

Astrophysical N-body/SPH TREE-GRAPE/GPU implementations.

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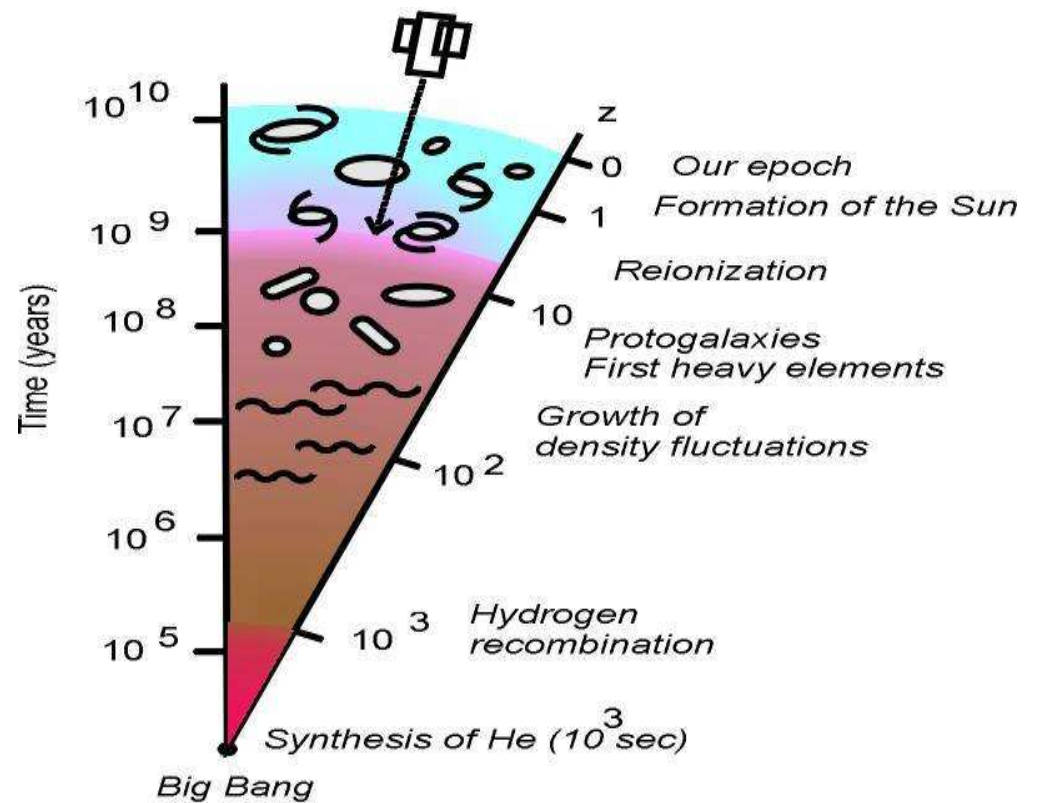
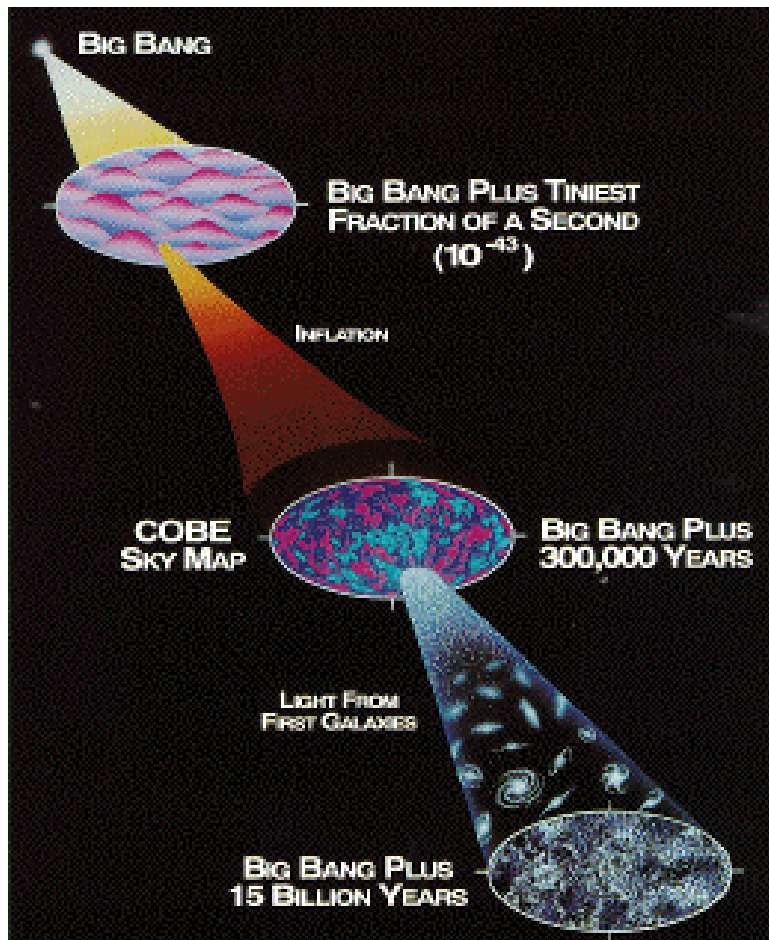
N-body 2008, Turku, Finland, 10 – 14 August 2008

Collaborators & Grants

- Naohito Nakasato
 - Keigo Nitadori
 - Ingo Berentzen & Rainer Spurzem
 - G. Marcus, G. Lienhart, A. Kugel, R. Maenner
 - M. Wetzstein, T. Naab, A. Burkert
- Univ. of Aizu, Japan
Tokyo Univ., Japan
Univ. Heidelberg
Univ. Heidelberg
Univ. Munich
- **DFG SFB: No. 439/B11: 2005 - 2008**
 - **Volkswagen/Baden-Württemberg, GRACE project: 2005 - 2008**

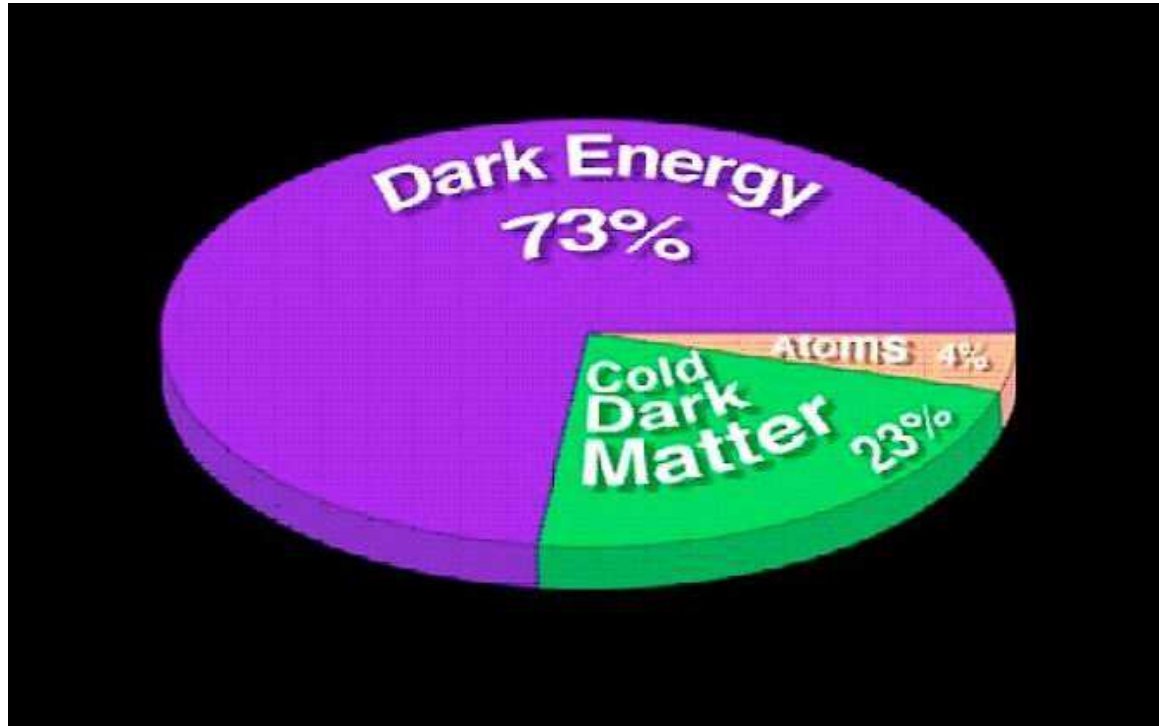


Formation of the Universe



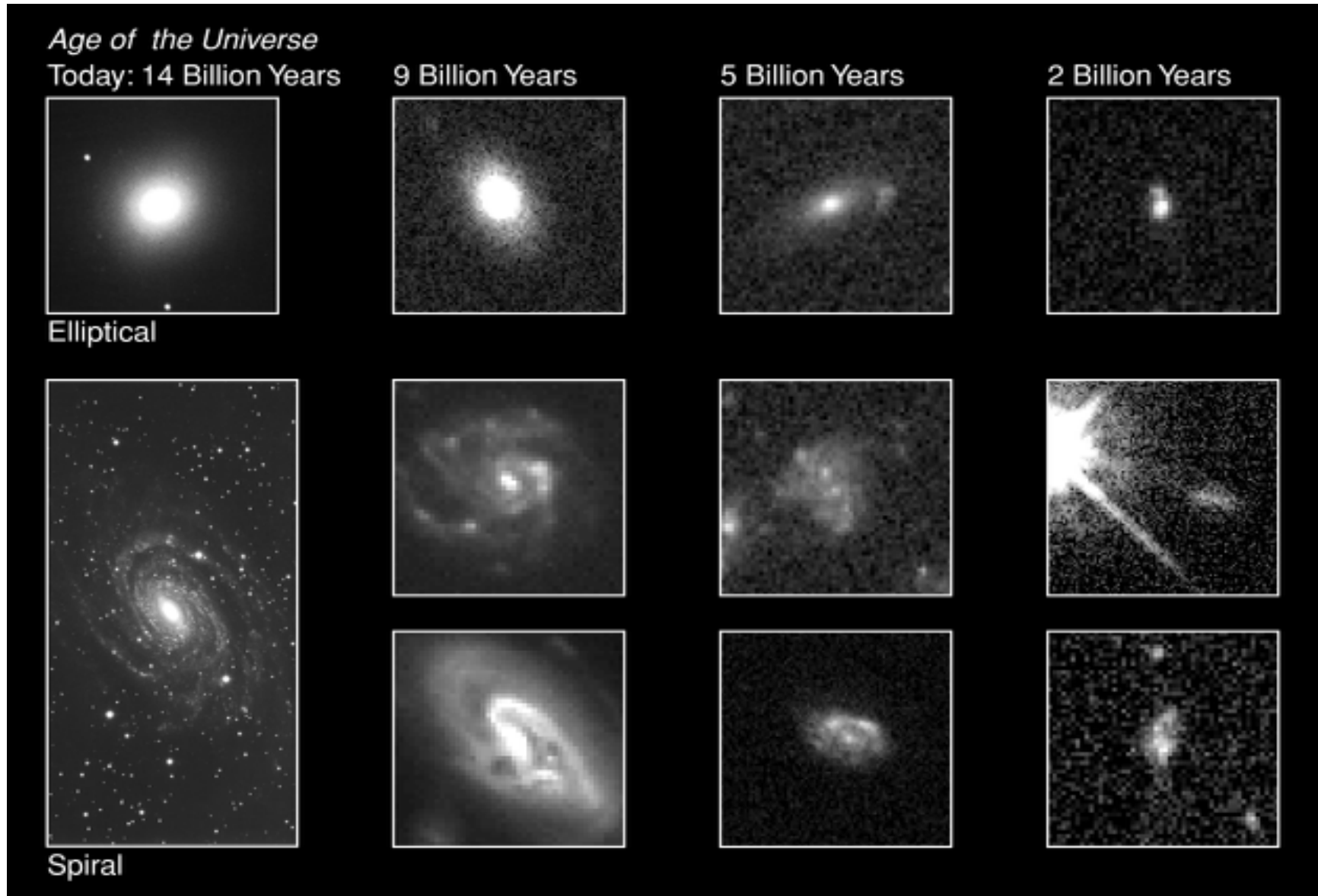
History of the Universe

Formation of the Universe

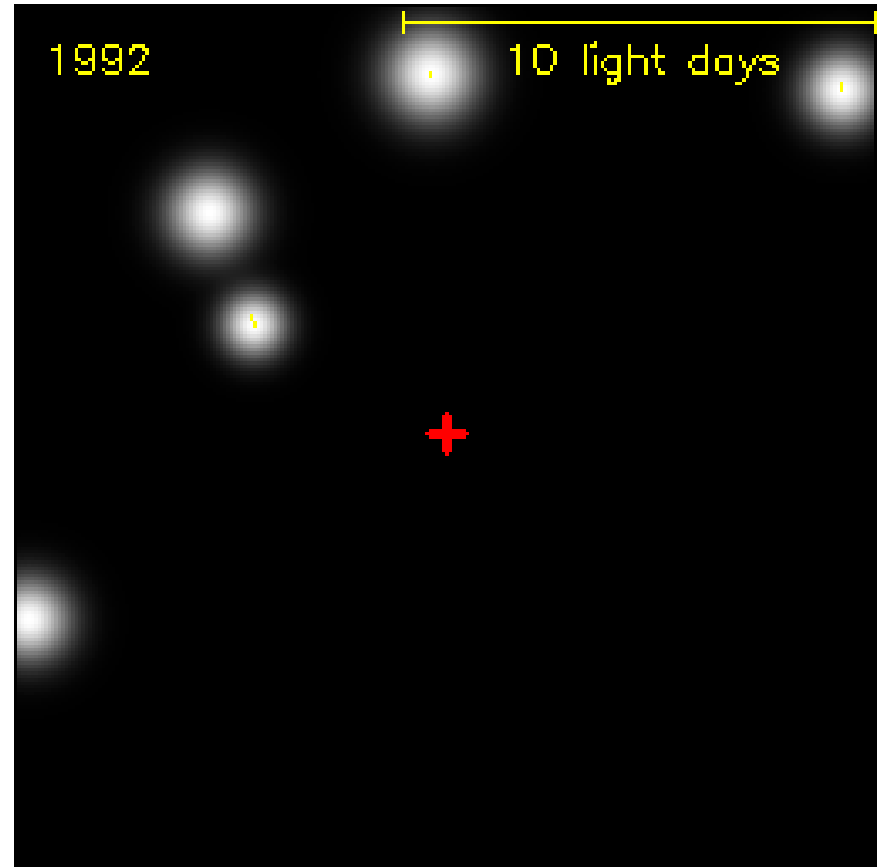
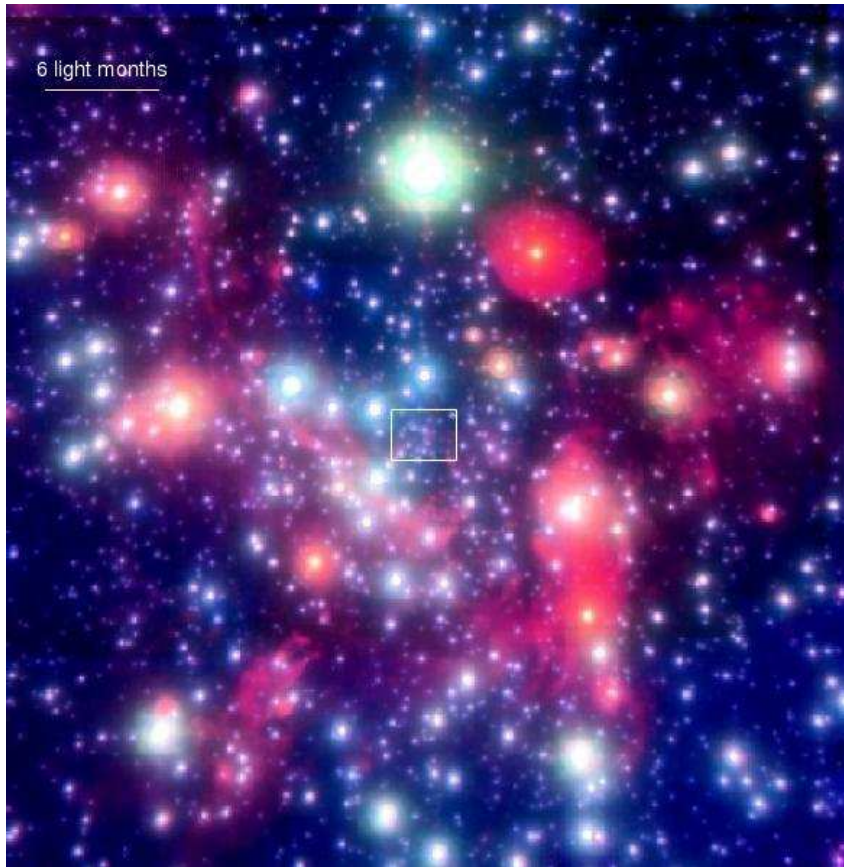


Darth Vader says:
Don't underestimate the power
of the dark side...

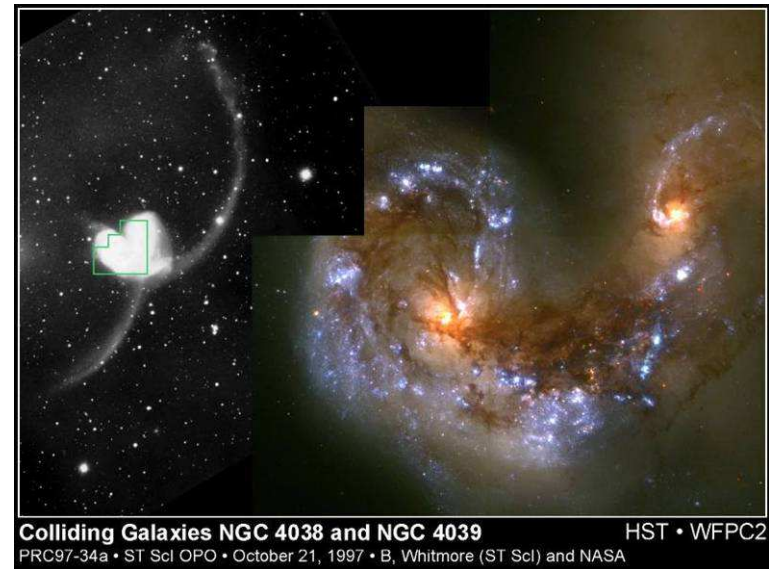
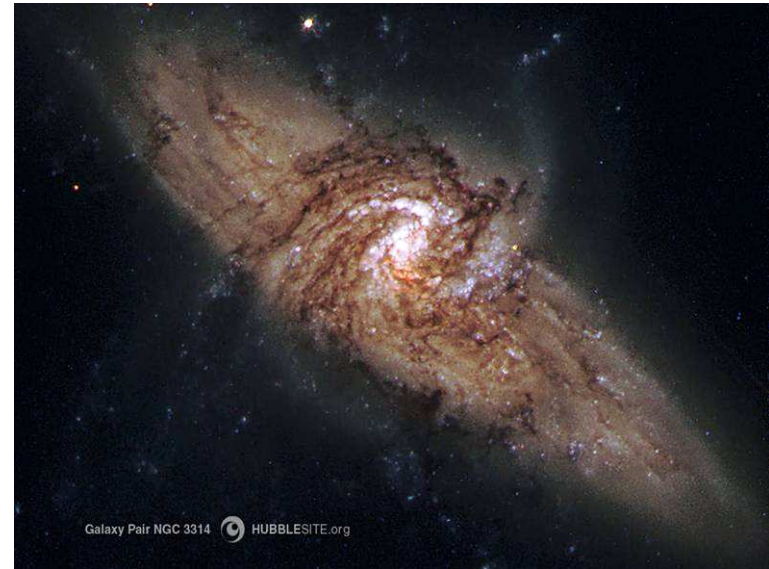
Observations



BH's in galaxies (MW - Sgr A*)



Galaxy Collisions



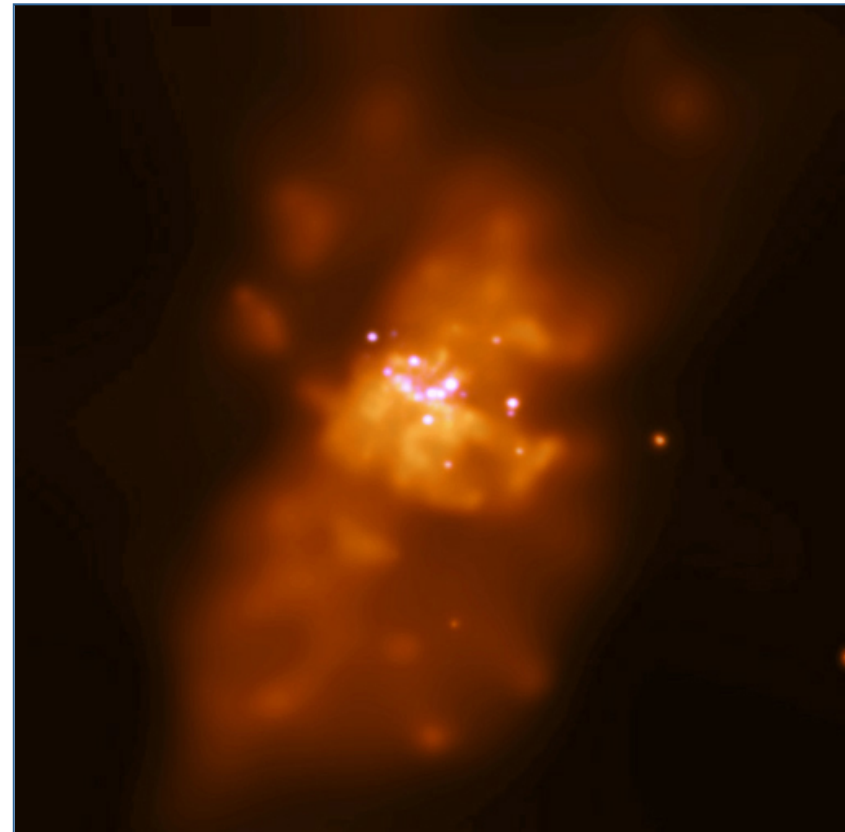
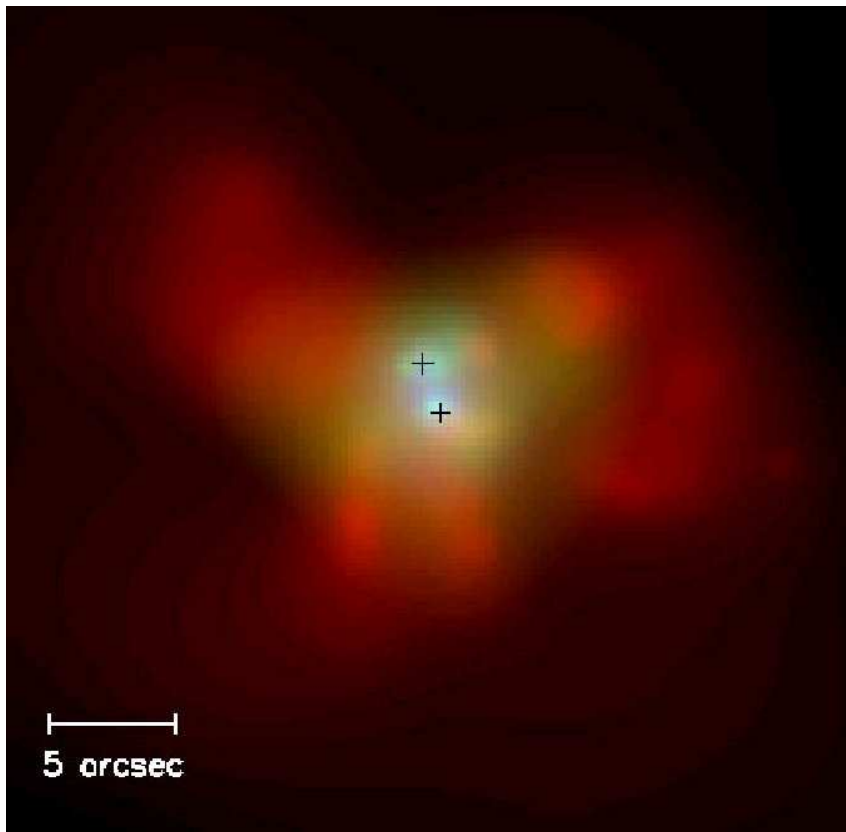
Galaxy Collisions \approx BH's collisions

Multiple Massive Black Holes
NGC6240

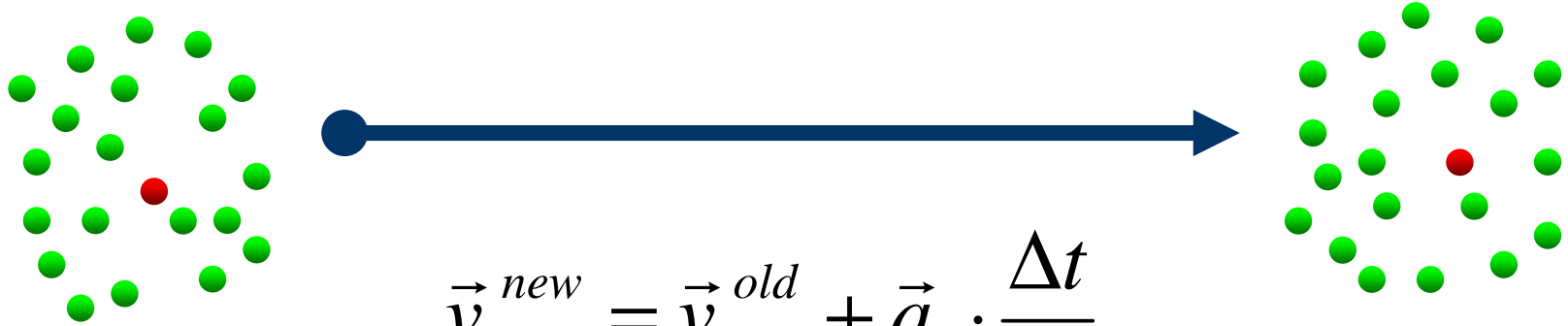
strong ongoing merger...
Komossa et al. 2002

