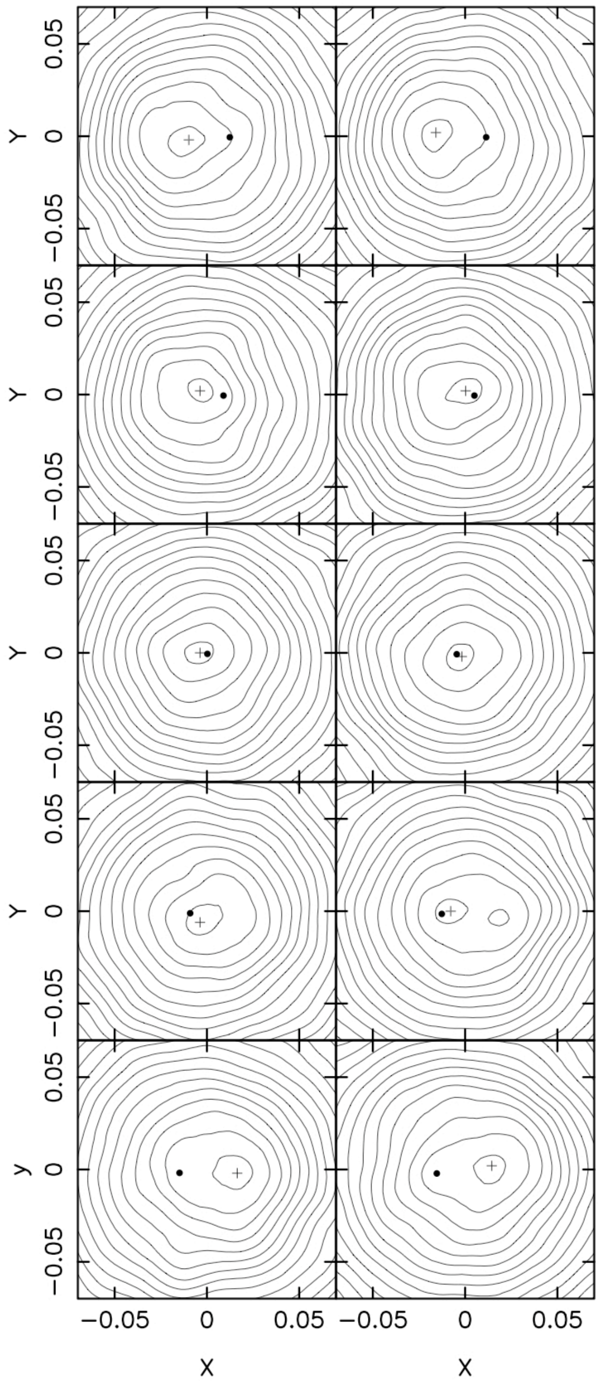
Full evolution of BH trajectories, after kicks of various sizes.