Two AGN in each of Nuclei separation
 ~ 1 kpc Chandra X-Ray

M82: The bright spots in the center are
supernova remnants and X-ray binaries. The
luminosity of the X-ray binaries suggests that
most contain a black hole. A close encounter
with a large galaxy, M81, in the last 100 Myr is
thought to be the cause of the starburst activity.
Ebisuzaki et al. 2002



Basic idea of any N-body code



$$\vec{v}_i^{new} = \vec{v}_i^{old} + \vec{a}_i \cdot \frac{\Delta t}{2}$$

$$t^{old} \quad \vec{r}_i^{new} = \vec{r}_i^{old} + \vec{v}_i^{new} \cdot \Delta t$$

$$t^{new} = t^{old} + \Delta t$$

$$\vec{r}_i^{old}$$

$$\vec{a}_i = - \sum_{j=1; j \neq i}^N \frac{G \cdot m_j}{(r_{ij}^2 + \epsilon^2)^{3/2}} \vec{r}_{ij}$$

$$\vec{r}_i^{new}$$

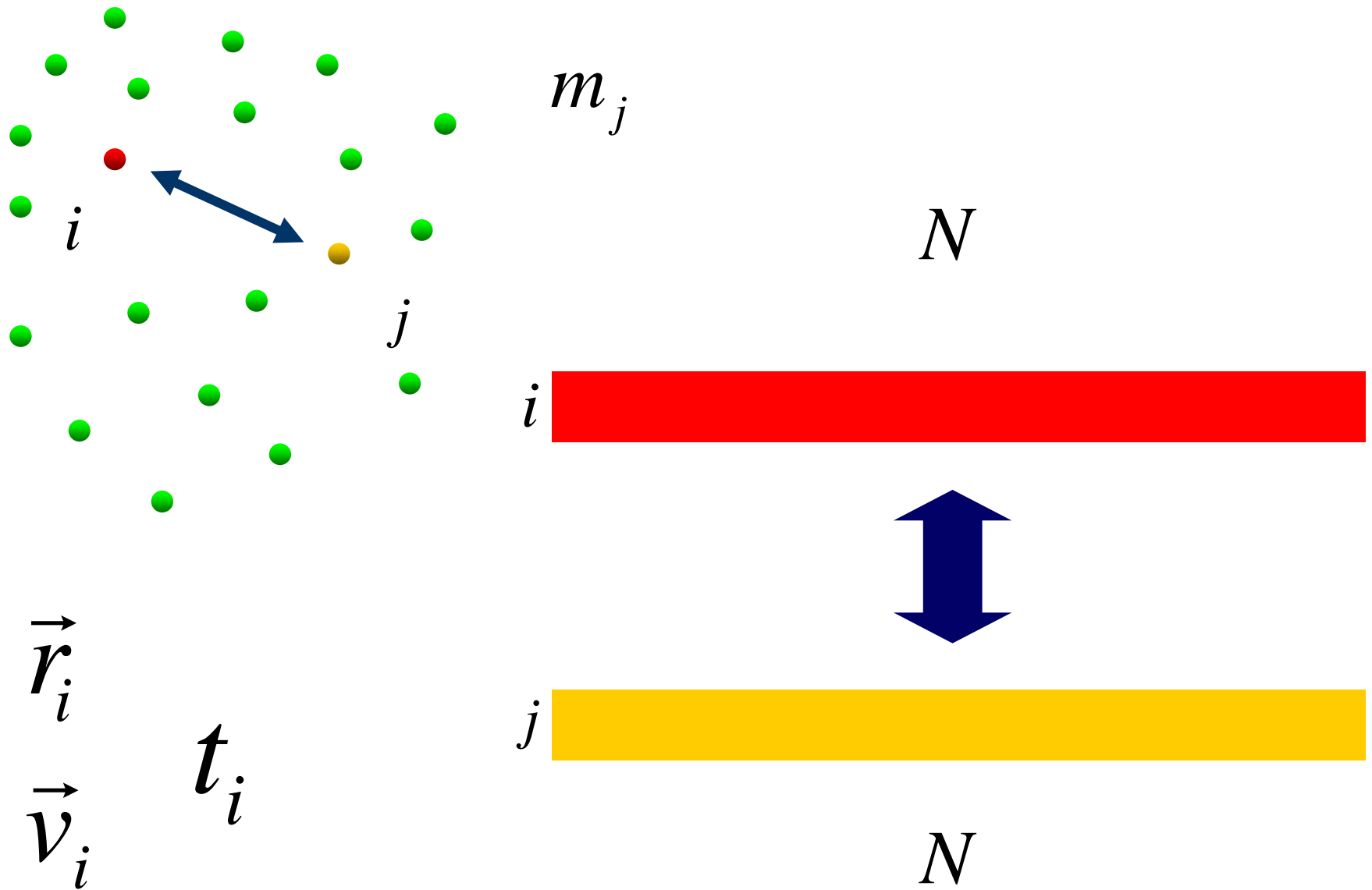
$$\vec{v}_i^{old}$$

$$\vec{v}_i^{new}$$

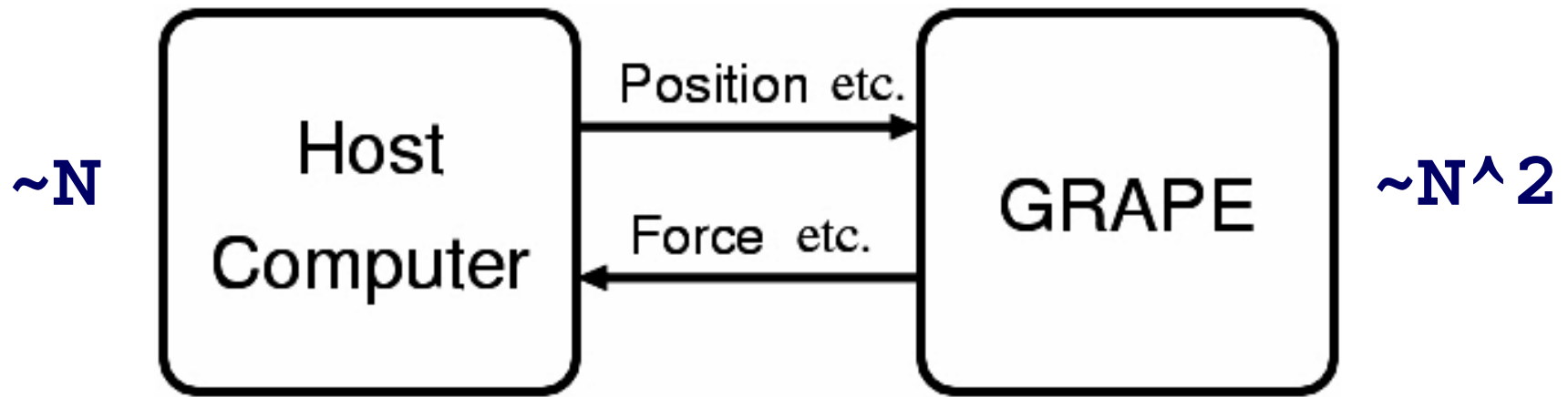
$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$$

$$\vec{v}_i^{new} = \vec{v}_i^{old} + \vec{a}_i \cdot \frac{\Delta t}{2}$$

Basic idea of any N-body code



Basic idea of any **GRAPE N-body code**



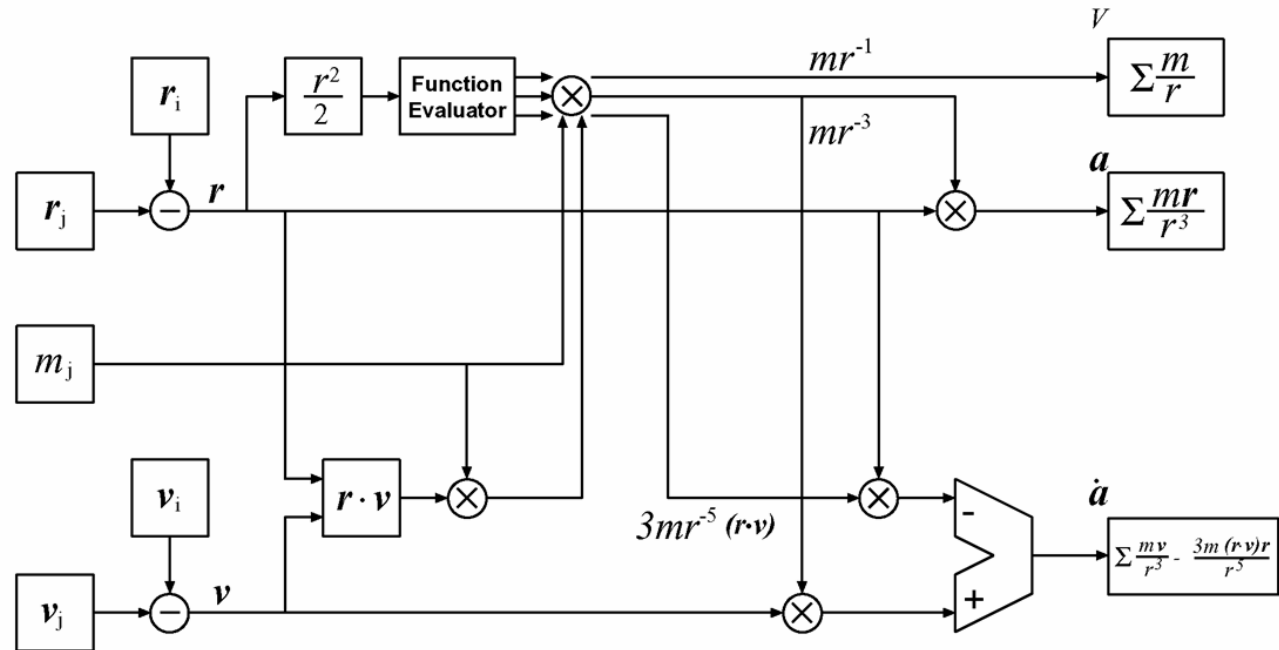
$$\vec{a}_i = \sum_{j=1; j \neq i}^N \vec{f}_{ij} \quad \vec{f}_{ij} = - \frac{G \cdot m_j}{(r_{ij}^2 + \epsilon^2)^{3/2}} \vec{r}_{ij}$$

GRAPE = GRAVity PipE – more detail...

$$m_i; \vec{r}_i; \vec{v}_i; t_i$$

48 pipelines

$$m_j; \vec{r}_j; \vec{v}_j; t_j$$



$$\phi_i; \vec{a}_i; \dot{\vec{a}}_i$$

Commerce GRAPE6a boards

<http://www.metrix.co.jp>

Profile



Hamamatsu **Metrix**
浜松メトリックス株式会社

Micro Grape 6Af

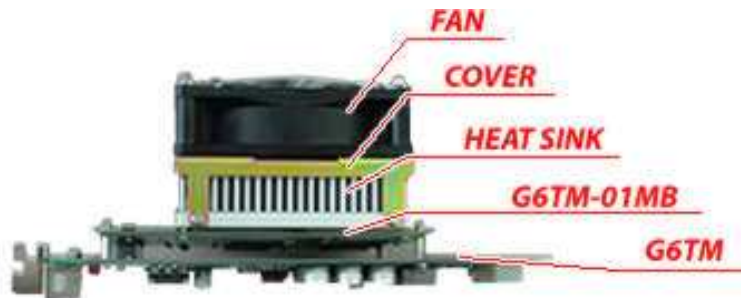
G6TM01



▲ Rear side



▲ Front Side



▲ Front side

Grape 6 BL4

The most suitable for a Cluster system

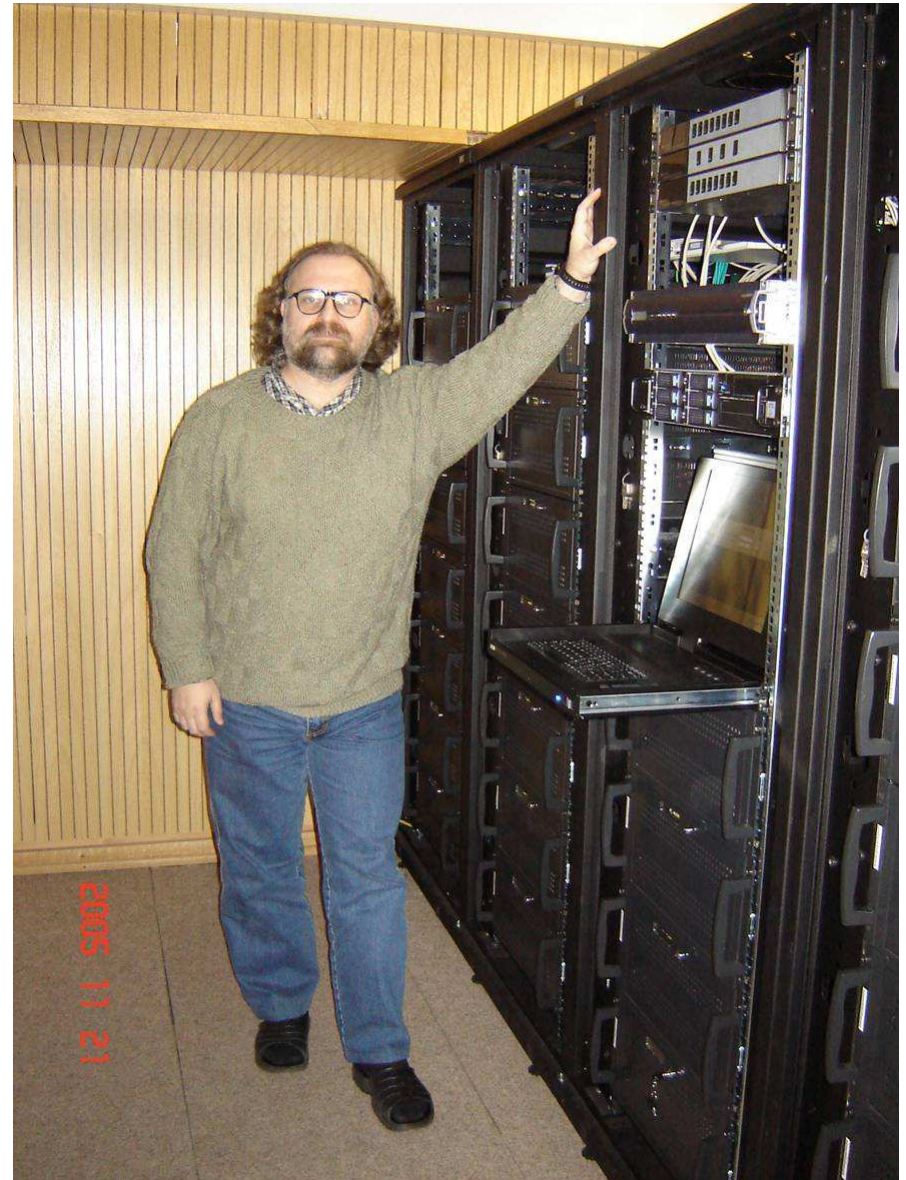


▲ Grape6 BL4 with Heatsink

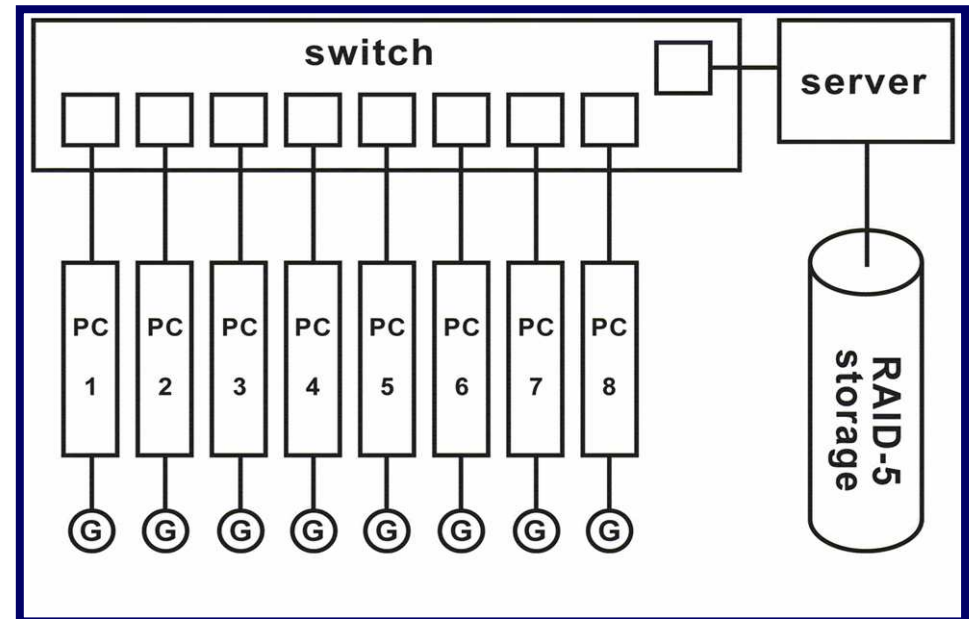


G6TM-01 assembly with
G6TM-01MB module

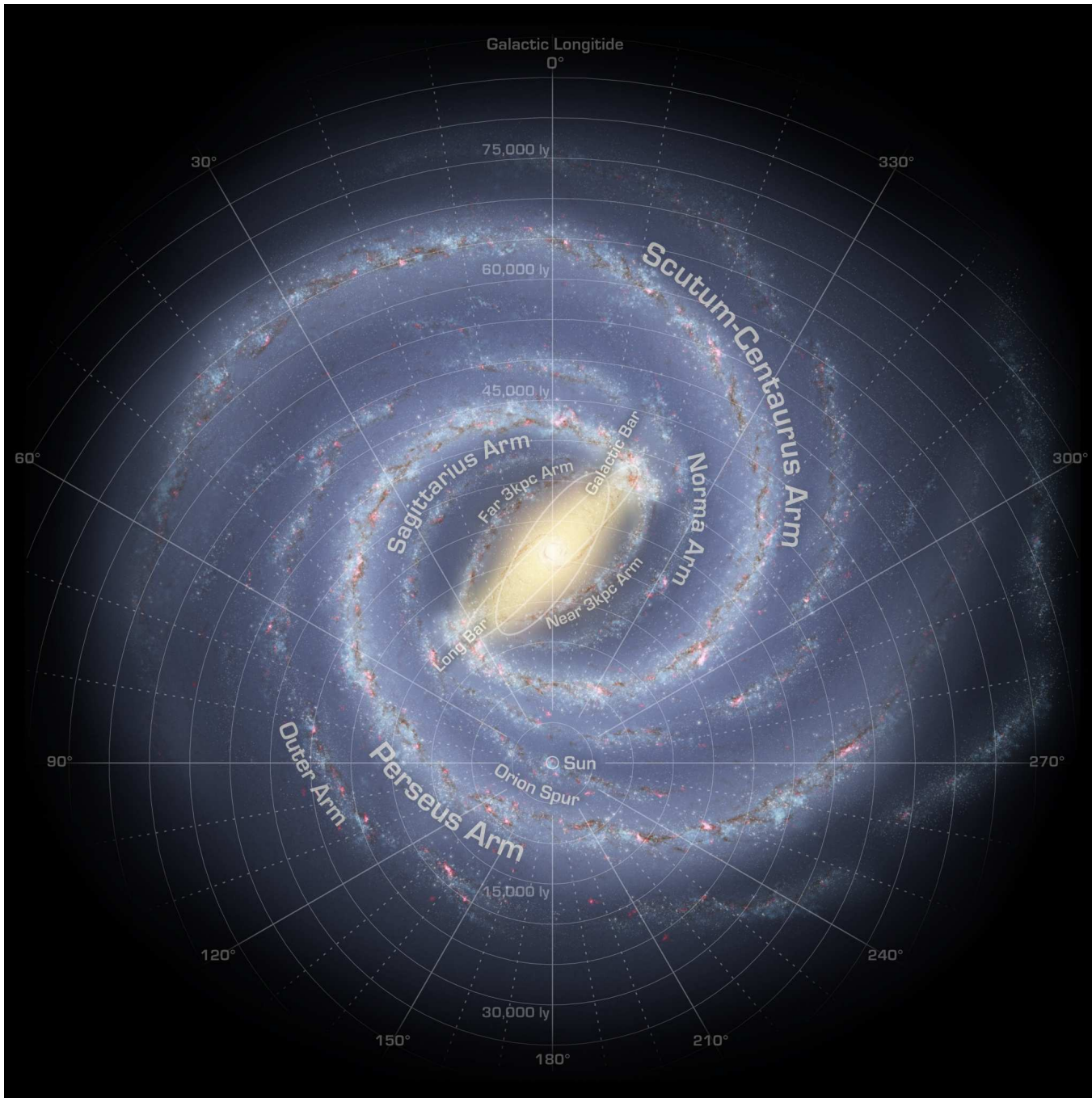
RIT & ARI 32 node GRAPE6a clusters



MAO 8+1 node GRAPE6 blx64 cluster



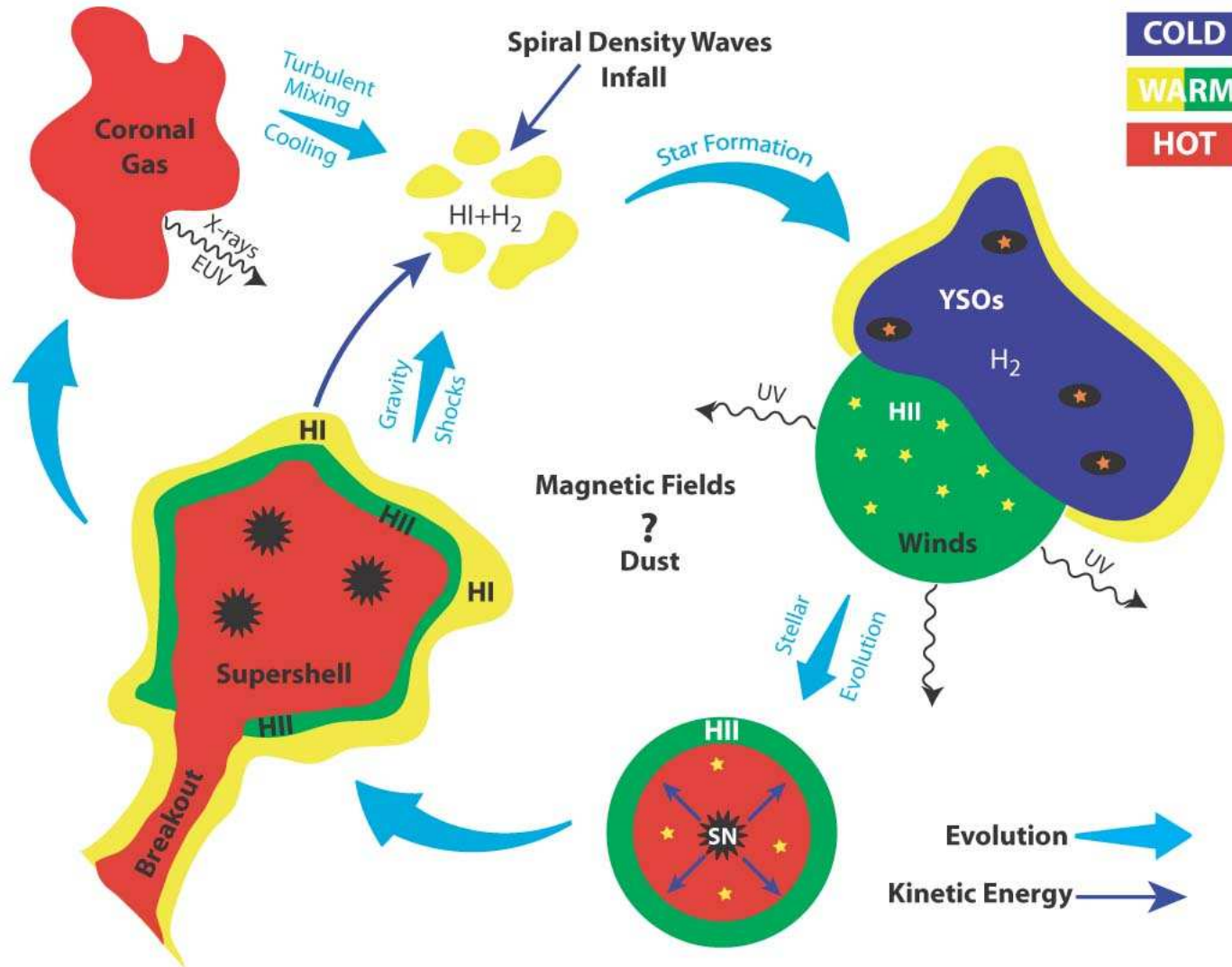
- 9 x2 dual-core Xeon 2.0 GHz
- 9 GRAPE6 blx64
- 5 TB RAID
- Infiniband switch (2x10 Gb/s)
- Speed: ~1 Tflops
- N up to 2M
- Cost: ~100k EUR
- Funding: NASU