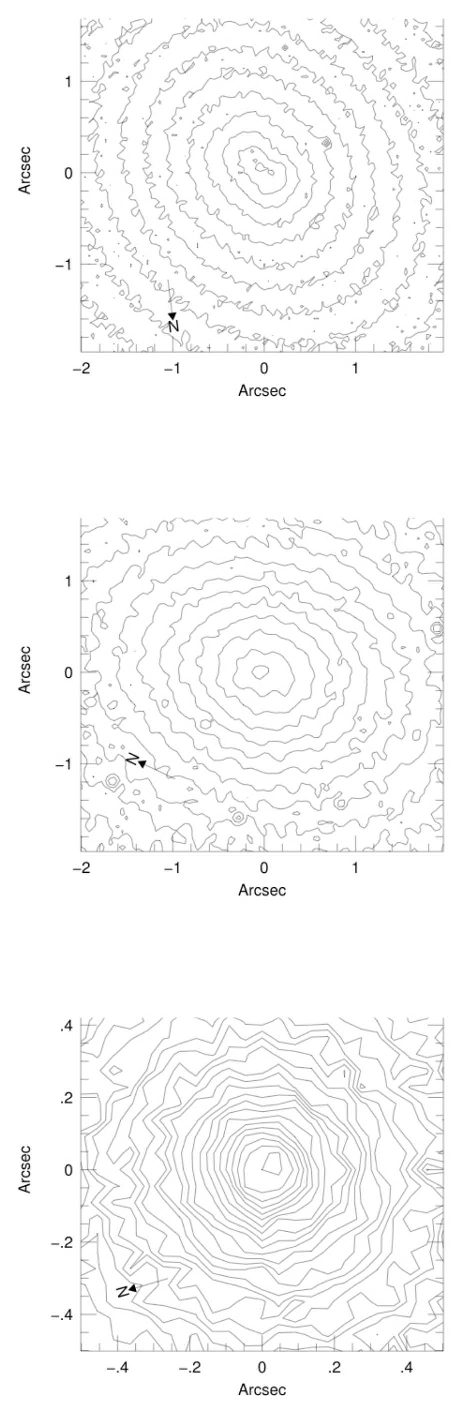
“Phase II” is indicated with solid lines.

“Phase III” (Brownian regime) is indicated with dashed lines.