~80% stars
~20% gas

**Highly
multi-phase**

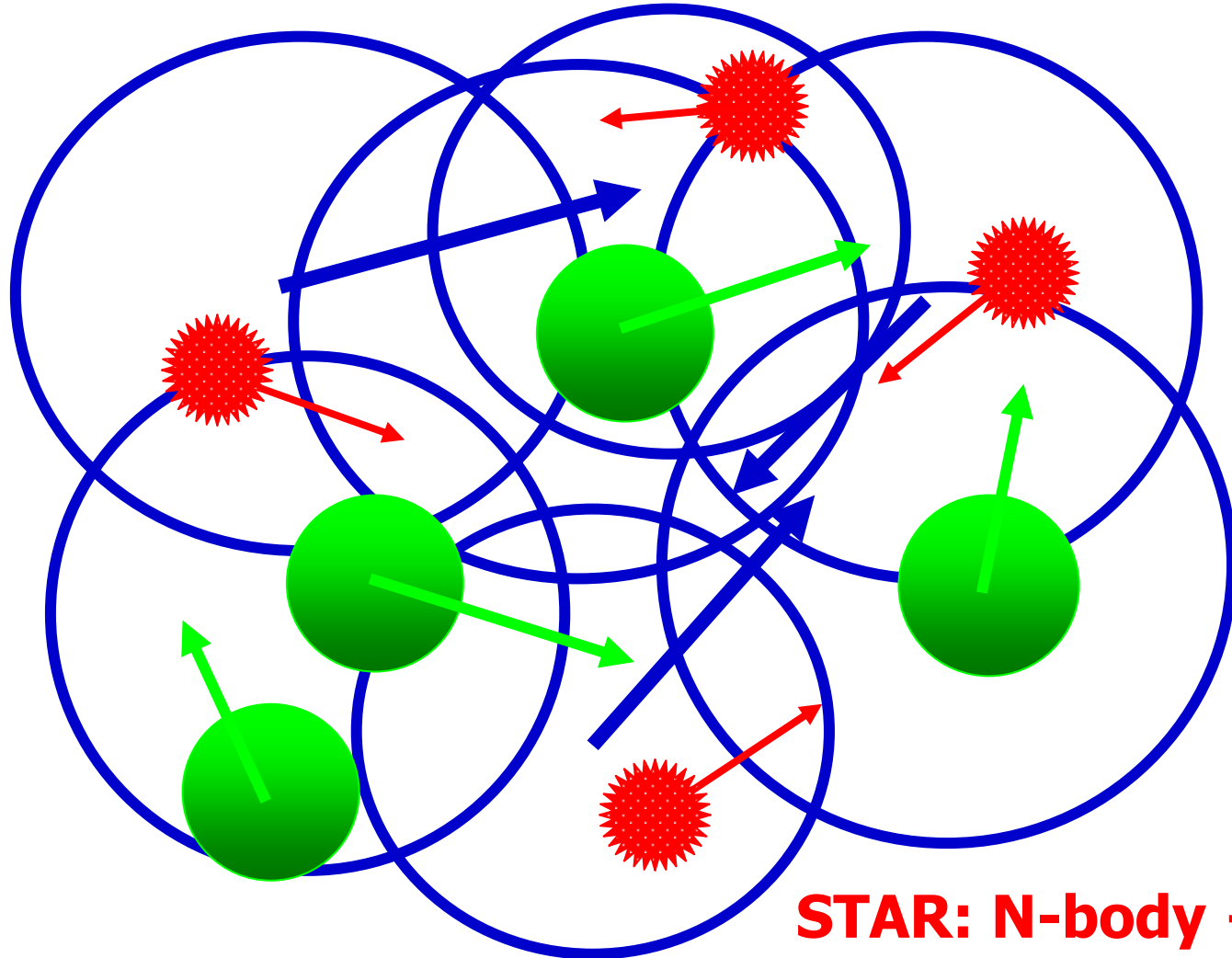
ISM "Ecology"



Tumlinson, 2004: astro-ph/0411249

Multi-Phase SPH code

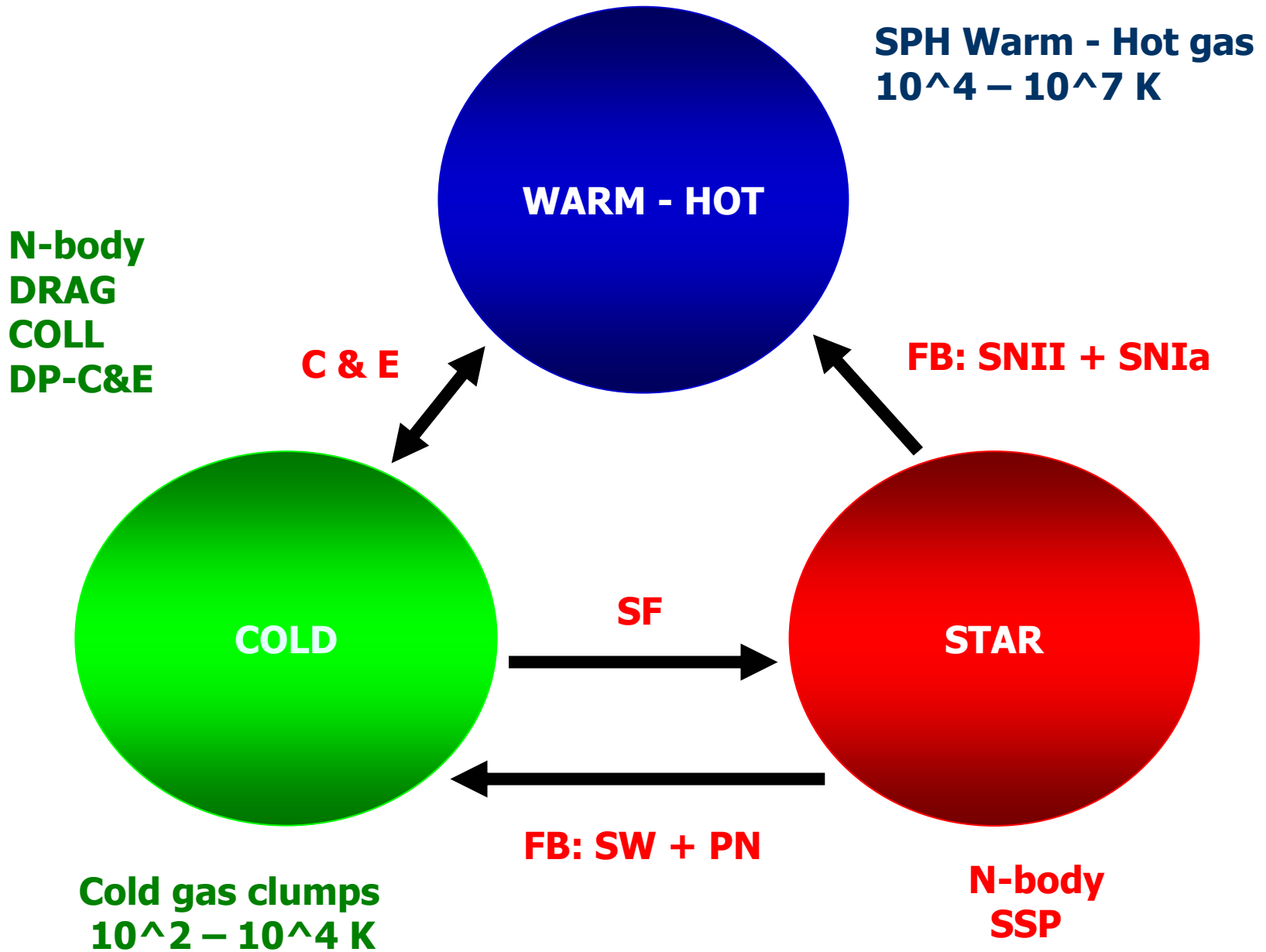
WARM - HOT: SPH



STAR: N-body + SSP

COLD: N-body + Viscosity

Multi-Phase SPH code



Basic Equations

Monaghan, 1977; Lucy, 1977 $\rho_i \equiv \sum_{j=1}^N m_j \cdot W_{ij}$

$$\frac{d\vec{v}_i}{dt} = - \sum_{j=1}^N m_j \cdot \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \tilde{\Pi}_{ij} \right) \cdot \vec{\nabla}_i W_{ij} - \vec{\nabla}_i \Phi_i - \vec{\nabla}_i \Phi_i^{ext}$$

$$\frac{du_i}{dt} = \frac{1}{2} \sum_{j=1}^N m_j \cdot \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \tilde{\Pi}_{ij} \right) \cdot (\vec{v}_i - \vec{v}_j) \cdot \vec{\nabla}_i W_{ij} + \frac{\Gamma_i - \Lambda_i}{\rho_i}$$

$$P_i = (\gamma - 1) \cdot \rho_i \cdot u_i$$

Basic Equations

Monaghan & Gingold, 1983

$$\Pi_{ij} = \begin{cases} [-\alpha \cdot c_{ij} \cdot \mu_{ij} + \beta \cdot \mu_{ij}^2] / \rho_{ij} & \text{if } (\vec{r}_{ij} \cdot \vec{v}_{ij}) < 0 \\ 0 & \text{else} \end{cases}$$

$$\mu_{ij} = \frac{h_{ij} \cdot (\vec{v}_i - \vec{v}_j) \cdot (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^2 + \varepsilon \cdot h_{ij}^2}$$

$\alpha = 1$
 $\beta = 2$
 $\varepsilon = 0.01$

$$\rho_{ij} = \frac{1}{2}(\rho_i + \rho_j) \quad h_{ij} = \frac{1}{2}(h_i + h_j) \quad c_{ij} = \frac{1}{2}(c_i + c_j)$$

Basic Equations

Monaghan & Lattanzio, 1985

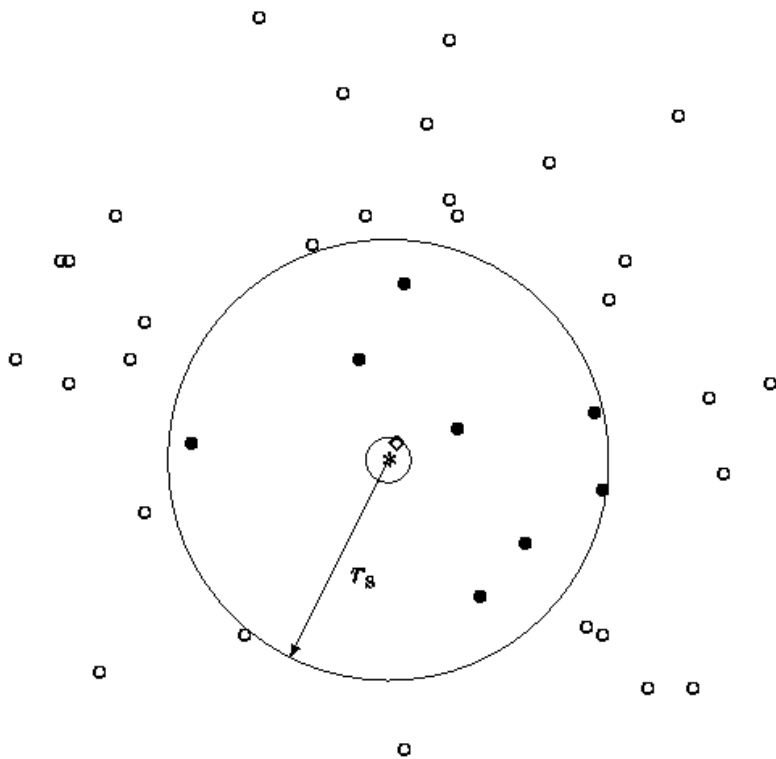
Hernquist & Katz, 1989

$$W(r;h) = \frac{1}{\pi \cdot h^3} \cdot \begin{cases} 1 - \frac{3}{2}(r/h)^2 + \frac{3}{4}(r/h)^3 & 0 \leq r/h < 1 \\ \frac{1}{4}(2 - r/h)^3 & 1 \leq r/h < 2 \\ 0 & 2 \leq r/h \end{cases} \quad W_{ij} = W(|\vec{r}_i - \vec{r}_j|; h_{ij})$$

Balsara, 1995; Steinmetz, 1996

$$\tilde{\Pi}_{ij} = \frac{1}{2} (f_i + f_j) \cdot \Pi_{ij} \quad f_i = \frac{|\vec{\nabla} \cdot \vec{v}|_i}{|\vec{\nabla} \cdot \vec{v}|_i + |\vec{\nabla} \times \vec{v}|_i + \varepsilon \cdot c_i / h_i}$$

Define smoothing length



$$|r_{ij}| \leq 2 \cdot \max(h_i; h_j)$$

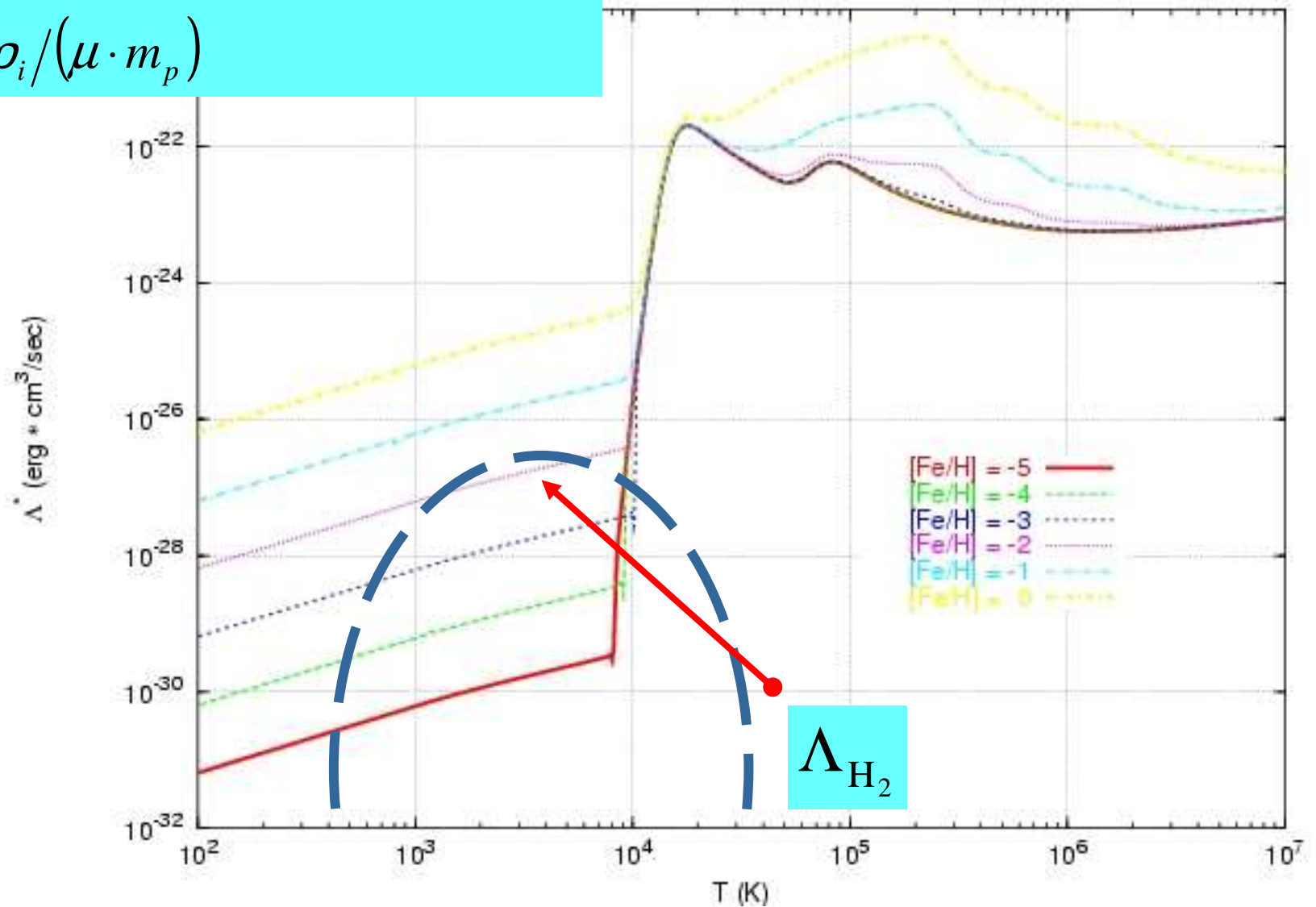
Inside "2•h" $N_B = \text{const} = 50$

Using the ANN with kdtree

<http://www.cs.umd.edu/~mount/ANN/>

Delgarno & McCray, 1972; Sutherland & Dopita, 1993

$$\Lambda \equiv \Lambda(\rho, u, Z, \dots) \cong \Lambda^*(T, [\text{Fe}/\text{H}]) \cdot n_i^2$$
$$n_i^2 = \rho_i / (\mu \cdot m_p)$$



Integrator

Predictor step:

$$\left\{ \begin{array}{l} \vec{v}_i^p = \vec{v}_i^n + \vec{a}_i^n \cdot \Delta t \\ \vec{r}_i^p = \vec{r}_i^n + (\vec{v}_i^n + \vec{v}_i^p) \cdot \frac{\Delta t}{2} \\ u_i^p = u_i^n + \dot{u}_i^n \cdot \Delta t \end{array} \right.$$

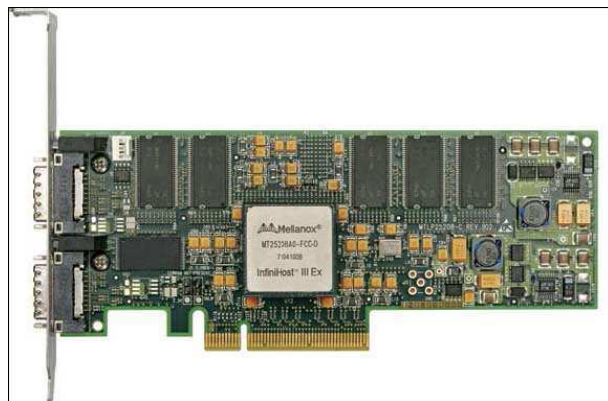
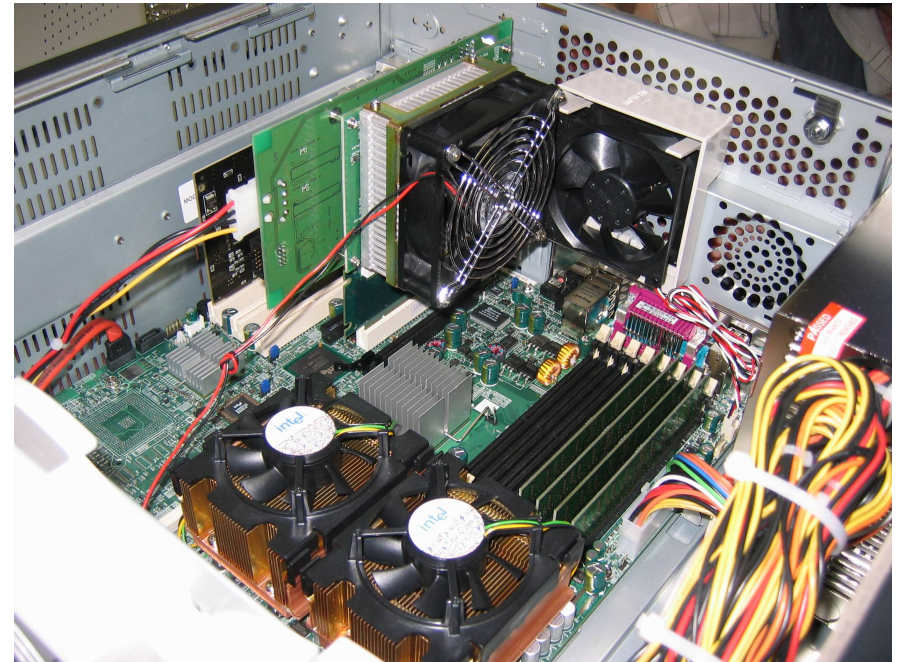
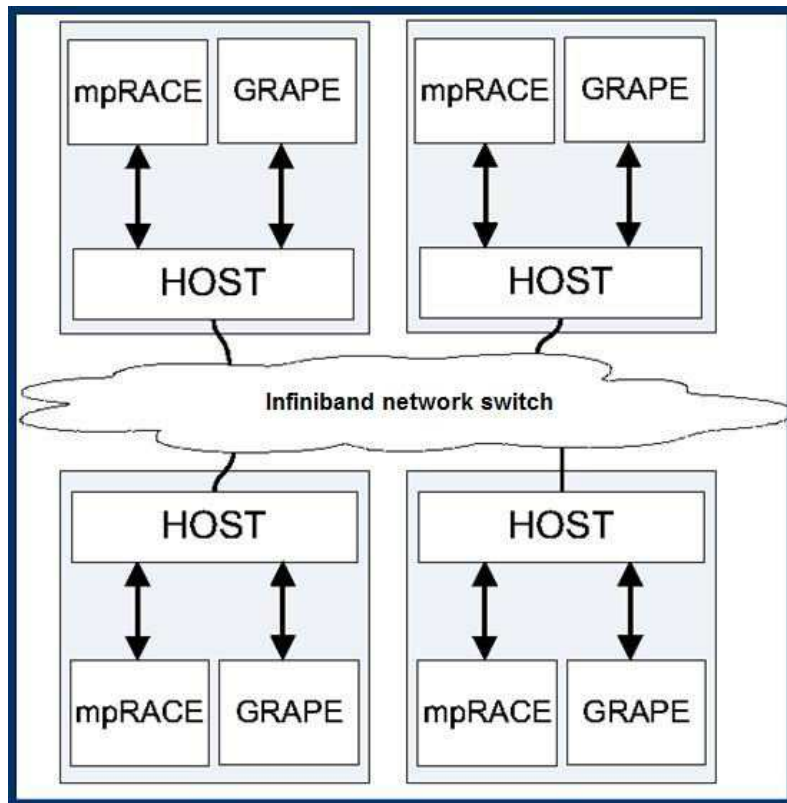
Corrector step:

$$\left\{ \begin{array}{l} \vec{v}_i^{n+1} = \vec{v}_i^n + (\vec{a}_i^n + \vec{a}_i^p) \cdot \frac{\Delta t}{2} \\ \vec{r}_i^{n+1} = \vec{r}_i^n + (\vec{v}_i^n + \vec{v}_i^{n+1}) \cdot \frac{\Delta t}{2} \\ u_i^{n+1} = u_i^n + (\dot{u}_i^n + \dot{u}_i^p) \cdot \frac{\Delta t}{2} \end{array} \right.$$

Time step:

$$\Delta t = 0.1 \cdot \min \left(\sqrt{\frac{2 \cdot h_i}{|\vec{a}_i|}}, \frac{h_i}{|\vec{v}_i|}, \frac{h_i}{c_i}, \frac{u_i}{\dot{u}_i} \right)$$

ARI 32 node GRAPE6a cluster



32x2 64 bit-Xeon P4, 3.2 GHz (~ 2 Gfps)

32 GRAPE6a (~ 120 Gfps)

32 FPGA-MPRACE (~ 20 Gfps)

3.5 TB RAID5 disk system

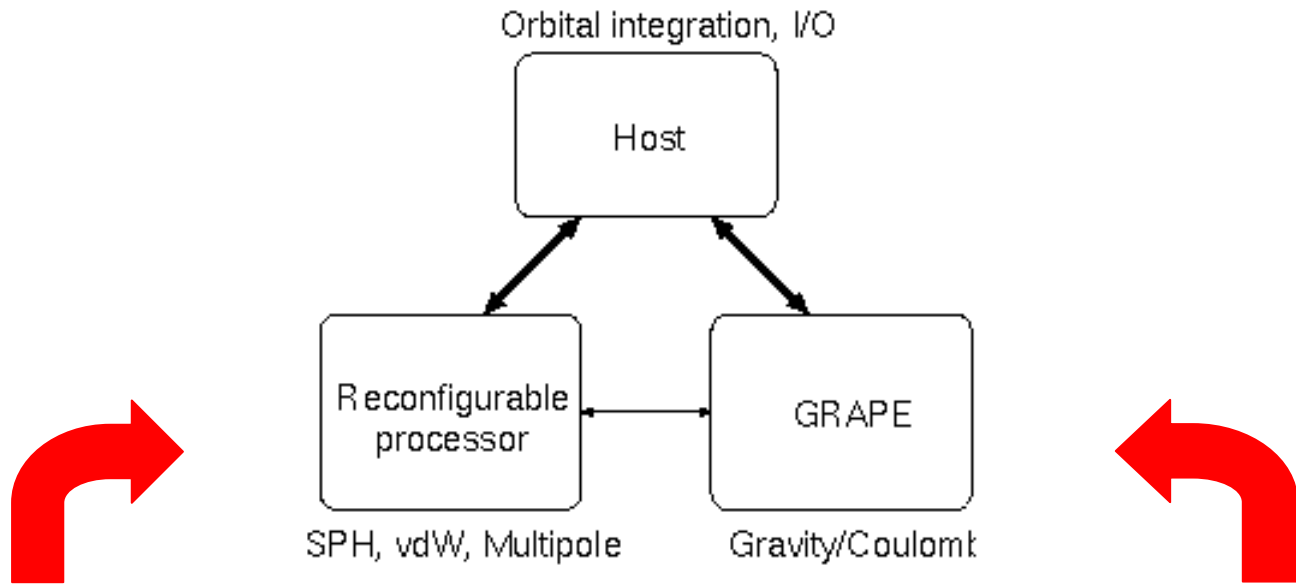
Infiniband, dual port network (~ 20 Gb/s)

Summary speed: ~ 4 Tfps

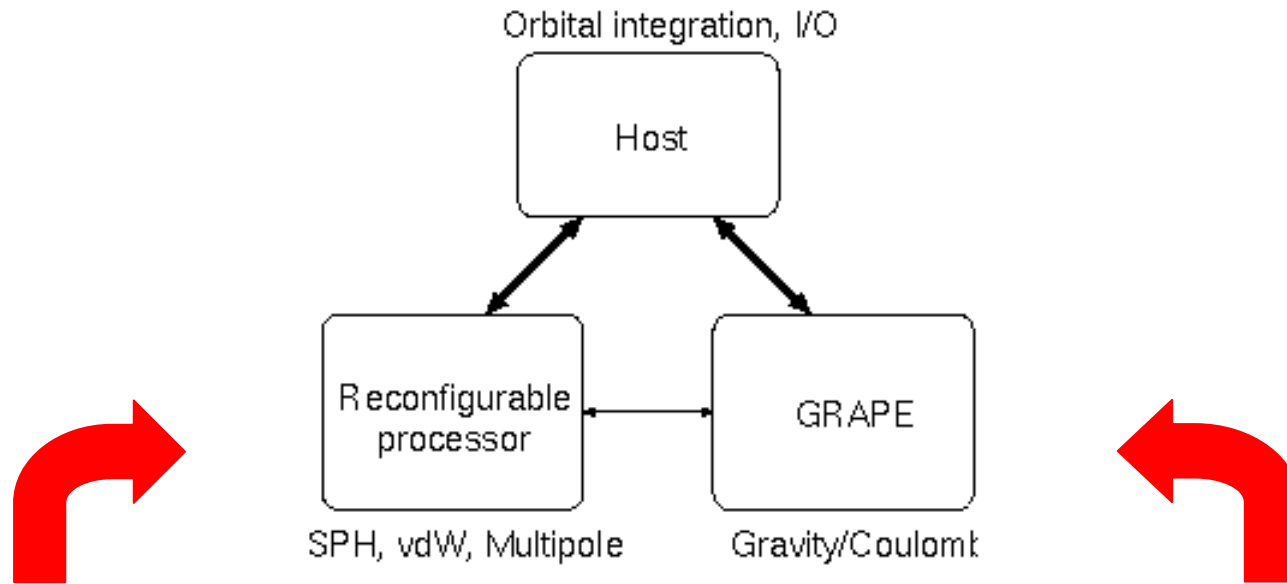
N (direct summation) up to 4M

Volkswagen/Baden-Württemberg ~ 400 k EUR

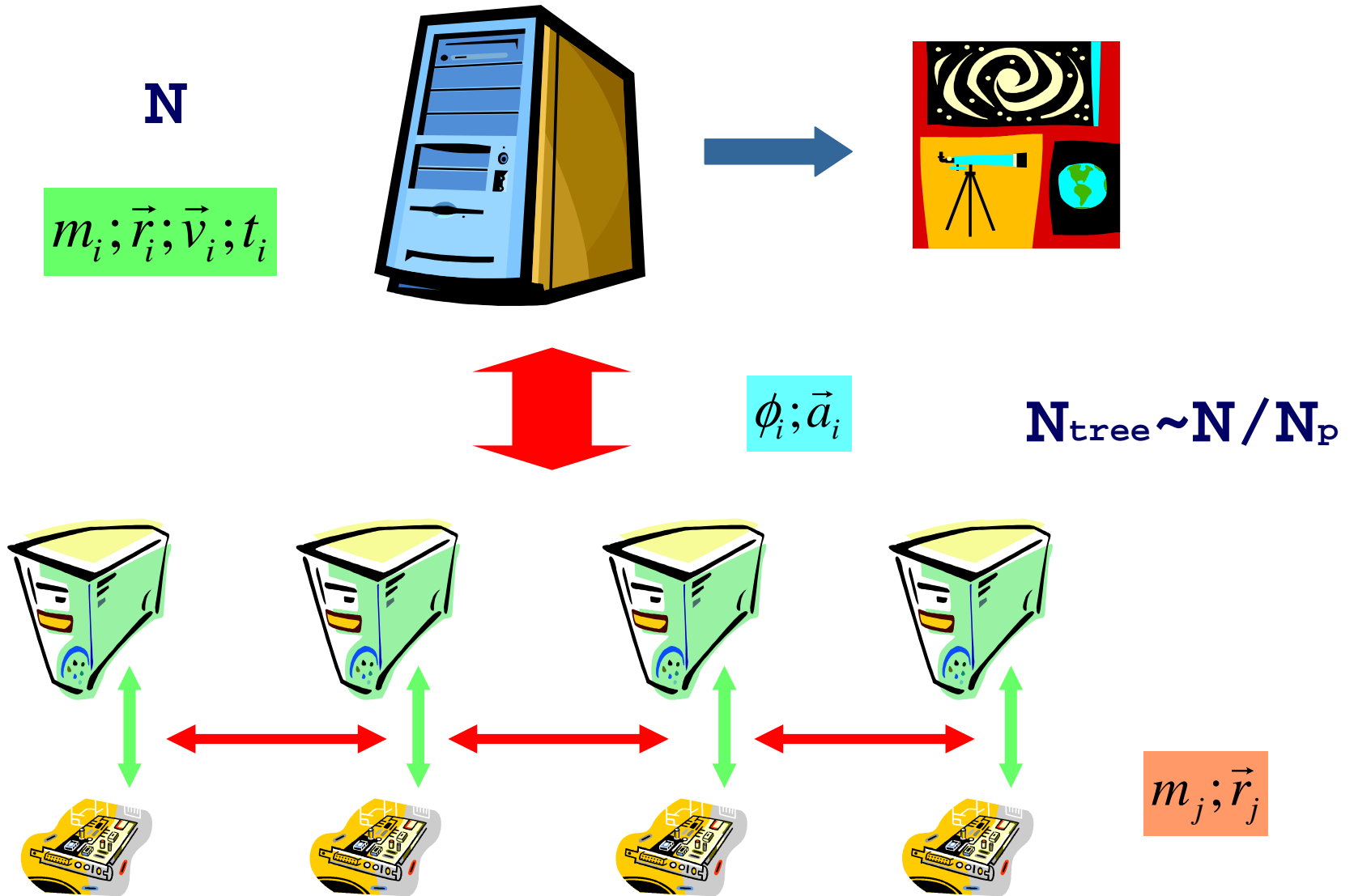
GRACE=GRAPE + MPRACE



GPU...

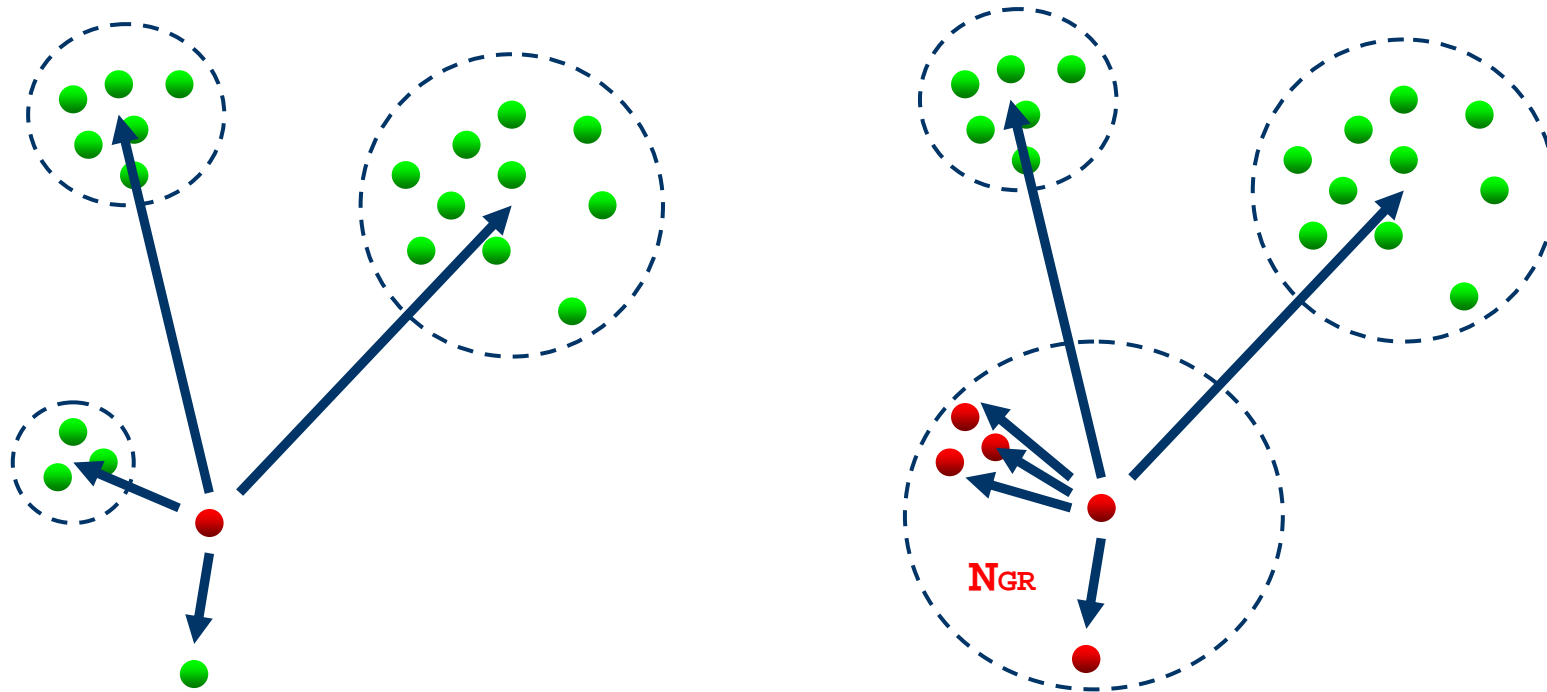


Parallel TREE gravity on the cluster



Parallel TREE gravity on the cluster

Jun Makino (pC++): TREE+GRAPE code



Makino, PASJ, 43, 621 (1991)

Inter. list on host $\sim N$
Inter. list length \rightarrow short...

Makino, PASJ, 56, 521 (2004)

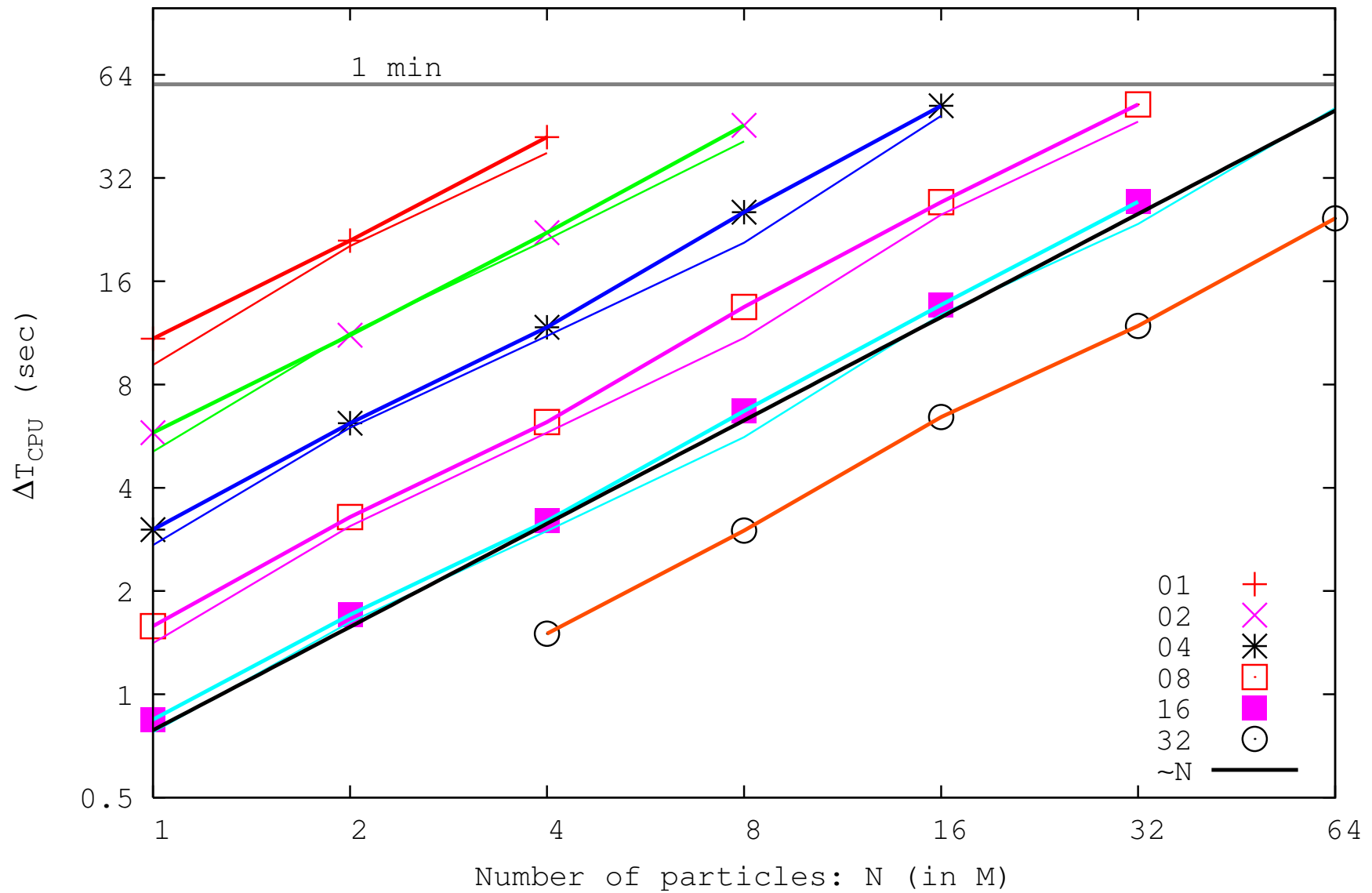
Fukushige, Makino & Kawai, PASJ, 57, 1009 (2005)

One interaction list is shared among
NGR particles!

Inter. list on host $\sim N/NGR$
Inter. list length \rightarrow larger...

Parallel TREE gravity on the cluster

Uniform & Plummer sphere, one full force calculation, $G=M=R=1$, $\epsilon=10^{-2}$



Parallel TREE gravity on the cluster

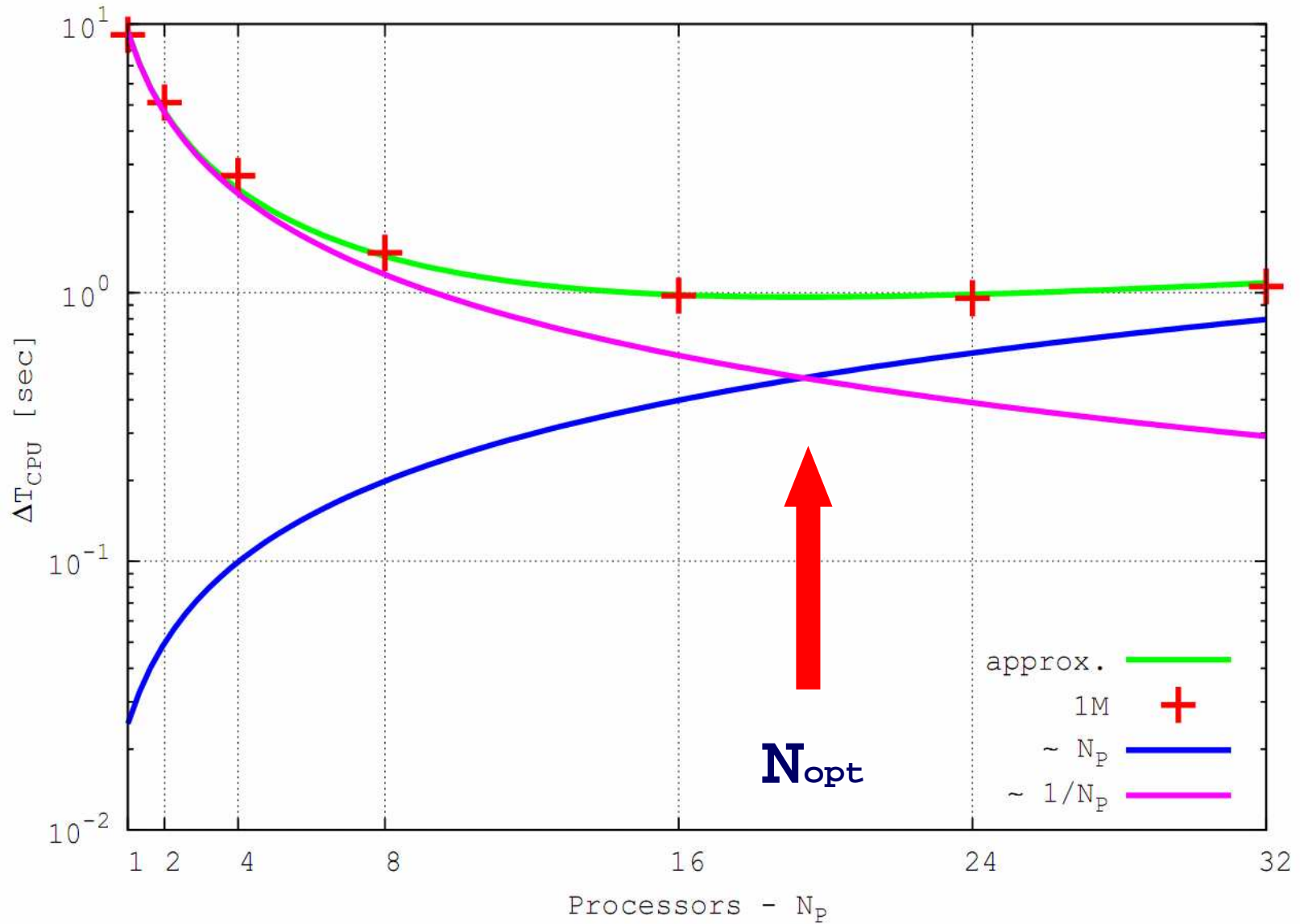
$$\varepsilon=0.001; \quad \theta=0.75; \quad 10 < N_{GR} < 3500$$

$$\Delta T_{total} = \Delta T_{constr} + \Delta T_{calc} + \Delta T_{comm}$$

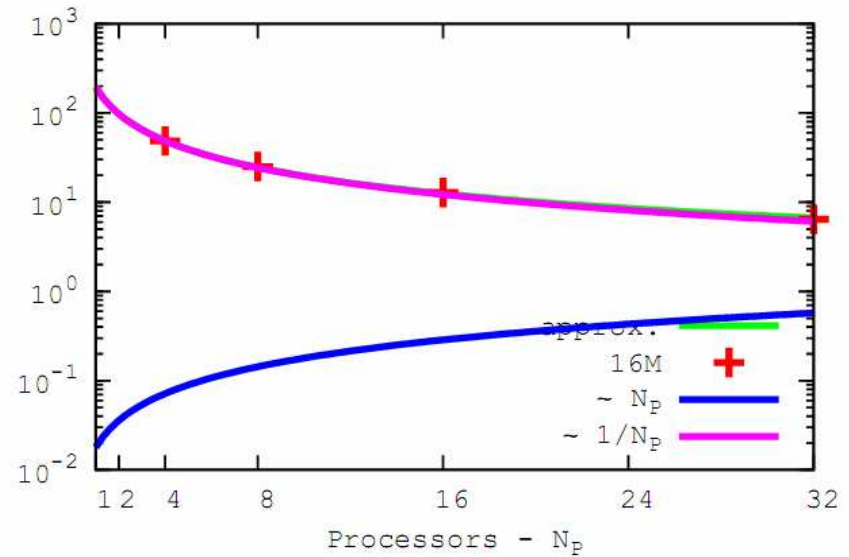
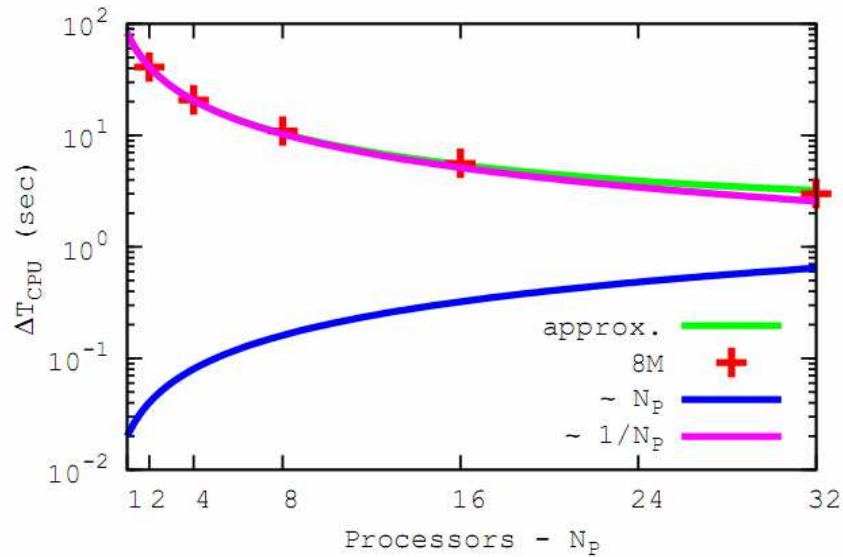
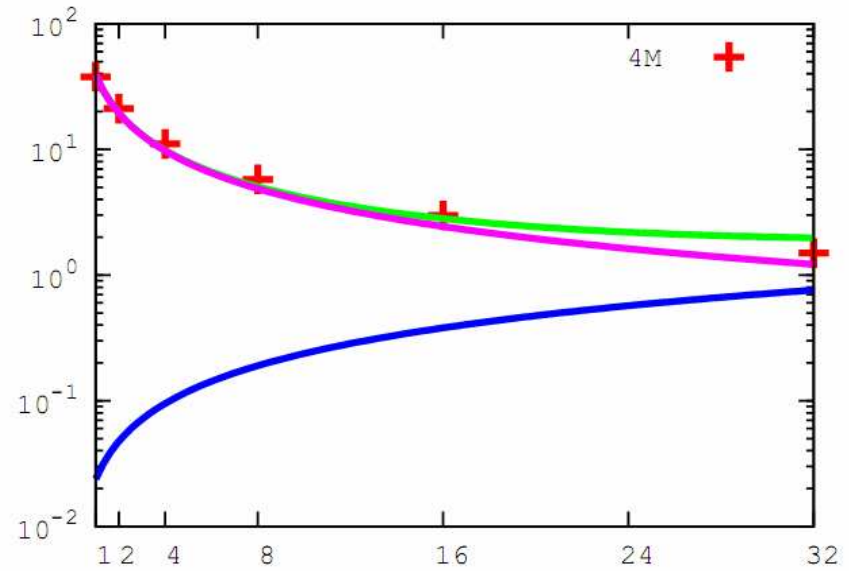
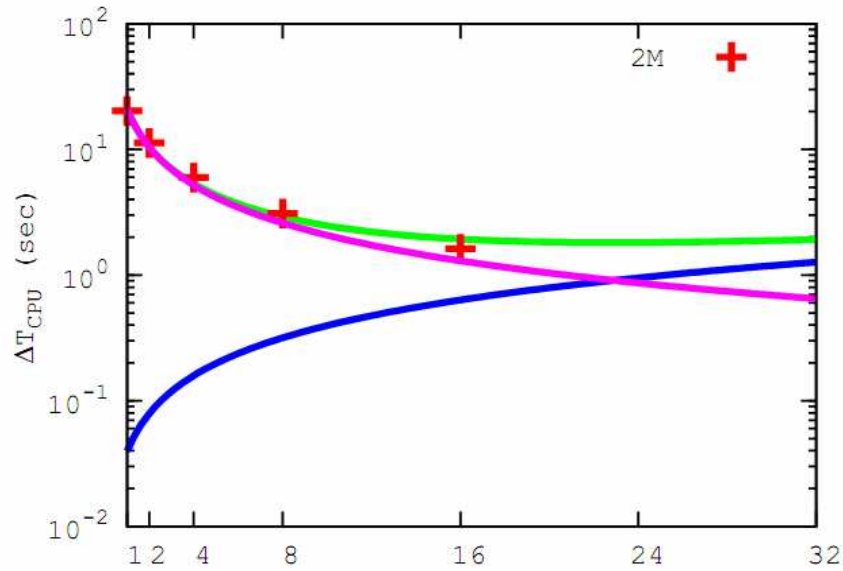
$$\Delta T_{constr} \propto \frac{N}{N_{proc}} \qquad \Delta T_{calc} \propto \frac{N \cdot \log(N)}{N_{proc}}$$