Gualandris & Merritt 2008

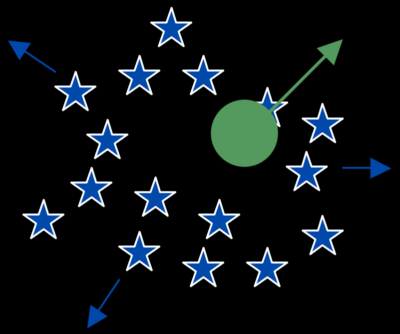
N-body oscillations



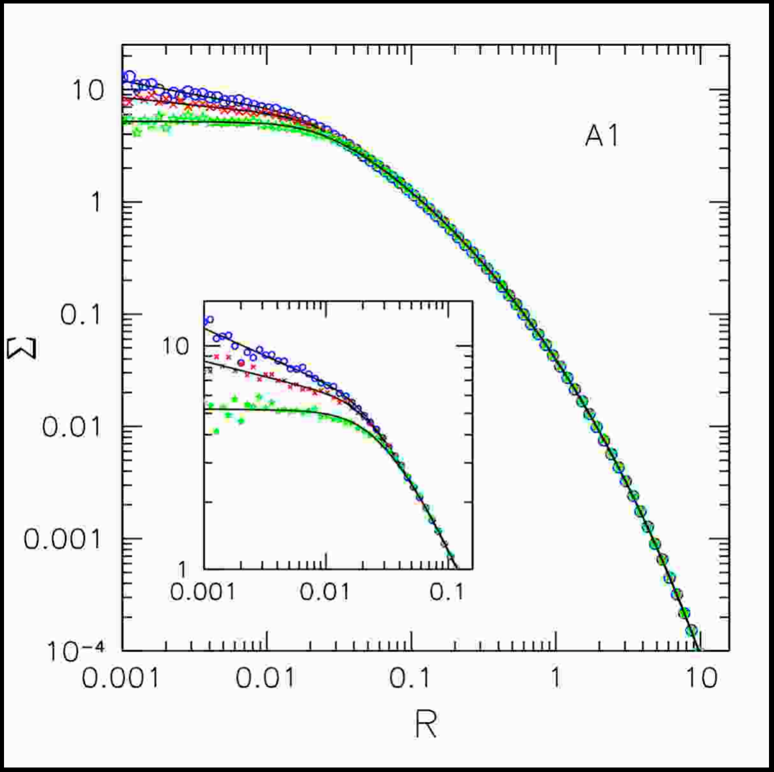


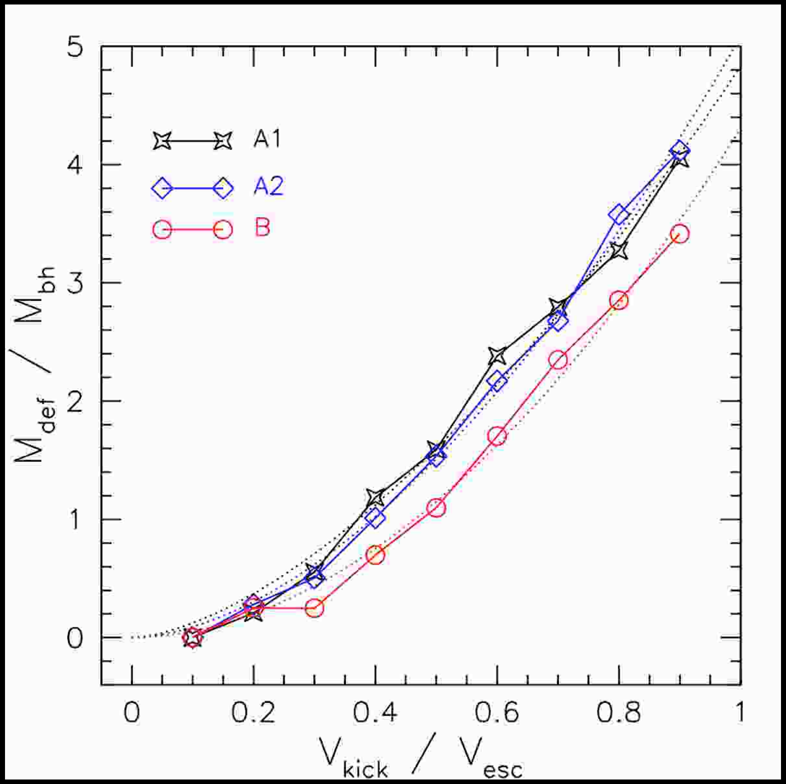
Offset/double nuclei

Lauer et al. 2005



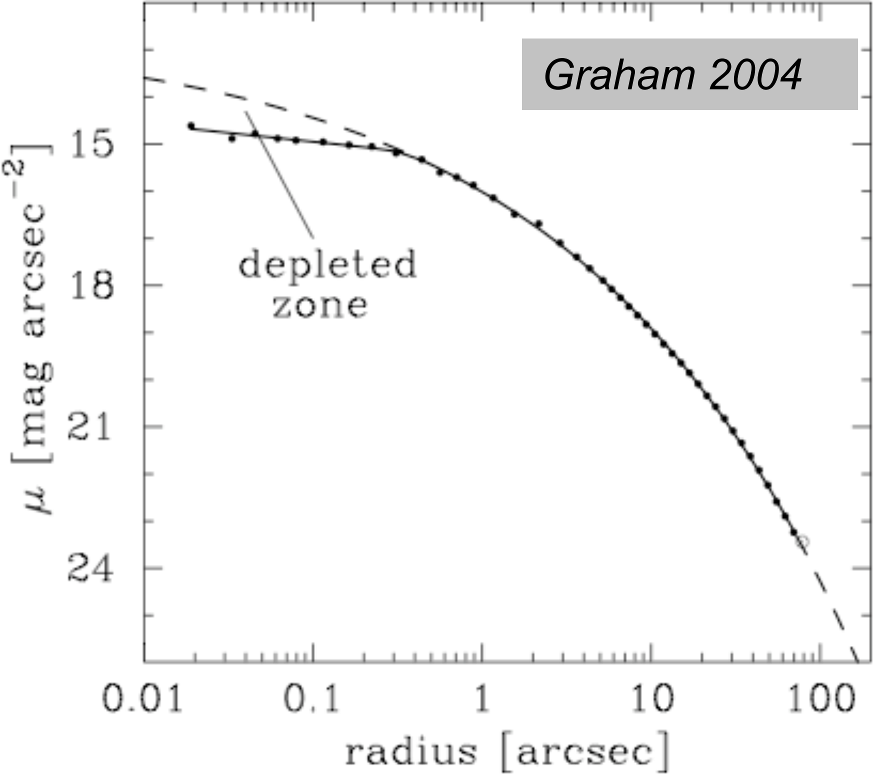
Mass deficits produced by kicked SMBHs.