$$\Delta T_{comm} \propto N \cdot N_{proc}$$

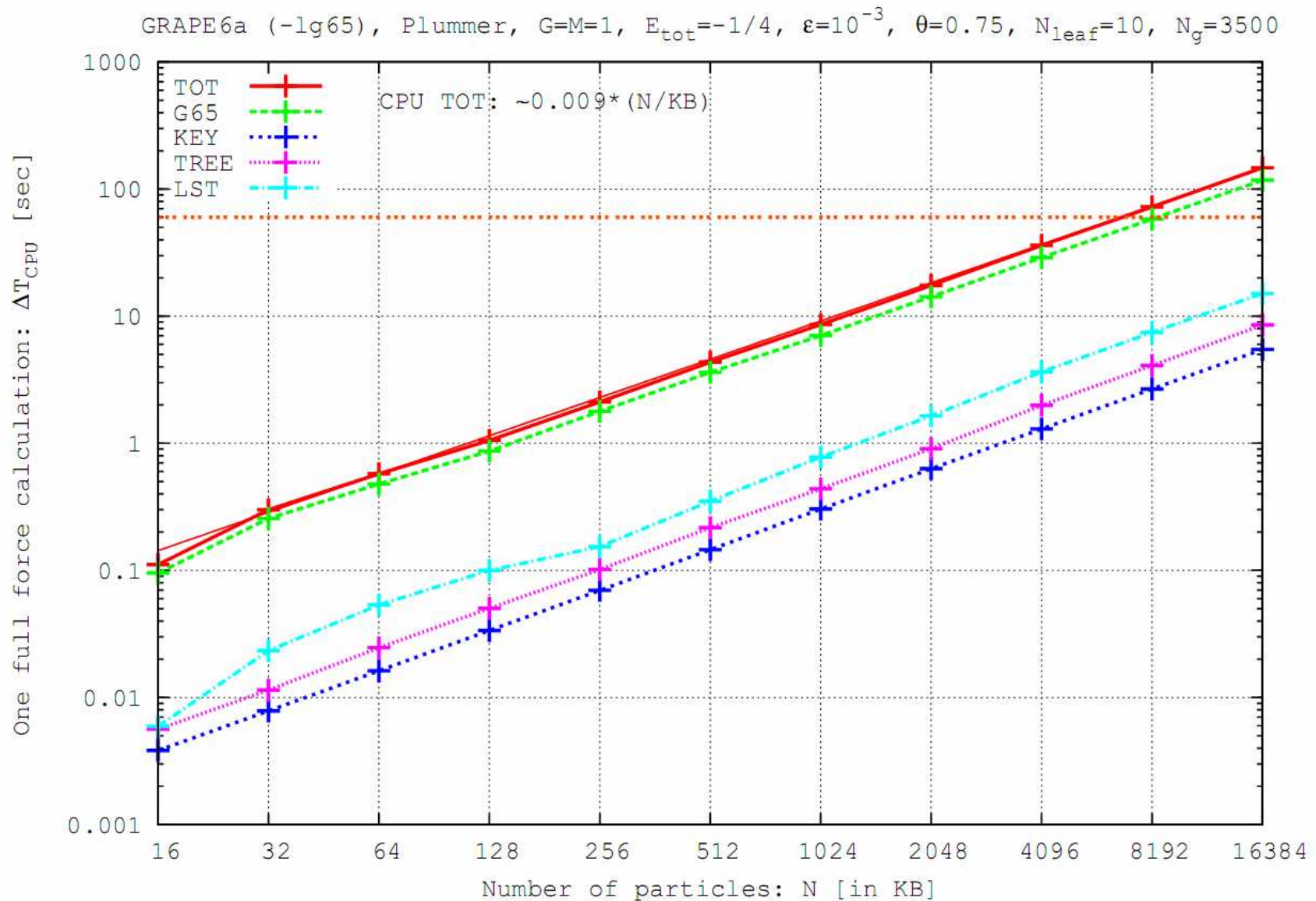
Parallel TREE gravity on the cluster



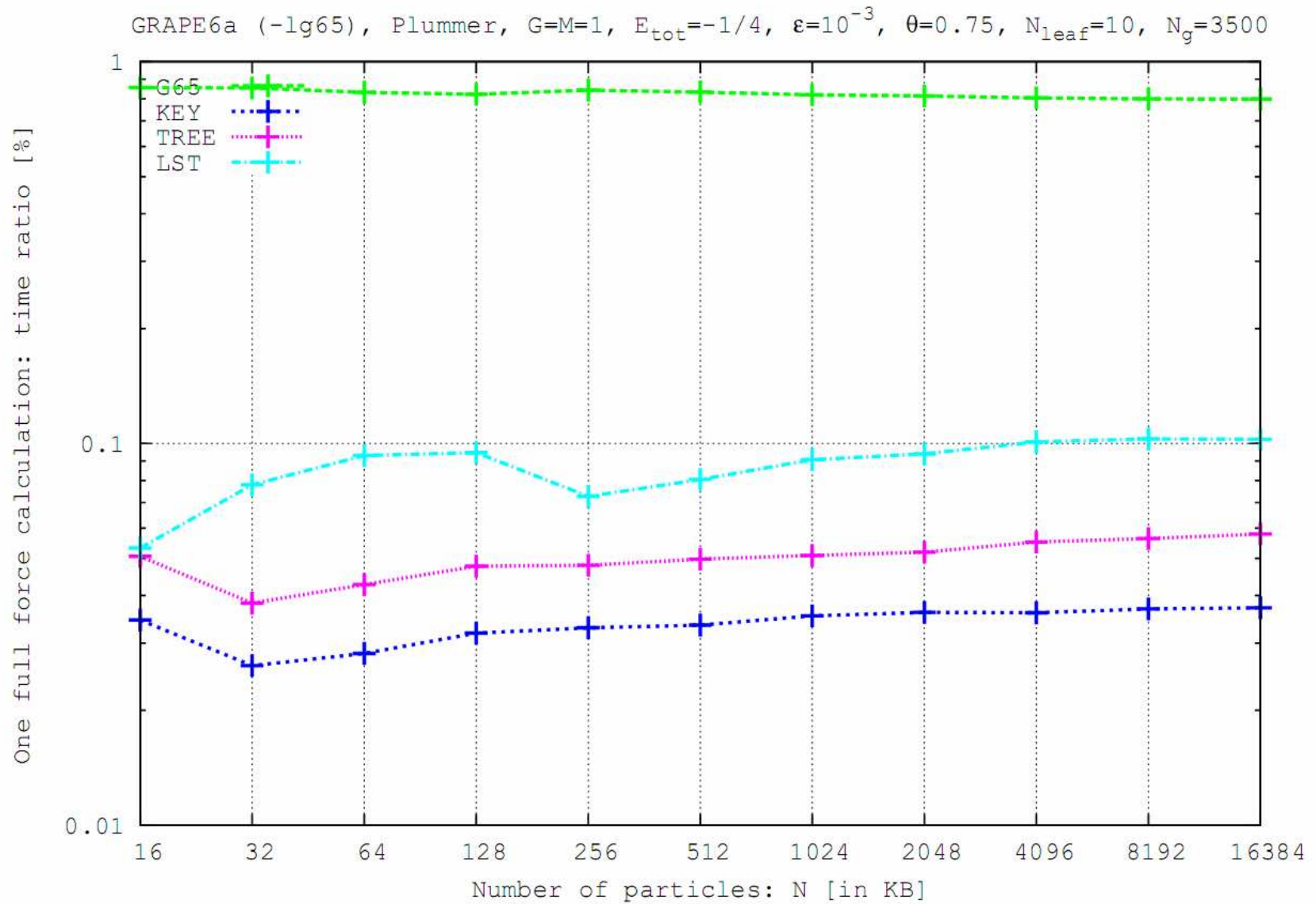
Parallel TREE gravity on the cluster



Parallel TREE gravity on the cluster



Parallel TREE gravity on the cluster



SPH - test

Adiabatic collapse of a cold gas sphere.

Evrard, 1988

Steinmetz & Muller, 1993

Carraro et al., 1998

Springel et al., 2001

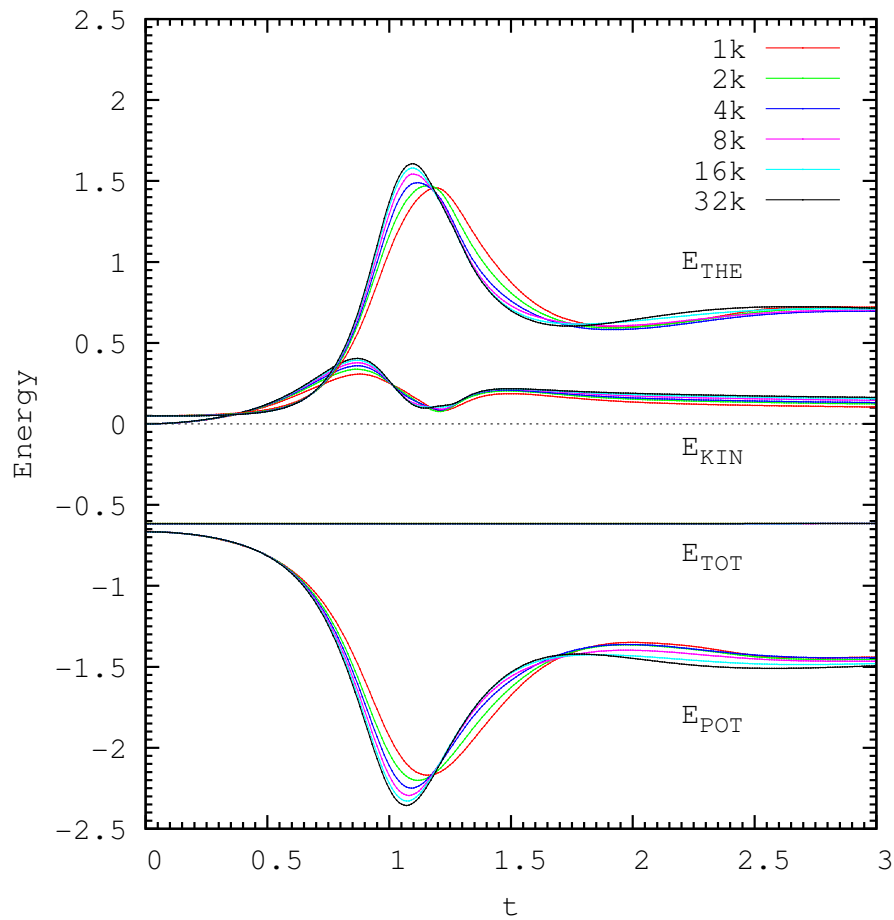
$$\rho = \frac{M}{2 \cdot \pi \cdot R^2} \cdot \frac{1}{r}$$

$$E_G = -\frac{2}{3} \cdot \frac{G \cdot M^2}{R}$$

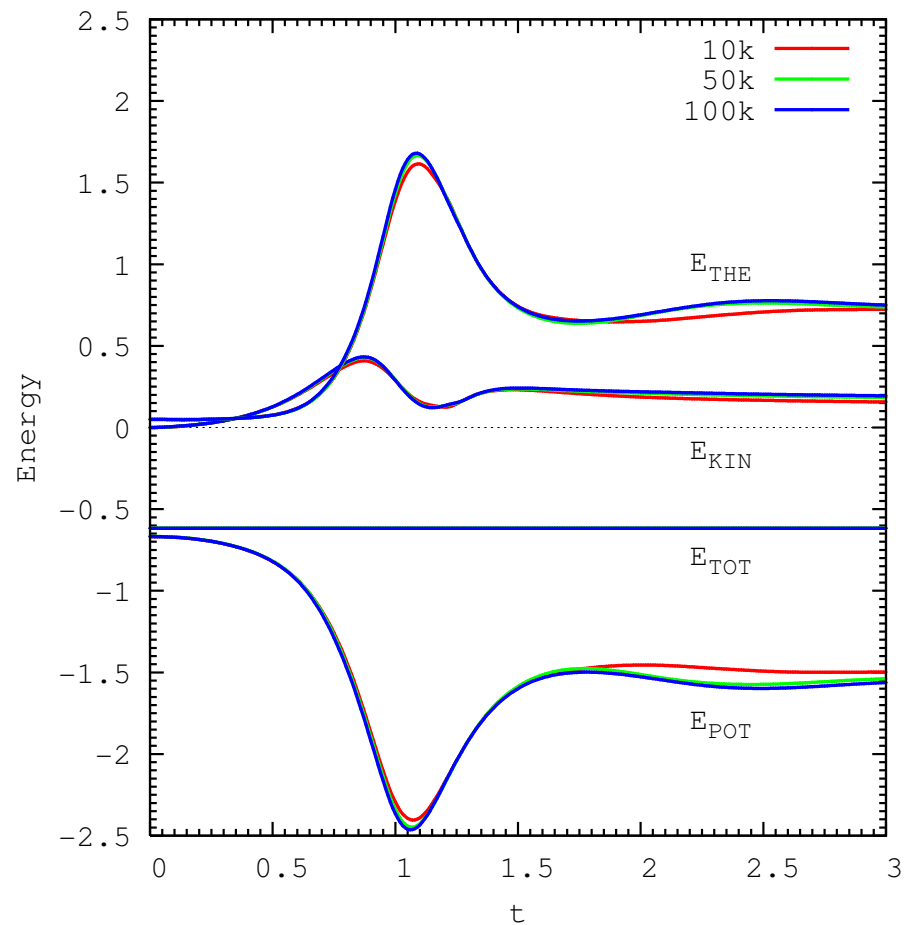
$$u = 0.05 \cdot \frac{G \cdot M}{R}$$

$$G = M = R = 1$$

SPH - test

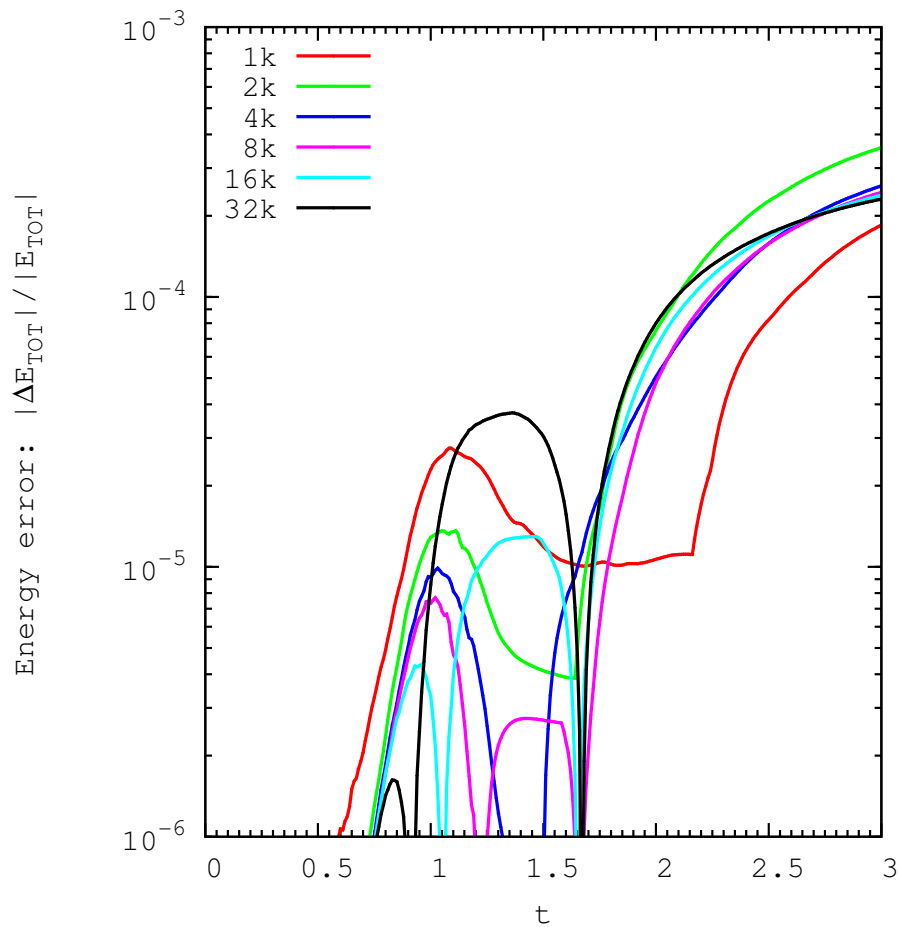


Berczik (N_{CPU}=1)

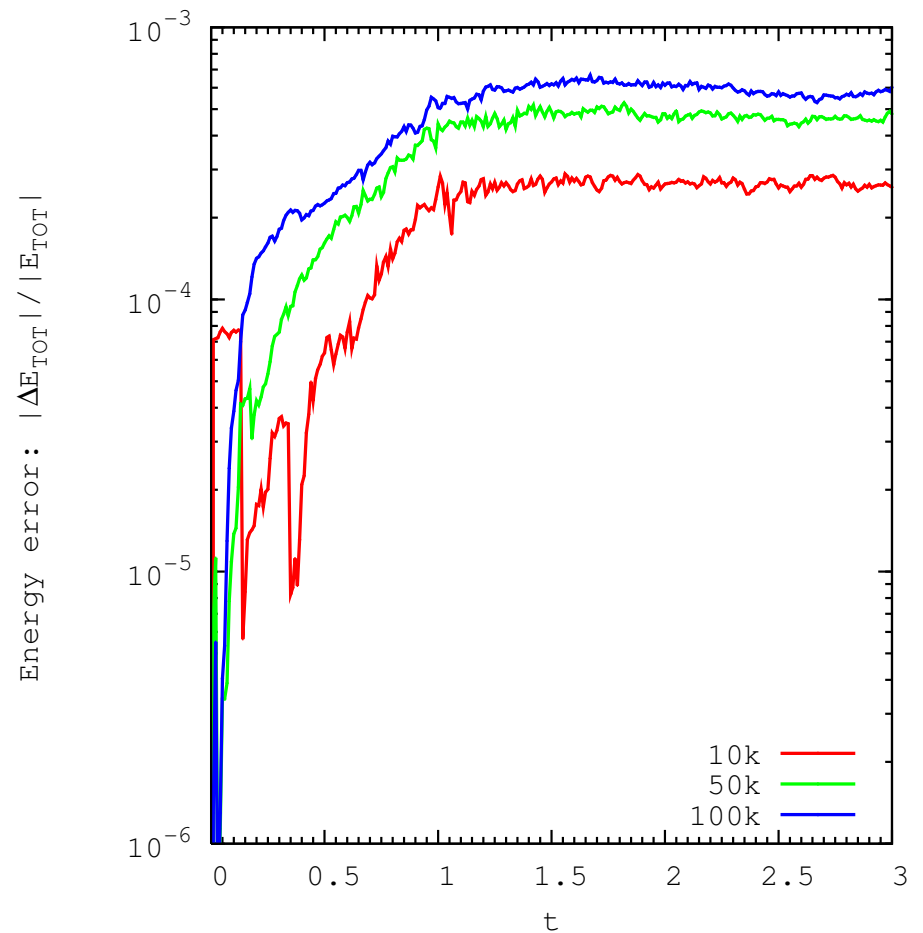


Nakasato (N_{CPU}=4)

SPH - test



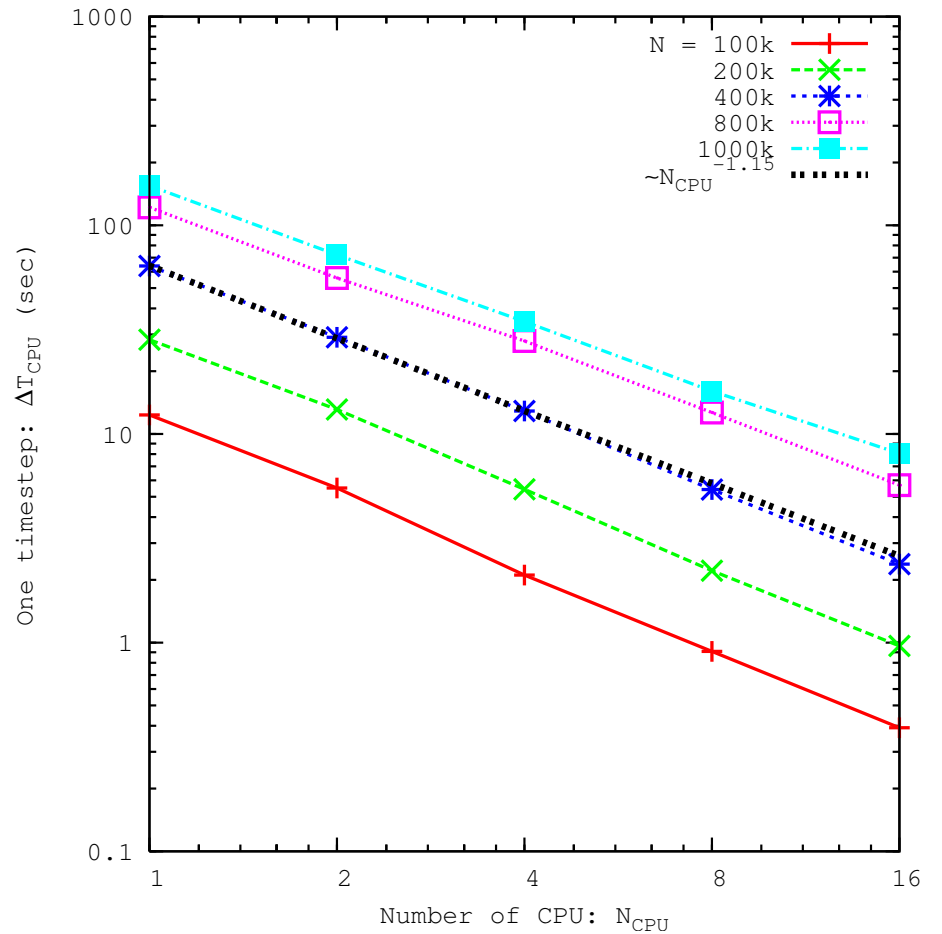
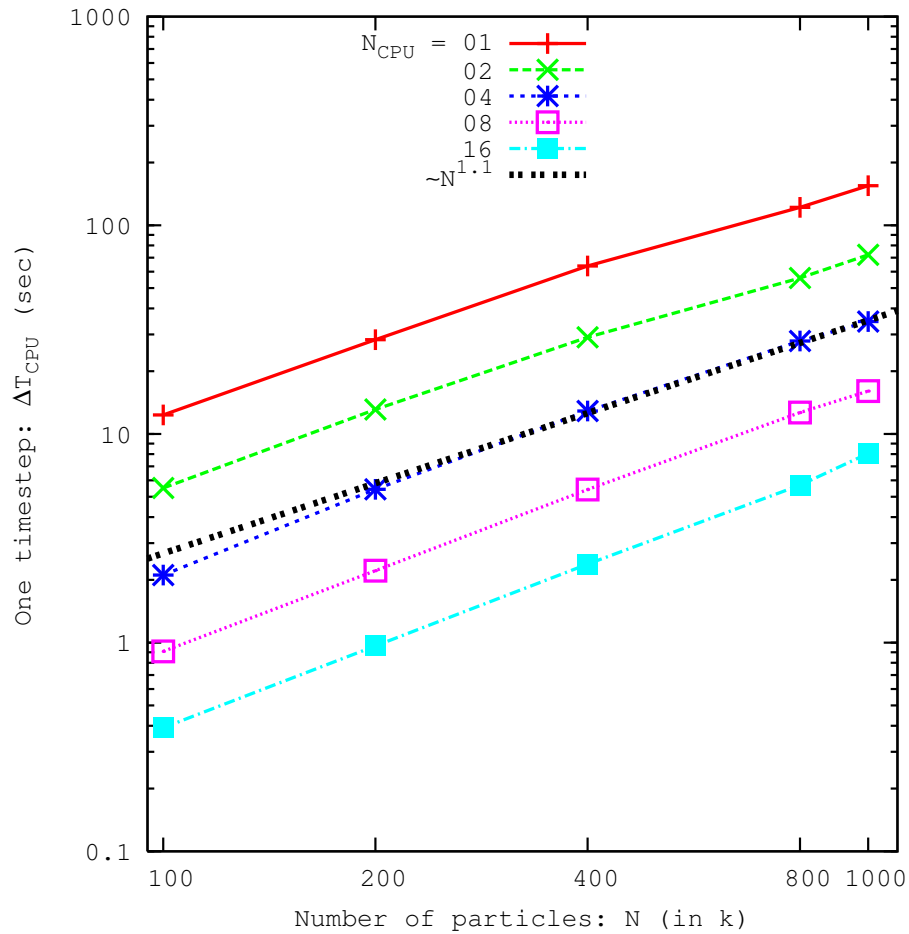
Berczik ($N_{CPU}=1$)



Nakasato ($N_{CPU}=4$)

Scaling results

GRAPE + SPH code: One timestep integration



GPU Hardware



<http://www.nvidia.com>

<http://gpgpu.org>

2007...

GeForce 8800 GTX, 128 Stream Proc., 768 MB

GeForce 8800 GTS, 128 Stream Proc., 512 MB

GeForce 8800 GT, 112 Stream Proc., 512 MB

2008...

GeForce 9800 GTX, 128 Stream Proc., 512 MB

GeForce 9800 GX2, 256 Stream Proc., 1 GB

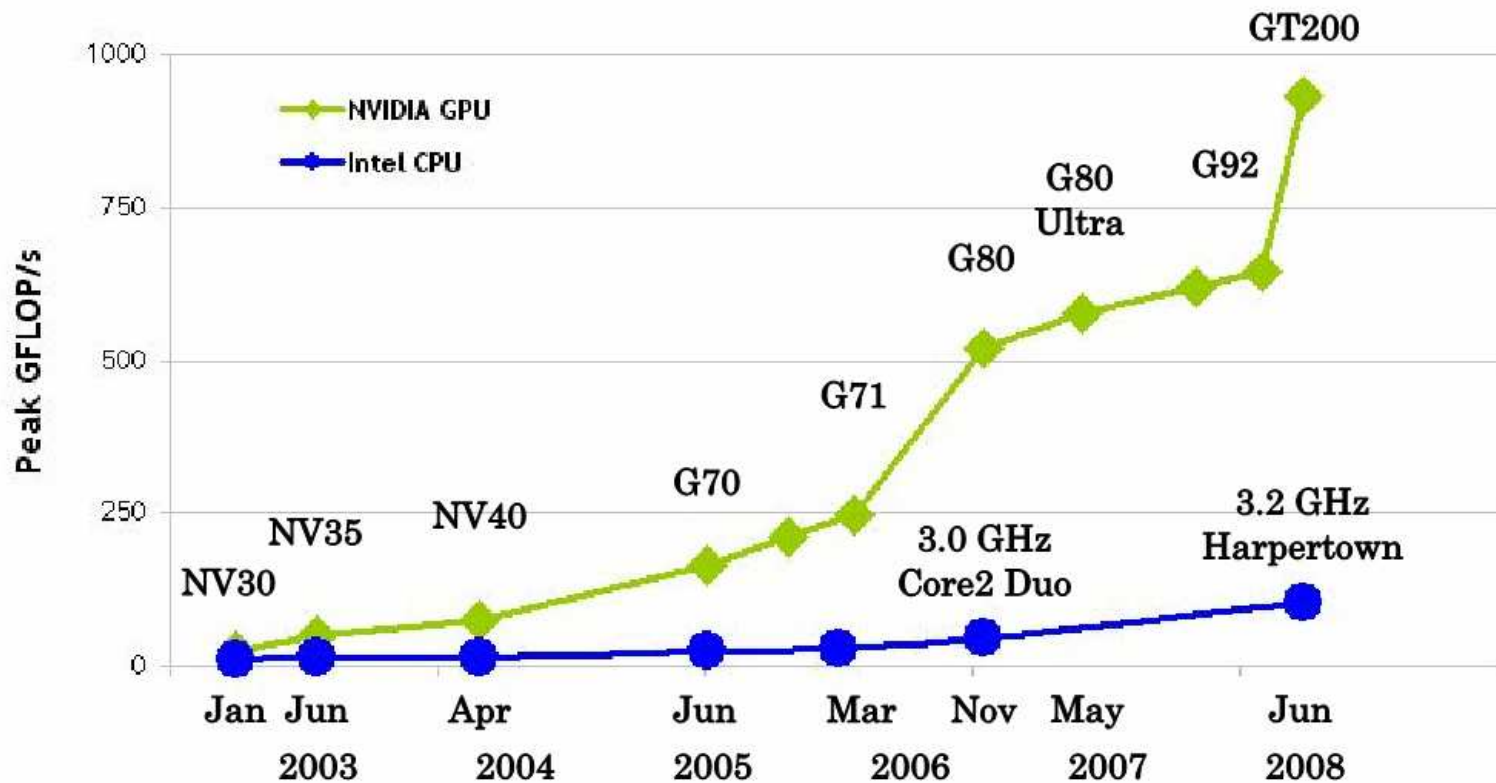
GeForce 9800 GT, 64 Stream Proc., 512 MB

GeForce GTX 280, 240 SP + 30 fp64, 1 GB

GeForce GTX 260, 192 SP + 24 fp64, 890 MB



CPU vs. GPU speedup timeline



GT200 = GeForce GTX 280	G71 = GeForce 7900 GTX	NV35 = GeForce FX 5950 Ultra
G92 = GeForce 9800 GTX	G70 = GeForce 7800 GTX	NV30 = GeForce FX 5800
G80 = GeForce 8800 GTX	NV40 = GeForce 6800 Ultra	