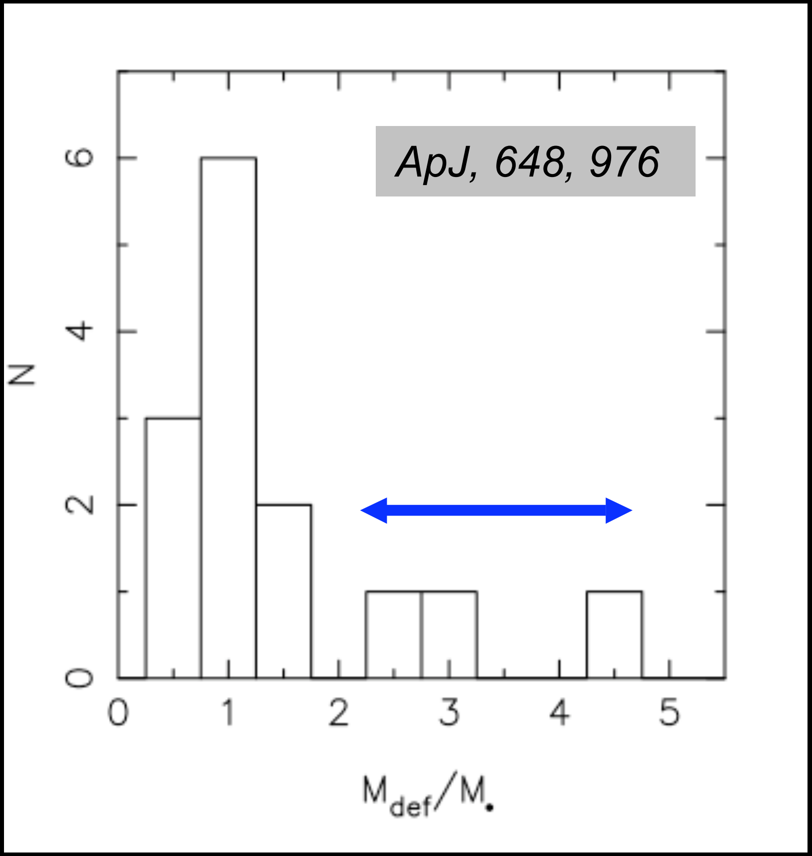




$$M_{def} \approx 5 M_{\bullet} \left(V_{kick} / V_{esc} \right)^{1.75}$$

Mass Deficits





Milosavjlevic et al. 2002
Ravindranath et al. 2002

Observing Recoiling SMBHs

- Offset QSO

  (Kapoor 1976; Madau & Qataert 2004; Loeb 2007)

- Interrupted accretion

  (Liu et al. 2003; Milosavljevic & Phinney 2005)

- UV / IR / X-ray flares

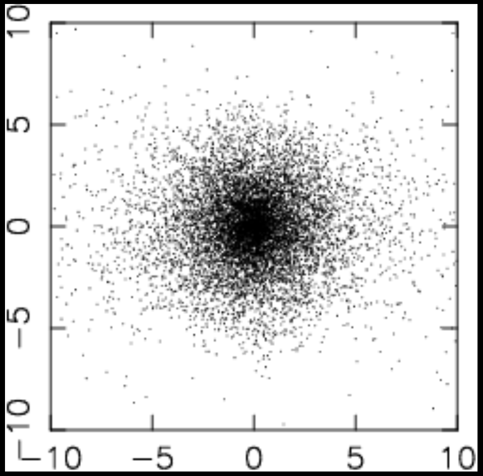
  (Lippai et al. 2008; Shields & Bonning 2008; Schnittman & Krolik 2008)