Hardware



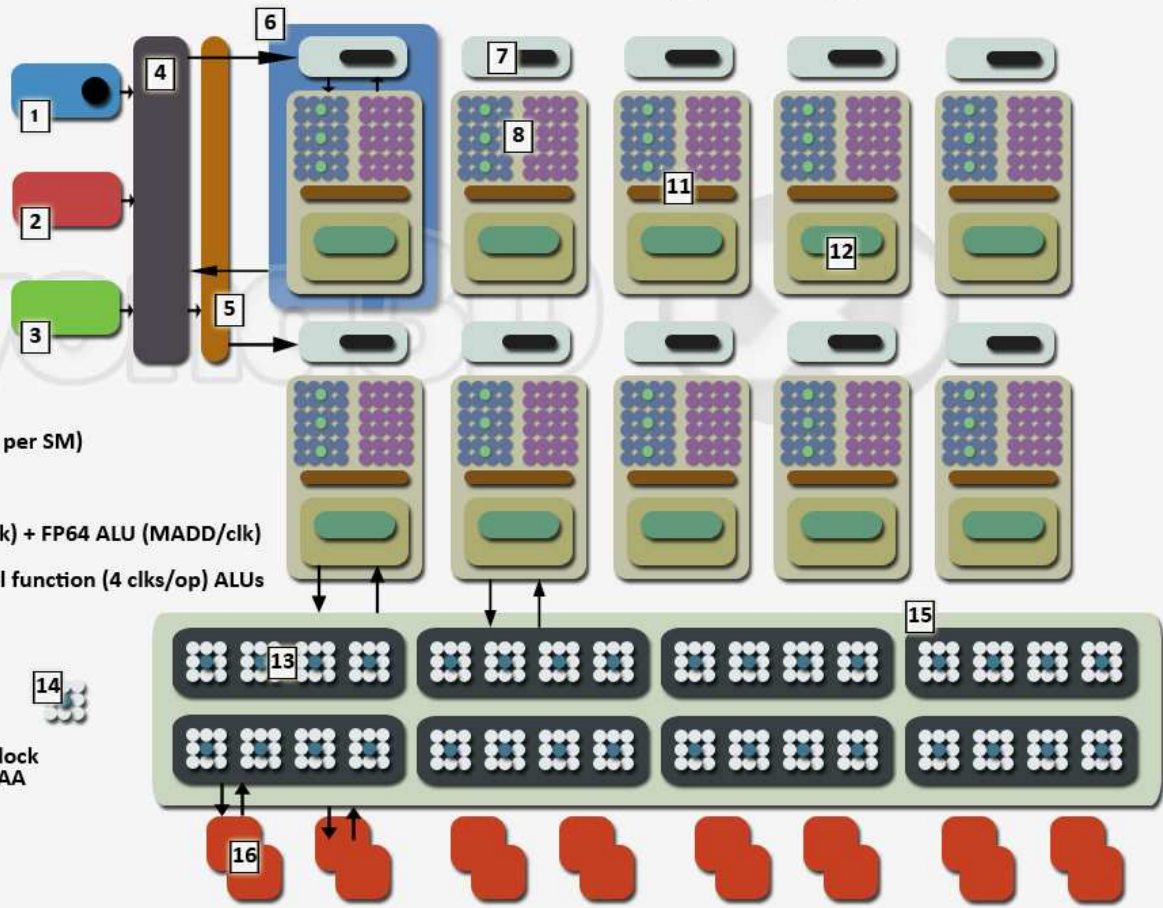
nb: not all datapaths are shown, and those that exist for illustration only
graphic revision 1.2 - 26th March 2008

64KiB per SM RF (1920KiB total)
256KiB shared L2 surface cache
16KiB per cluster L1 cache (160KiB total)
16KiB per SM shared memory (480KiB total)
6x G80 stream output buffer

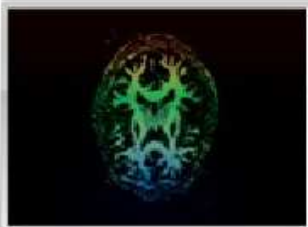
Major memory pools



- 1 Vertex thread setup and input assembler
- 2 Geometry thread setup
- 3 Pixel thread setup
- 4 Global thread scheduler
- 5 Triangle setup (1 triangle/clock), rasterisation and Z-cull
- 6 Thread processing cluster
- 7 Per cluster scheduler and register files (16K FP32 registers per SM)
- 8 SP and interpolator/special ALU groups
- 9 3 x 8-way scalar FP32 SP ALUs (MADD + MUL dual-issue/clock) + FP64 ALU (MADD/clock)
- 10 3 x 8-way FP32 scalar interpolator (1 attrib/clock) and special function (4 clks/op) ALUs
- 11 8 pixels/clock data address and setup
- 12 8 INT8 bilerp/clock filtering + L1 local store (16KiB)
- 13 ROP partition
- 14 ROP with 8 depth or colour samples/clock, 1 FP16 blend/clock
AA: 0x = 8Z/clock, 4x = 1Z + 1C/clock; max 8xMSAA, 16xCSAA
- 15 L2 shared data store (256KiB*)
- 16 DRAM pair (2 x 32-bit)

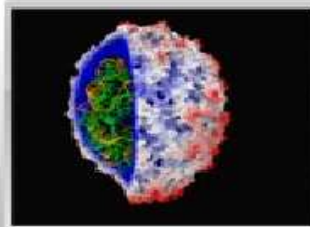


Speedups using GPU vs. CPU



146X

Interactive visualization of volumetric white matter connectivity¹



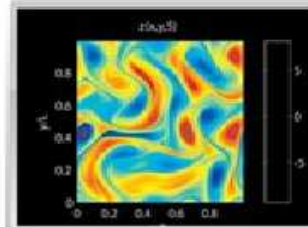
36X

Ionic placement for molecular dynamics simulation on GPU²



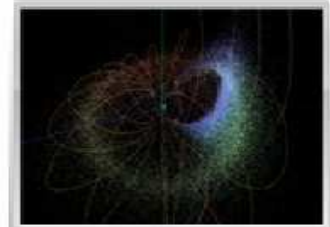
18X

Transcoding HD video stream to H.264 for portable video³



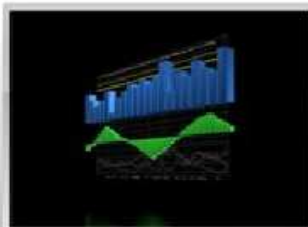
17X

Simulation in Matlab using mex file CUDA function⁴



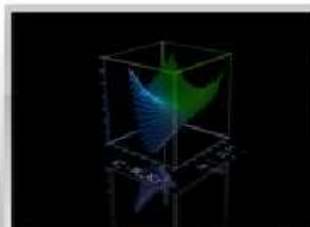
100X

Astrophysics N-body simulation⁵



149X

Financial simulation of LIBOR model with swaptions⁶



47X

GLAME@lab: M-script API for linear Algebra operations on GPU⁷



20X

Ultrasound medical imaging for cancer diagnostics⁸



24X

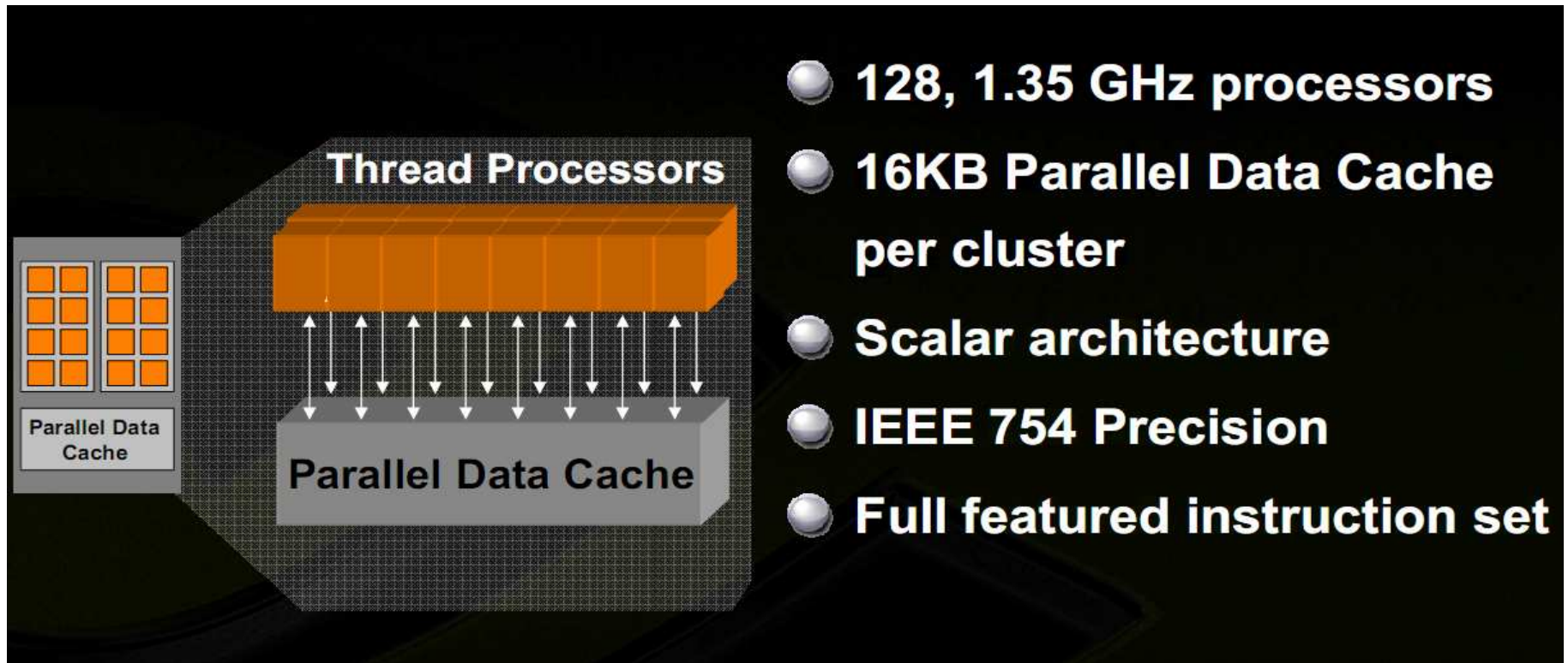
Highly optimized object oriented molecular dynamics⁹



30X

Cmatch exact string matching - find similar proteins & gene sequences¹⁰

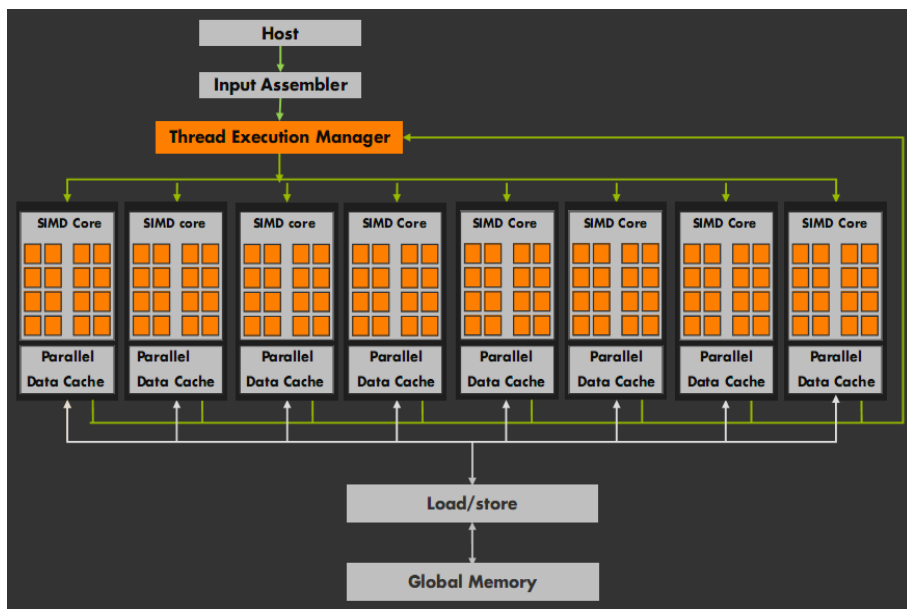
Hardware



GeForce 8800 GTX:

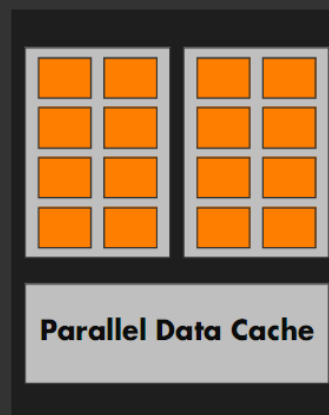
$575 \text{ MHz} * 128 \text{ processors} * 2 \text{ flop/inst} * 2 \text{ inst/clock} = 333 \text{ Gflops}$

Hardware



Each core

- 8 functional units
- SIMD 16/32 "warp"
- 8-10 stage pipeline
- Thread scheduler
- 128-512 threads/core
- 16 KB shared memory



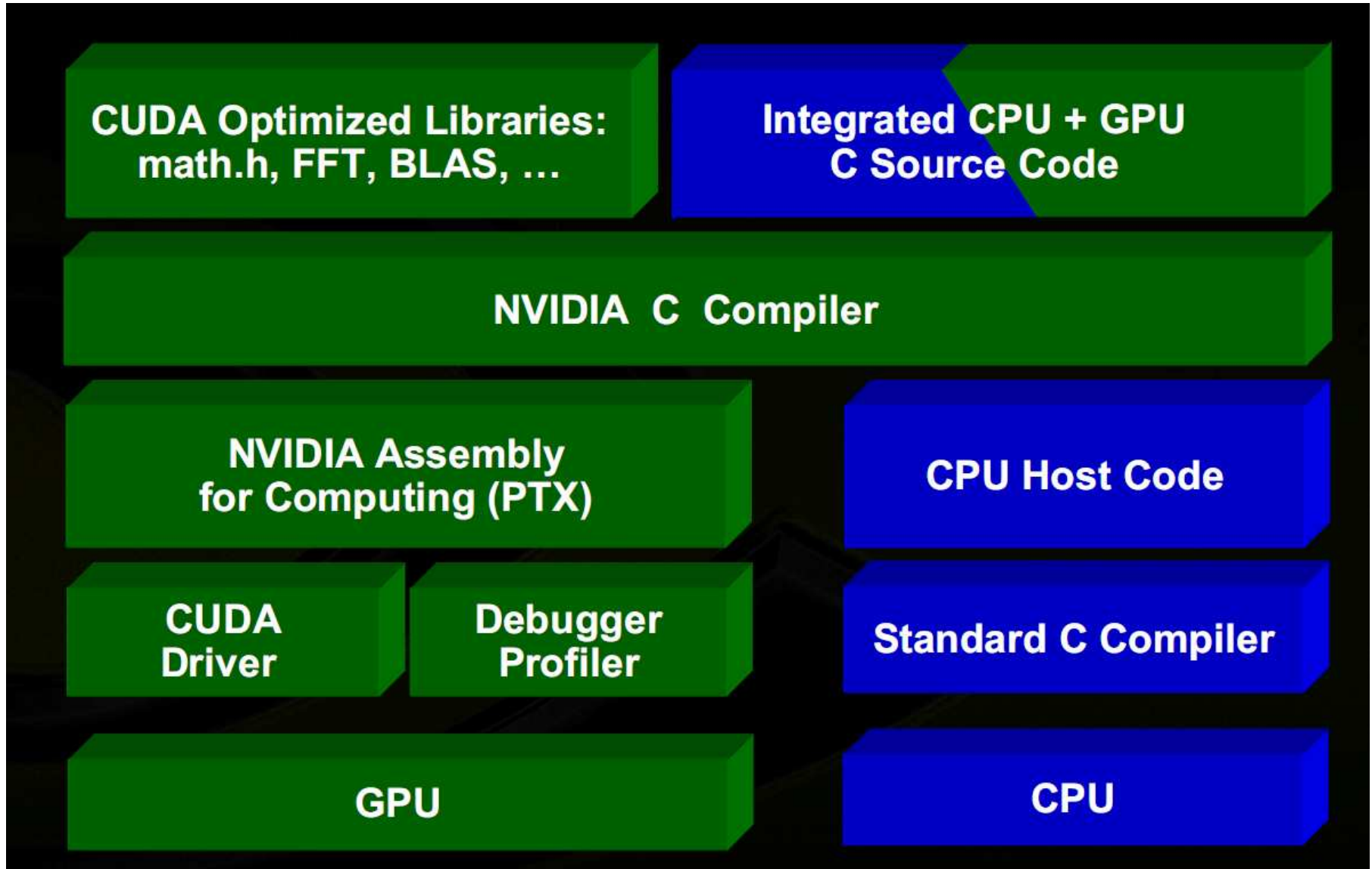
Total #threads/chip

$$16 * 512 = 8K$$

GeForce 8800 GTX:

$$575 \text{ MHz} * 128 \text{ processors} * 2 \text{ flop/inst} * 2 \text{ inst/clock} = 333 \text{ Gflops}$$

CUDA



Simple CUDA example

CPU C program

```
void addMatrix(float *a, float *b,
              float *c, int N)
{
    int i, j, index;
    for (i = 0; i < N; i++) {
        for (j = 0; j < N; j++) {
            index = i + j * N;
            c[index]=a[index] + b[index];
        }
    }
}

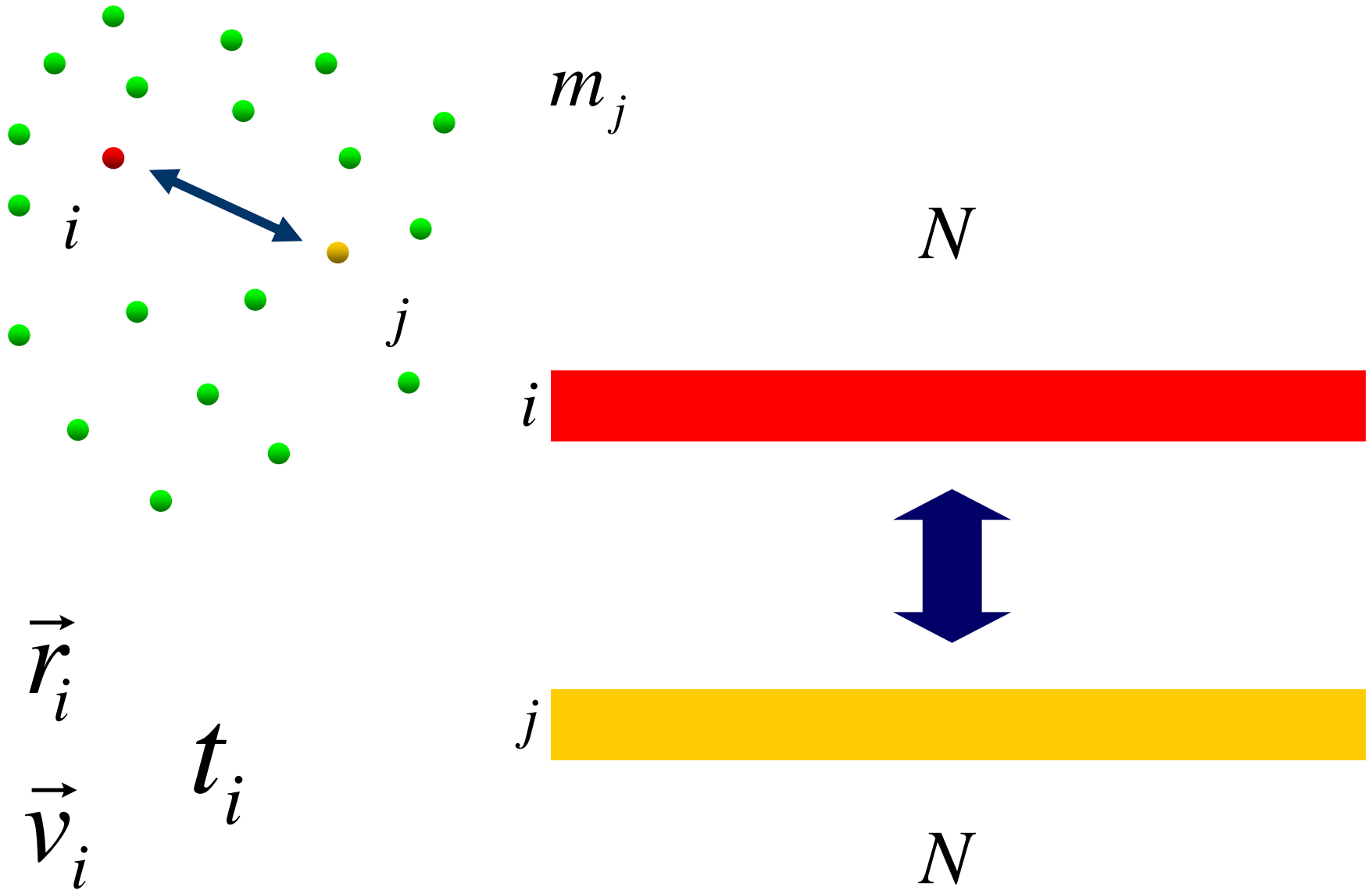
void main()
{
    .....
    addMatrix(a, b, c, N);
}
```

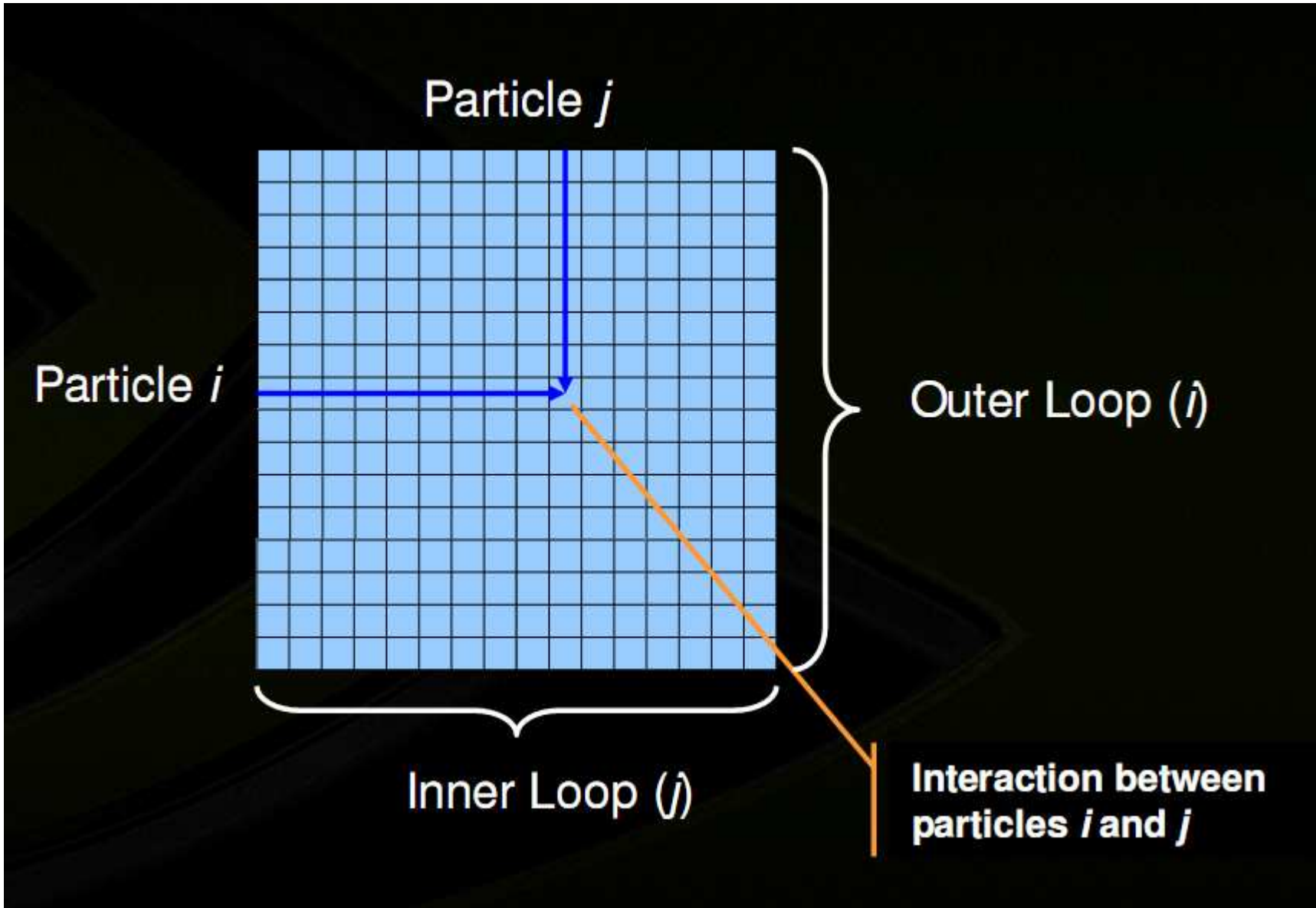
CUDA C program

```
__global__ void addMatrix(float *a, float *b,
                          float *c, int N)
{
    int i=blockIdx.x*blockDim.x+threadIdx.x;
    int j=blockIdx.y*blockDim.y+threadIdx.y;
    int index = i + j * N;
    if ( i < N && j < N)
        c[index]= a[index] + b[index];
}

void main()
{
    ..... // allocate & transfer data to GPU
    dim3 dimBlk (blocksize, blocksize);
    dim3 dimGrd (N/dimBlk.x, N/dimBlk.y);
    addMatrix<<<dimGrd,dimBlk>>>(a, b, c,N);
}
```