- Features in the hot gas

  (Devecchi et al. 2008)

All of these require the presence of gas

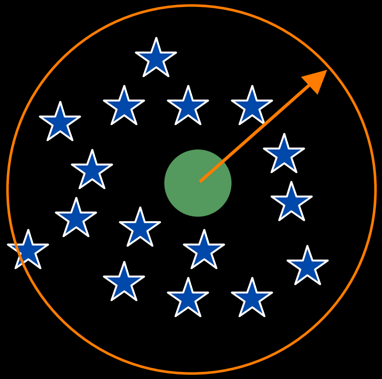
Stars Bound to a Recoiling SMBH



Stars initially within a radius:

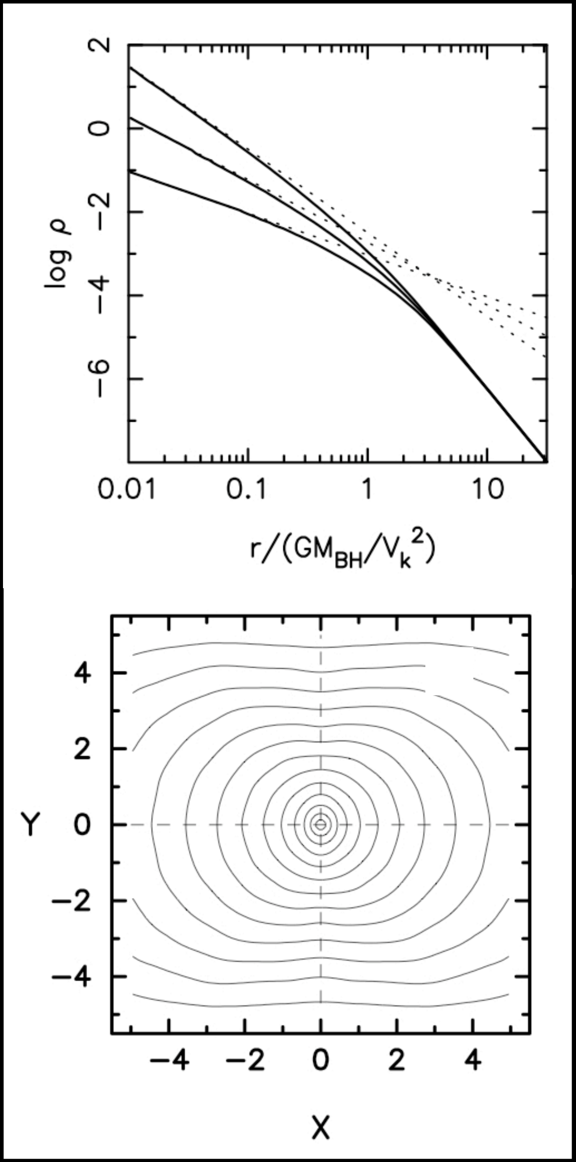
$$r_{\text{kick}} = GM_\bullet / V_{\text{kick}}^2$$

remain bound to the BH after the kick.

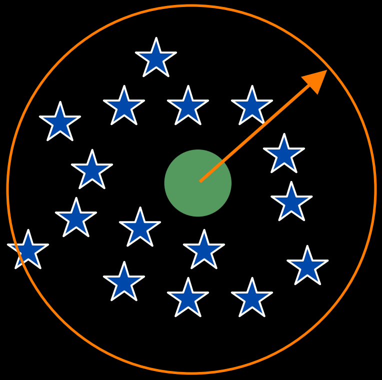


Komossa & Merritt 2008

Stars Bound to a Recoiling SMBH



Komossa & Merritt 2008



Stars initially within a radius:

$$r_{\rm kick} = GM_\bullet / V_{\rm kick}^2$$

remain bound to the BH after the kick.

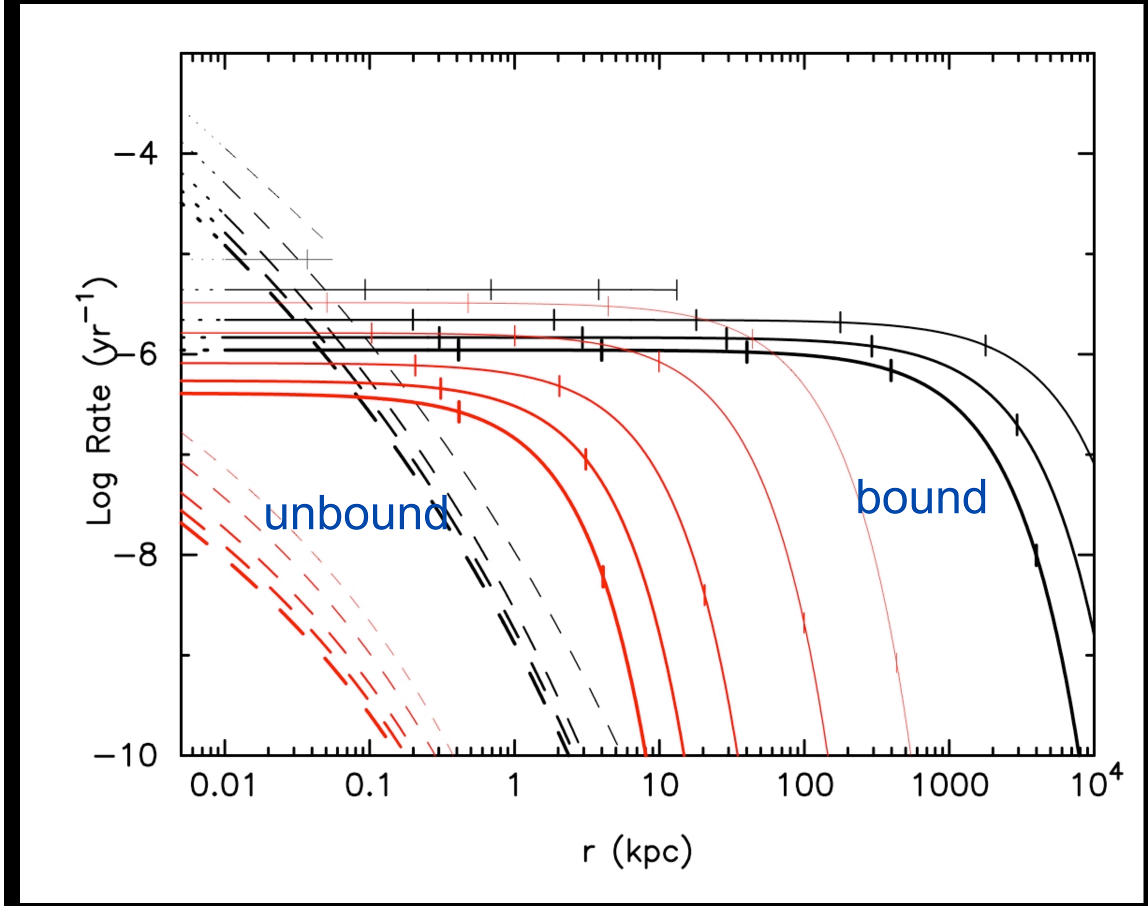
The total bound mass is:

$$M_{\rm bound} \approx \rho(r_{\rm kick}) r_{\rm kick}^3$$

$$\propto V_{\rm kick}^{2(\gamma-3)} \qquad (\rho \propto r^{-\gamma})$$

and is of order 1% $M_\bullet$ for $V_{\rm kick} = 10^3$ km s^{-1}.

Recoil Flares?



A recoiling SMBH disrupts both bound, and unbound, stars.

Disruption rates are only moderately lower than those of nuclear SMBHs.

Komossa & Merritt 2008

END