Basic idea of any N-body code





```

__device__ float3
bodyBodyInteraction(float3 ai, float4 bi, float4 bj) {
    float3 r;
    r.x = bi.x - bj.x;           // r_ij [3 FLOPS]
    r.y = bi.y - bj.y;
    r.z = bi.z - bj.z;

    // distSqr = dot(r_ij, r_ij) + EPS^2 [6 FLOPS]
    float distSqr = r.x * r.x + r.y * r.y + r.z * r.z;
    distSqr += softeningSquared;

    // invDistCube = 1/distSqr^(3/2) [4 FLOPS (2 mul, 1 sqrt, 1 inv)]
    float distSixth = distSqr * distSqr * distSqr;
    float invDistCube = 1.0f / sqrtf(distSixth);

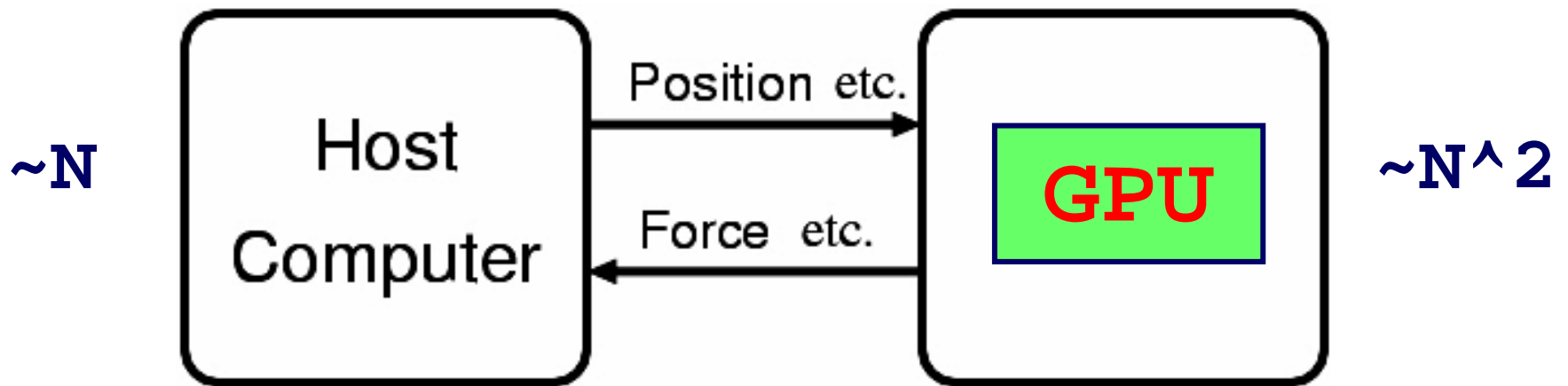
    float s = bj.w * invDistCube; // s = m_j * invDistCube [1 FLOP]

    ai.x += r.x * s;           // a_i = a_i + s * r_ij [6 FLOPS]
    ai.y += r.y * s;
    ai.z += r.z * s;
    return ai;
}

```

Total: 20 FLOPS

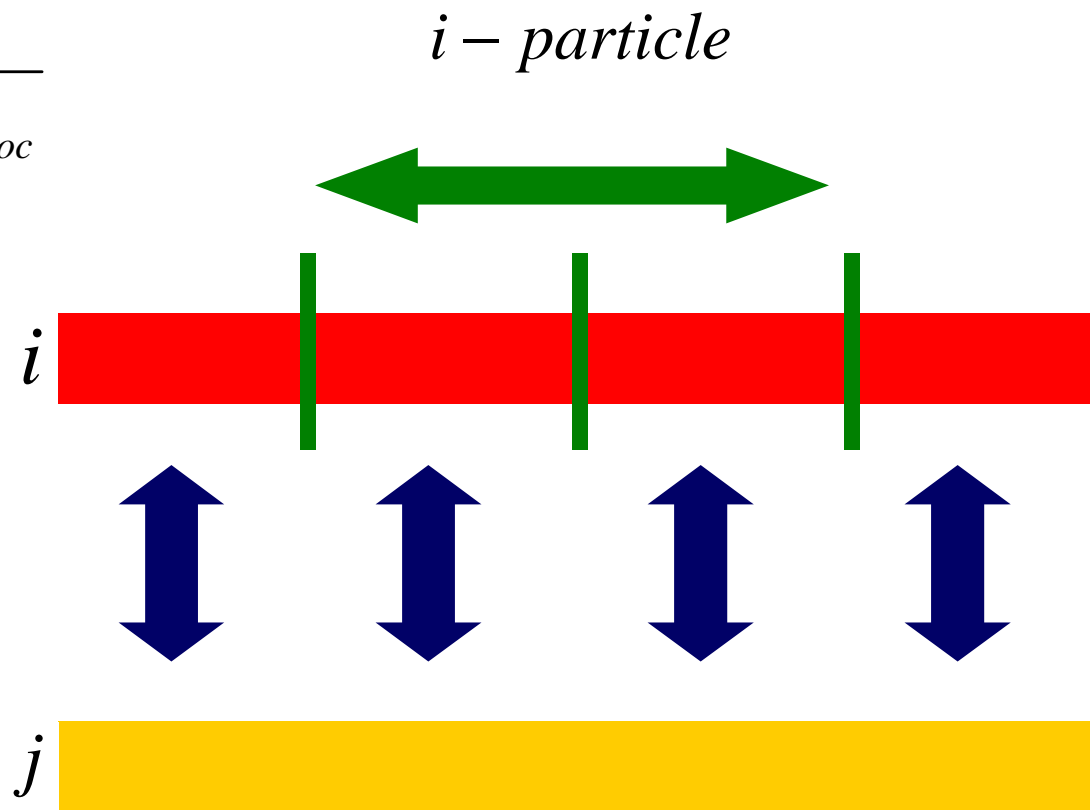
Basic idea of GRAPE/GPU N-body code



$$\vec{a}_i = \sum_{j=1; j \neq i}^N \vec{f}_{ij} \quad \vec{f}_{ij} = - \frac{G \cdot m_j}{(r_{ij}^2 + \epsilon^2)^{3/2}} \vec{r}_{ij}$$

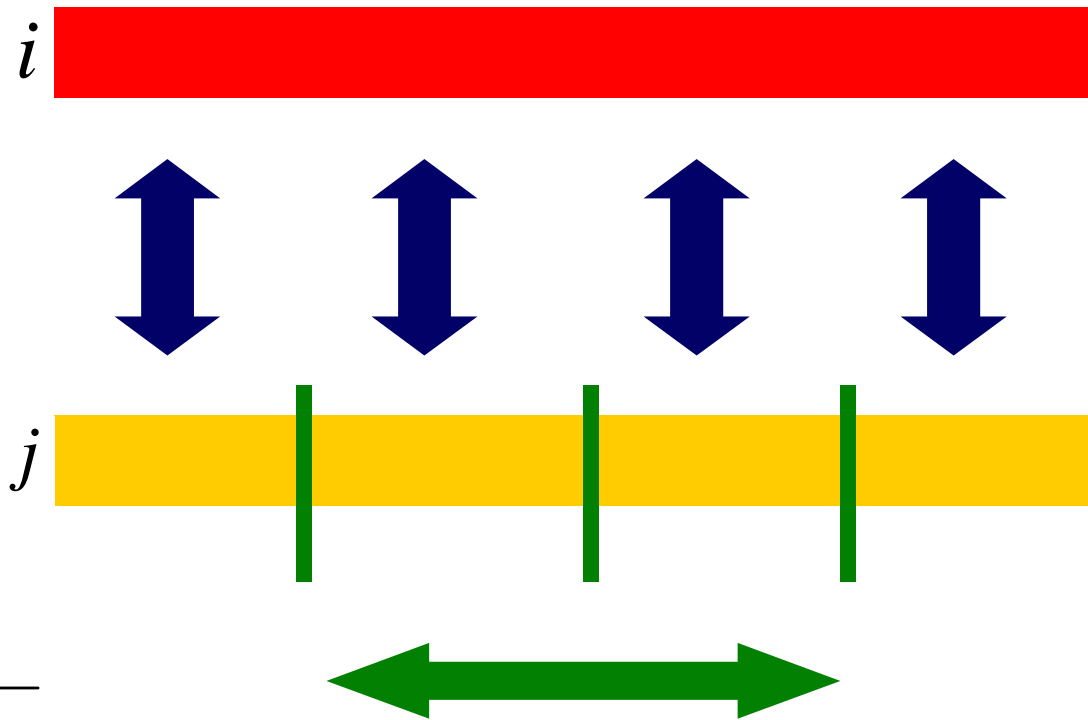
Basic idea of any parallel N-body code

$$N_{loc} = \frac{N}{N_{proc}}$$



Basic idea of any parallel N-body code

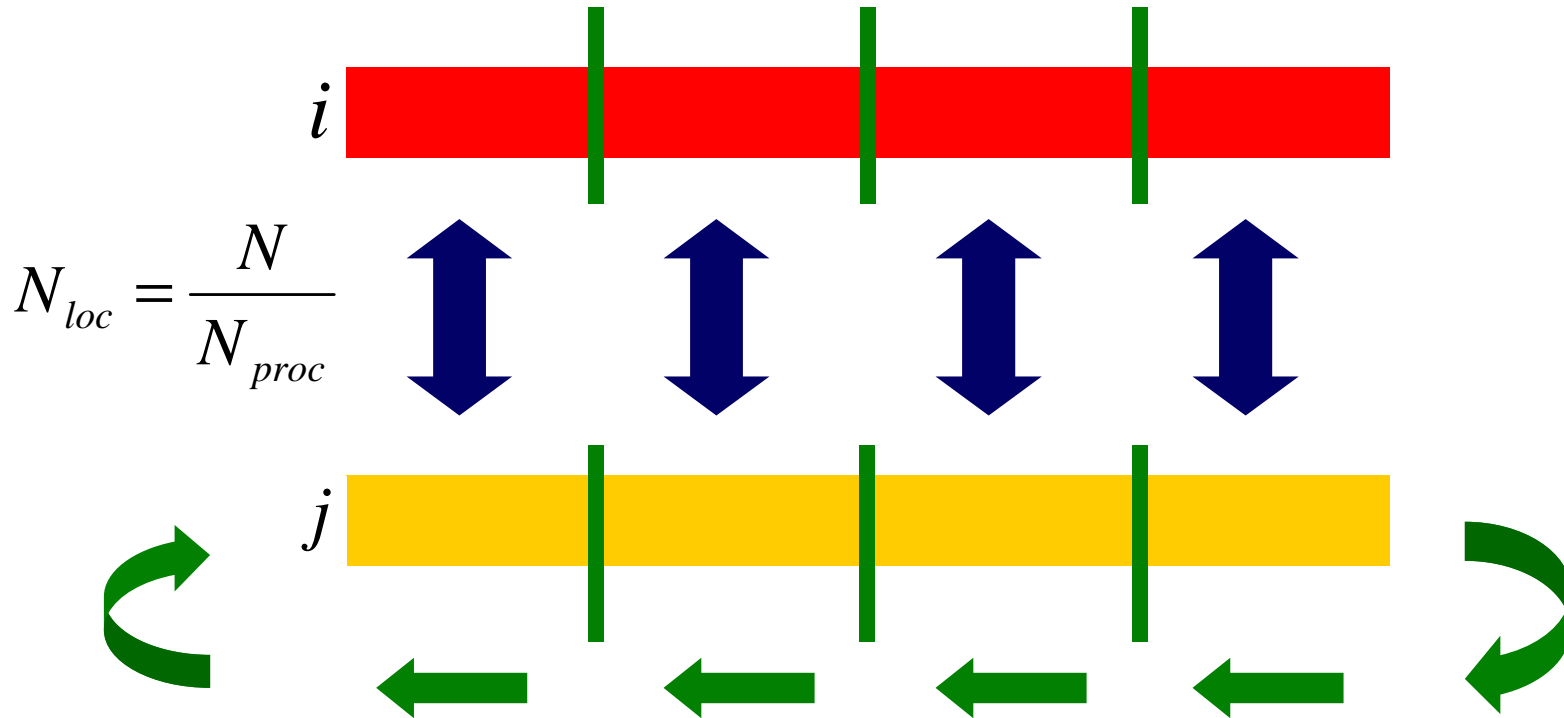
j - particle



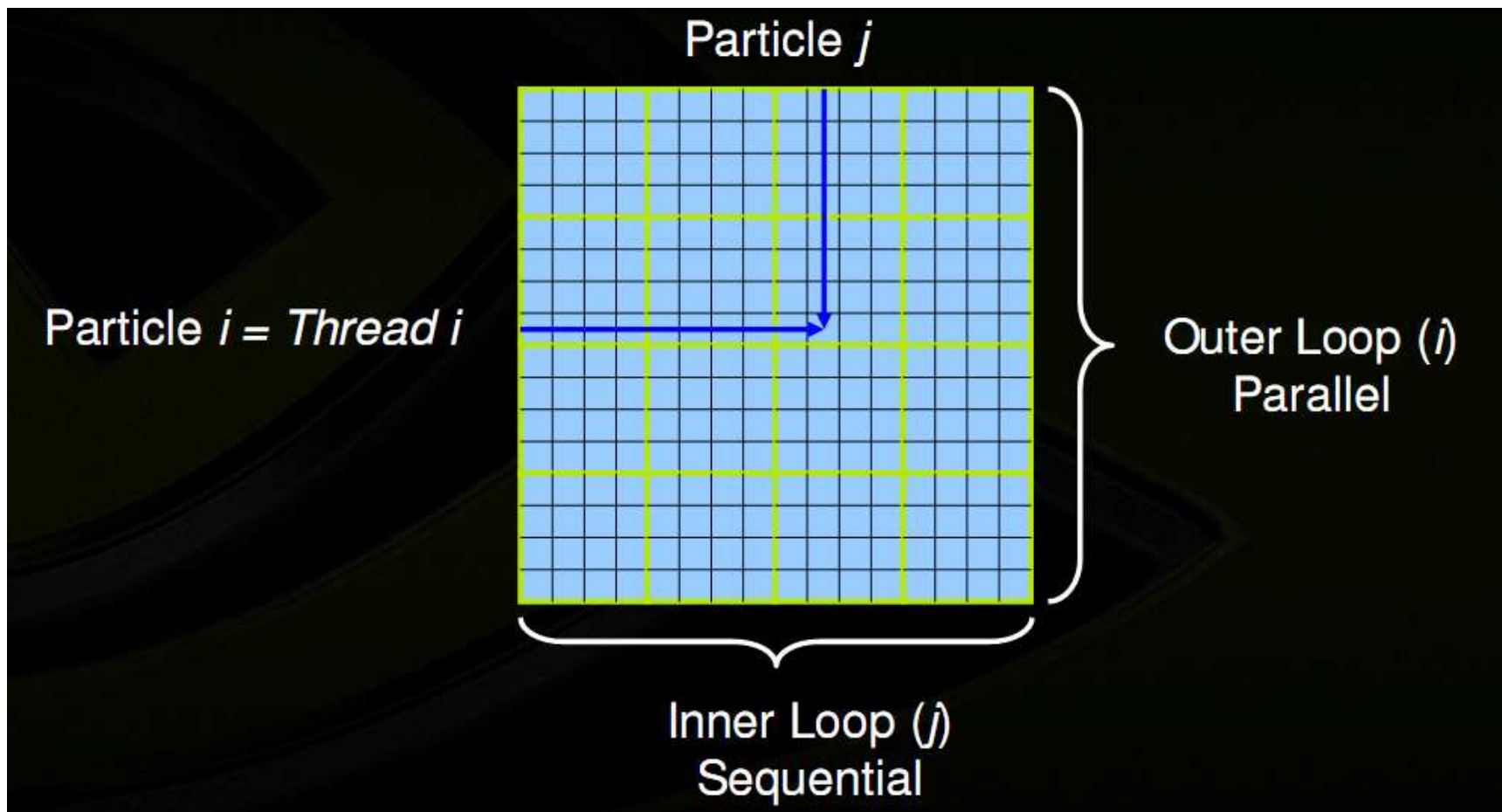
$$N_{loc} = \frac{N}{N_{proc}}$$

Basic idea of any parallel N-body code

$i, j - \text{particle}$

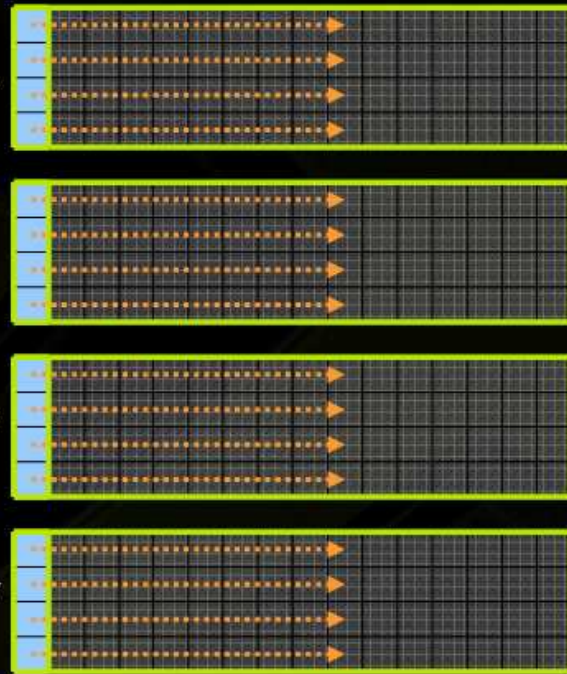


Some communication scheme...



N/p Thread Blocks
of p threads each

Particle j



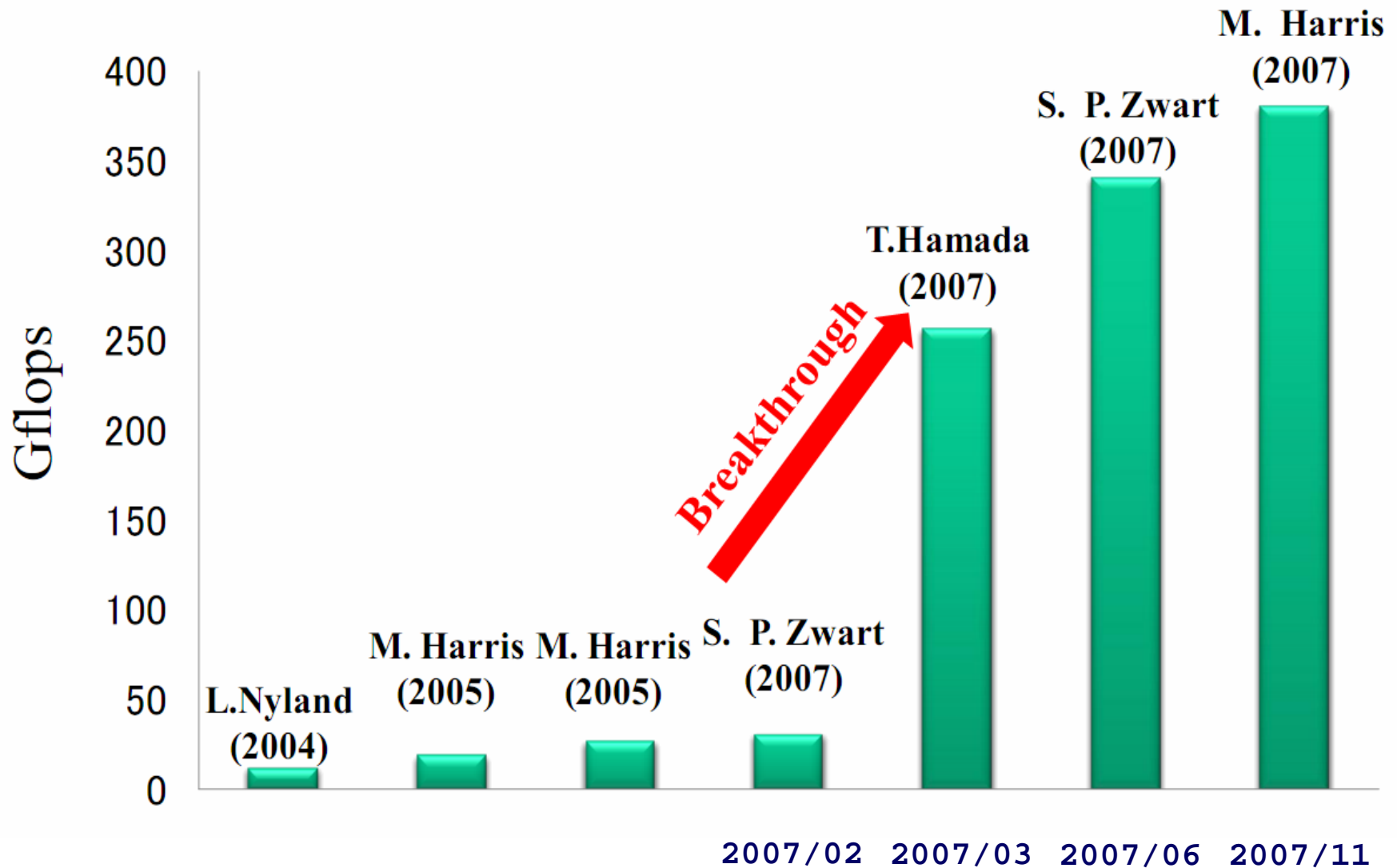
Outer Loop (i)
Parallel

Inner Loop (j)
Sequential

```
forall bodies i in parallel {  
    accel = 0;  
    pos = position[i]  
    foreach tile q {  
        forall threads p in thread block in parallel {  
            shared[p] = position[q*tile_size + p]  
        }  
        synchronize threads in block  
        foreach body j in tile q {  
            accel +=  
                computeAcceleration(pos, position[j])  
        }  
        synchronize threads in block  
    }  
}
```

GPU N-body speedup timeline

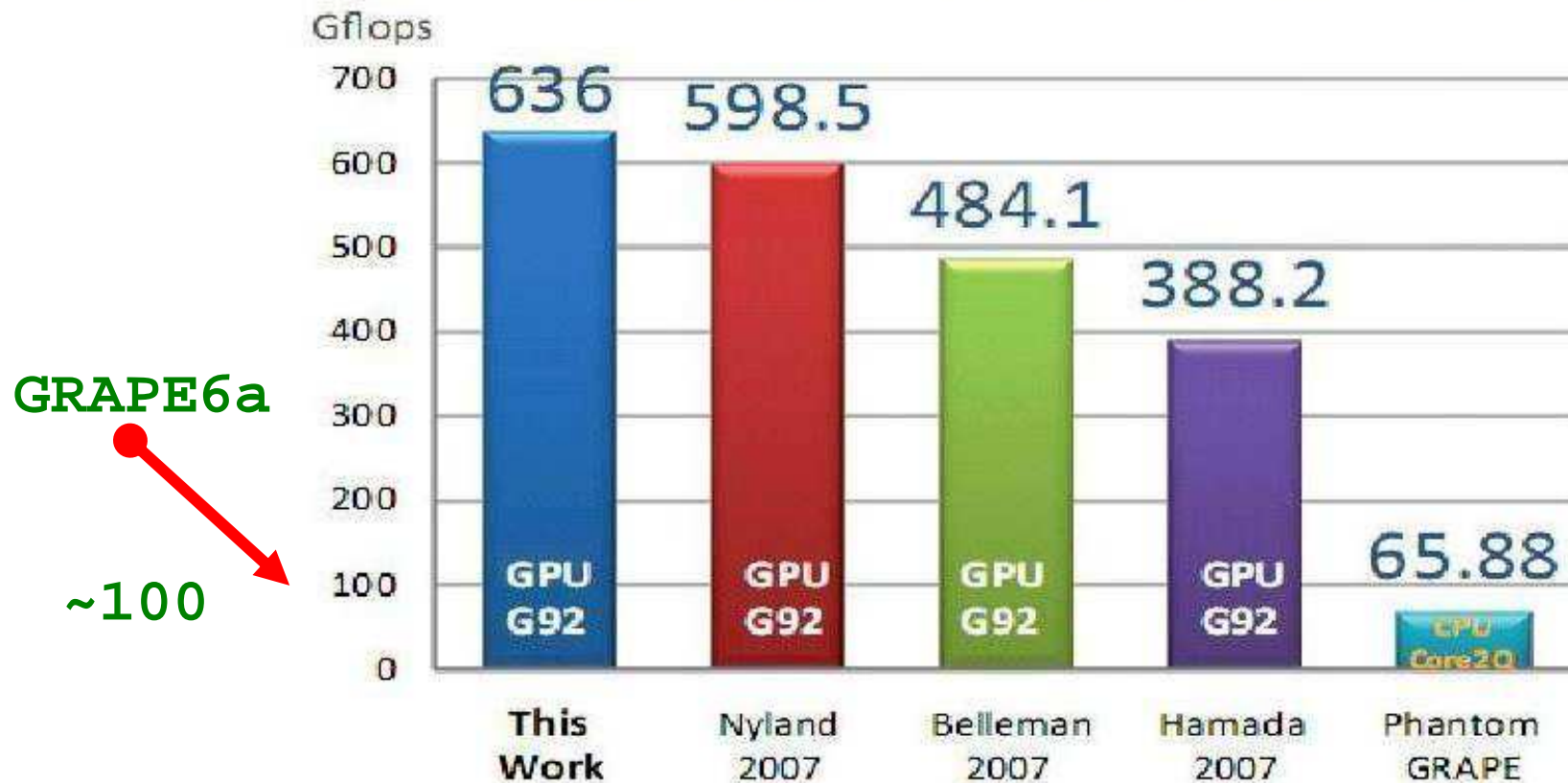
All on a same GPU: 8800 GTX (G80)



GPU N-body gravity

Hamada et al. 2008: Direct GPU code

$O(N^2)$ kernel demonstrations

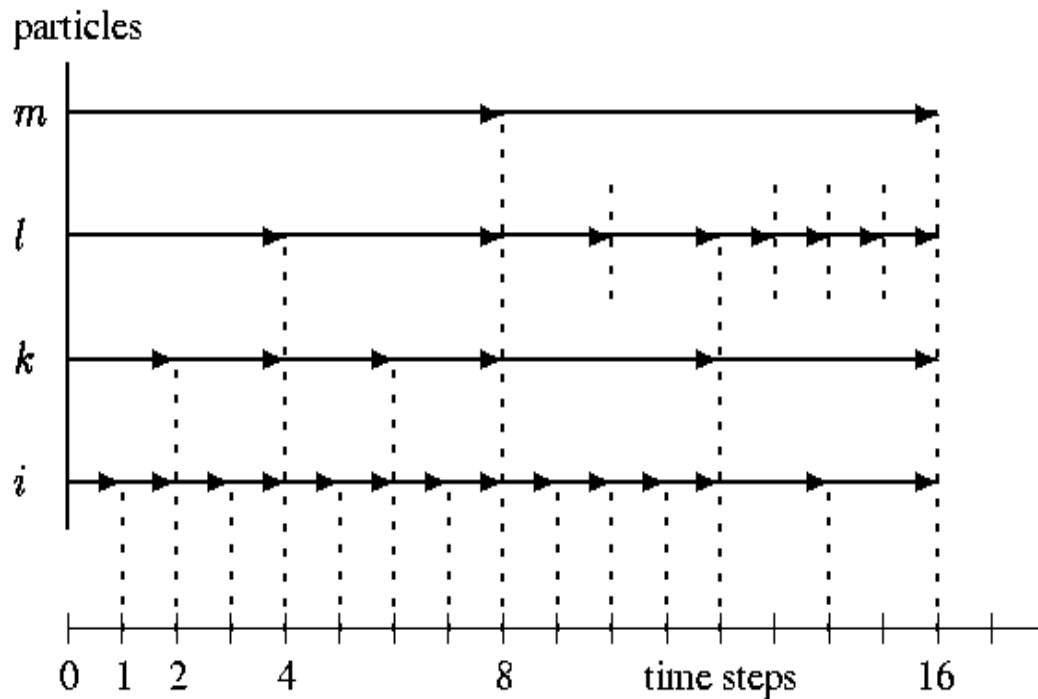


Results for 8800 GTS 512MB (G92)

Our own ϕ GRAPE/GPU N-body code

Harfst et al, NewA, 12, 357 (2007) [astro-ph/0608125]

Hierarchical Individual Block Time Steps



$$\Delta t = \sqrt{\eta \frac{|\vec{a}| |\vec{a}^{(2)}| + |\vec{a}|^2}{|\vec{a}| |\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}}$$

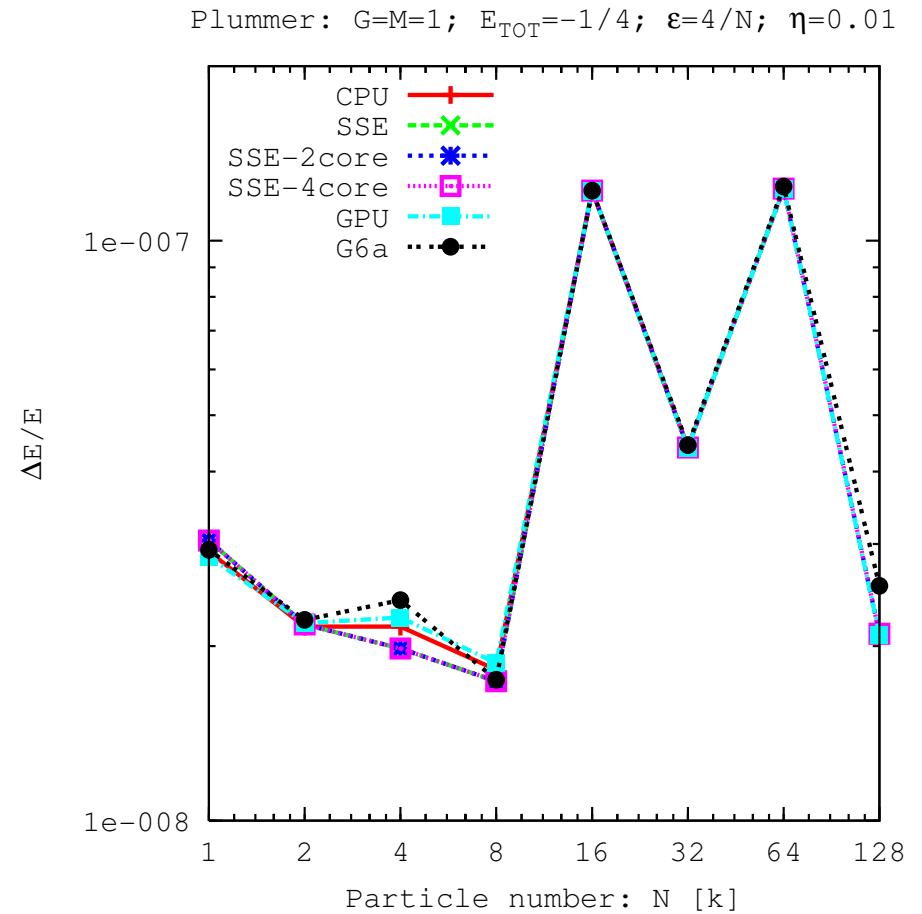
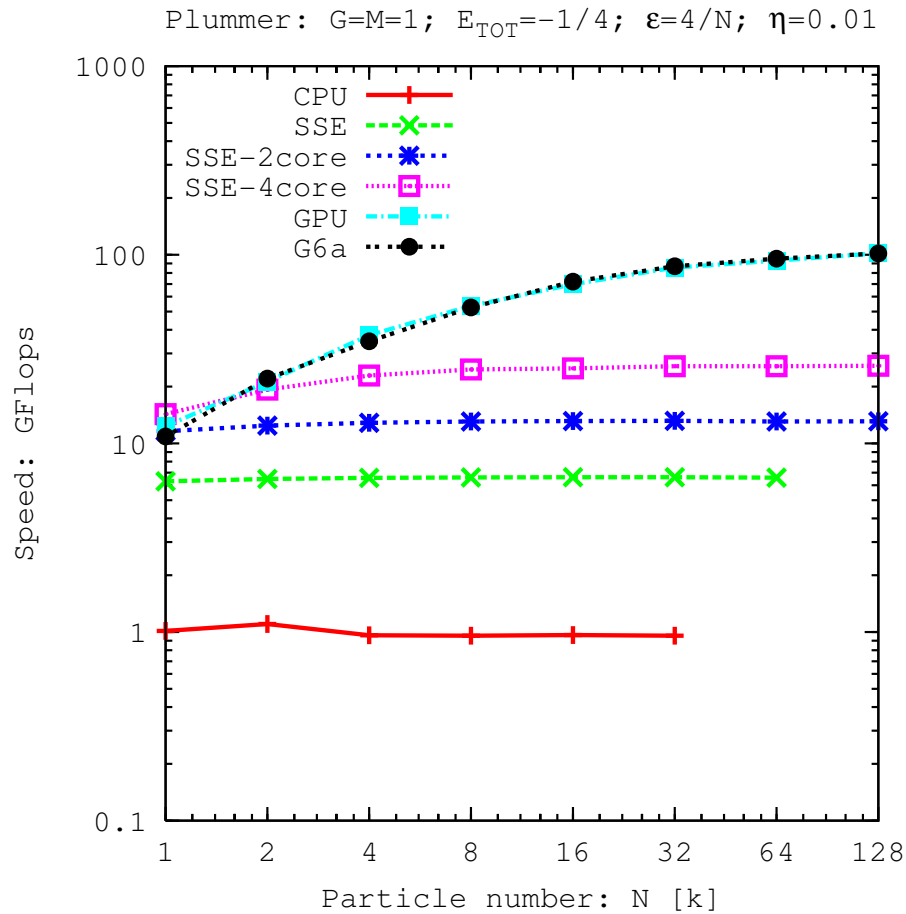
4th order Hermite scheme

$$\frac{d^2 \vec{r}_i}{dt^2} = \vec{a}_i$$

<ftp://ftp.ari.uni-heidelberg.de/pub/staff/berczik/phi-GRAPE/>

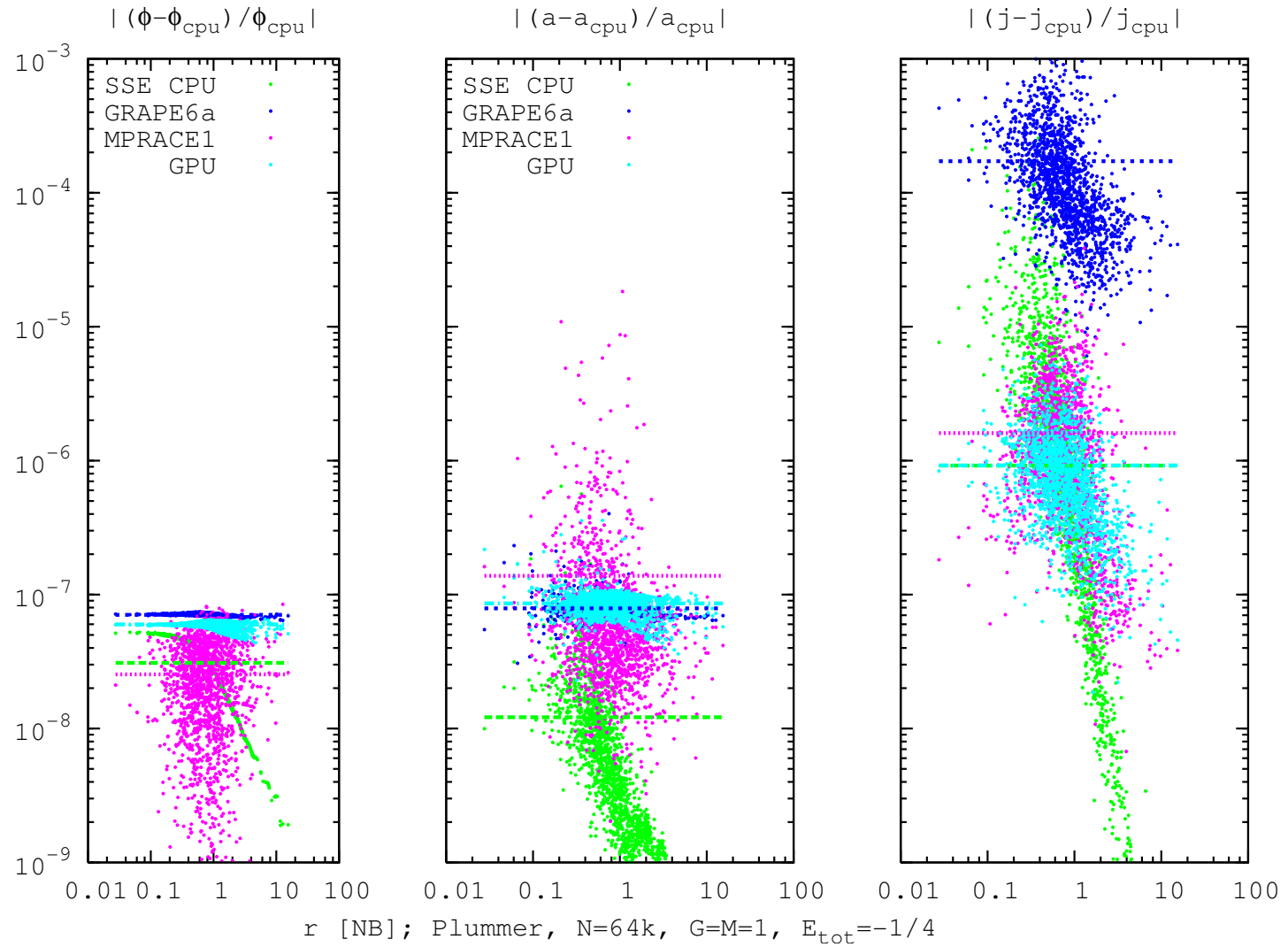
GPU Hermite 4th results

Nitadori, Berczik et al. 2007.11



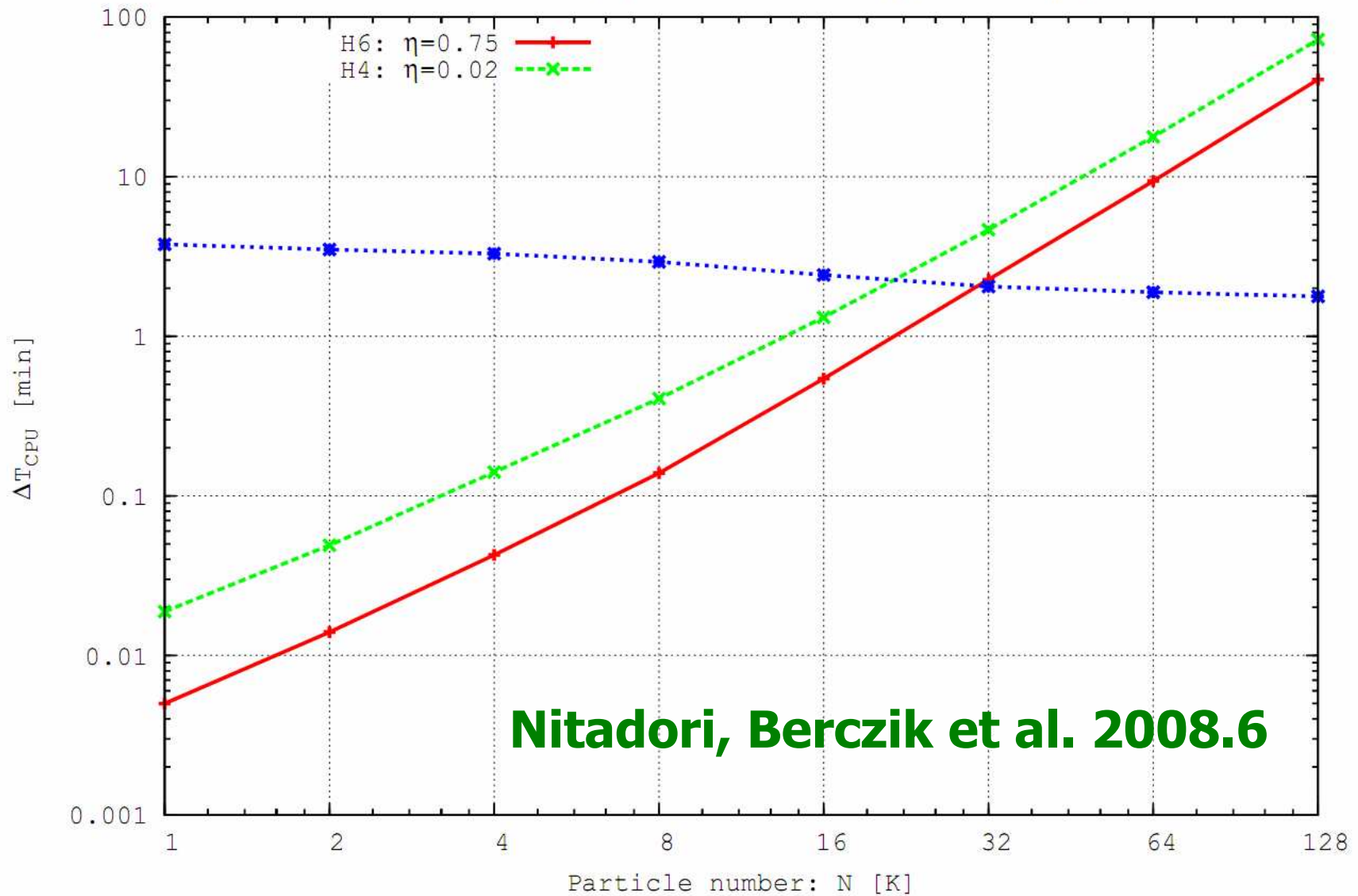
GPU Hermite 4th results

Nitadori, Berczik et al. 2007.11



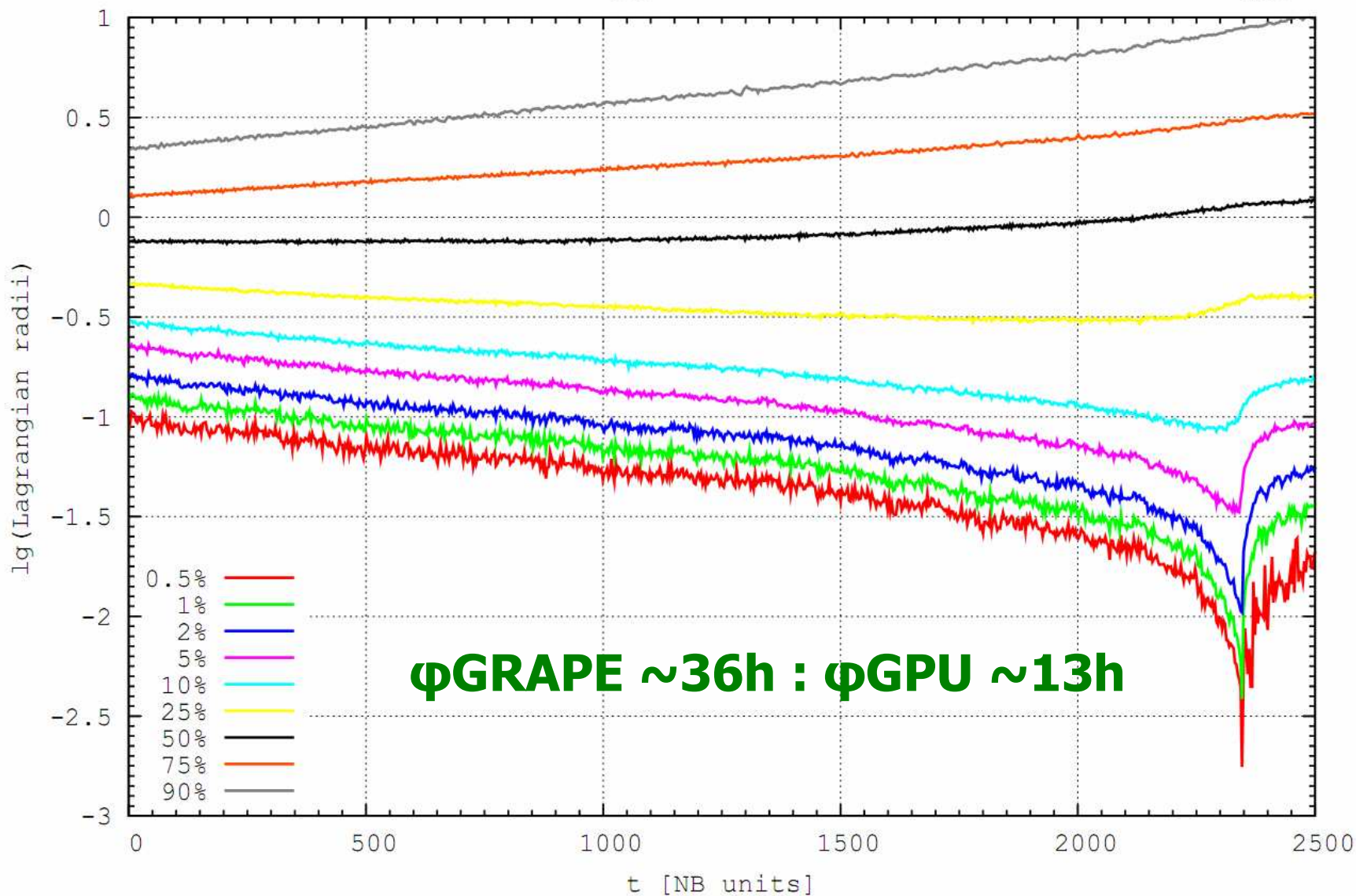
Hermite 4th vs. 6th results

Plummer, $G=M=1$, $E_{\text{tot}}=-1/4$, $\epsilon=10^{-4}$, $\tau_{\text{end}}=1$, $dt_{\text{min}}\approx 10^{-9}$



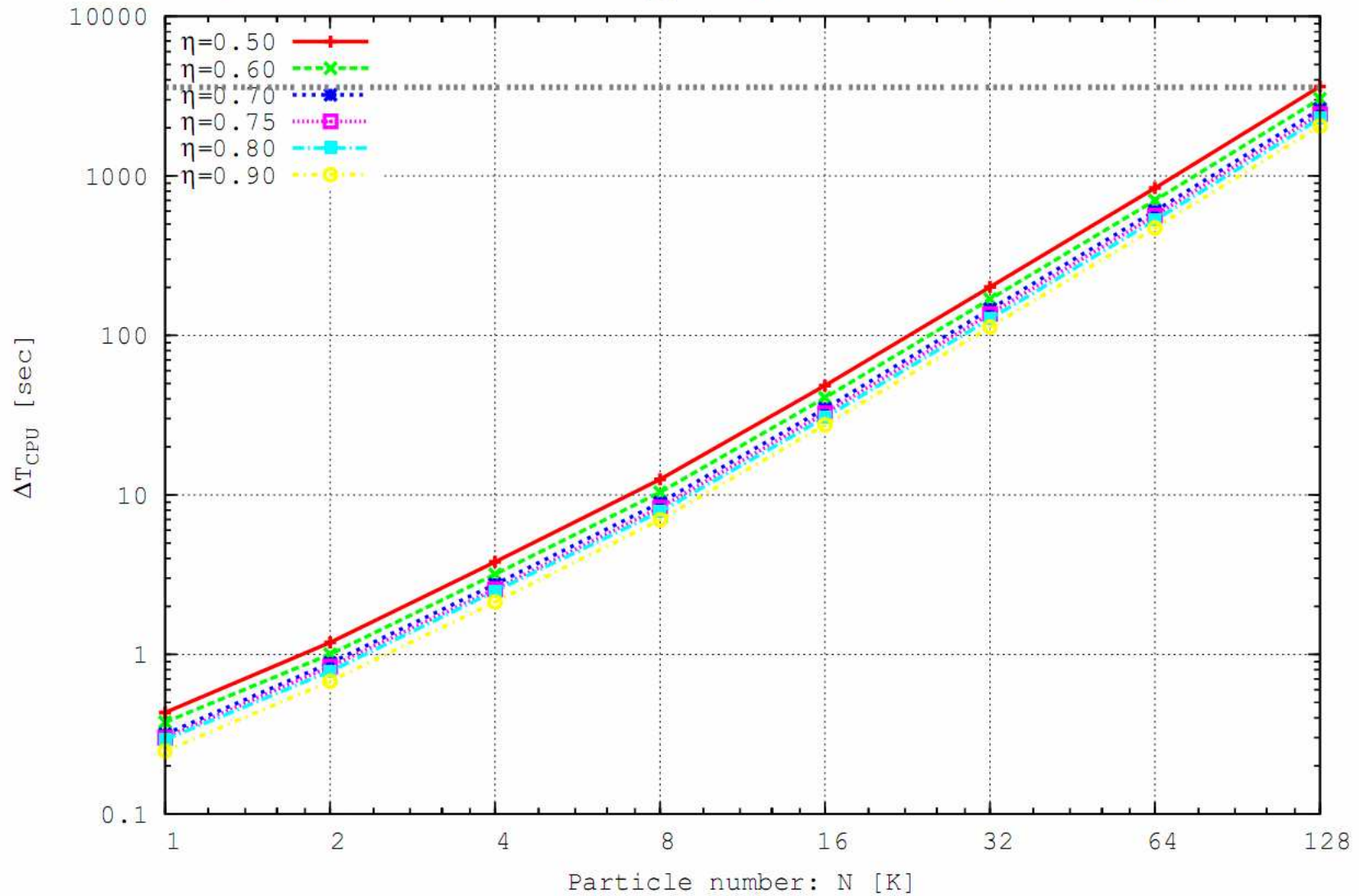
ϕ GPU Hermite 6th results

phi-GPU: Plummer, N=10k, G=M=1, $E_{\text{tot}}=-1/4$, $\epsilon=10^{-4}$, Hermite 6th, $\eta=0.75$, $dt_{\text{min}}\approx 10^{-9}$

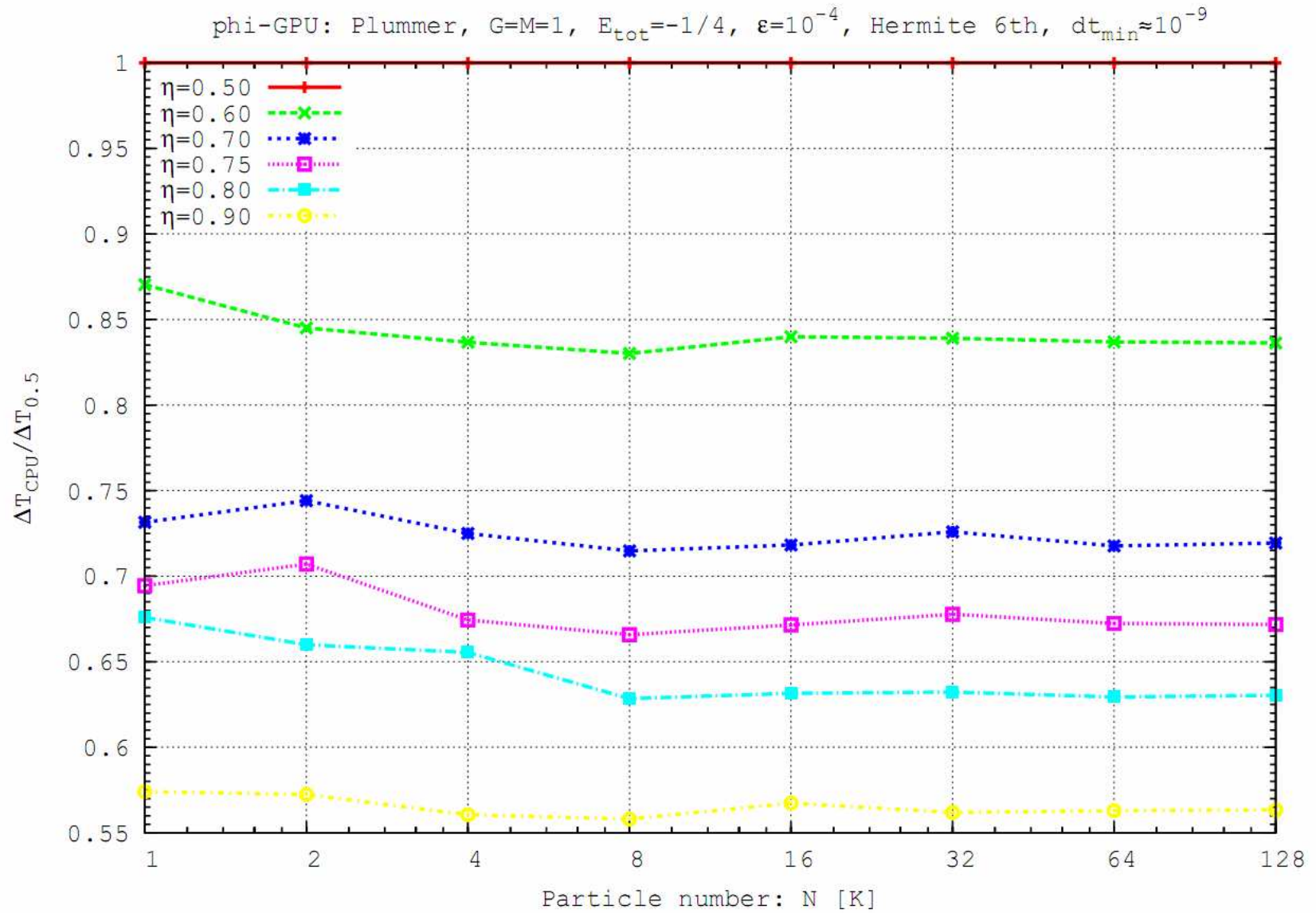


ϕ GPU Hermite 6th results

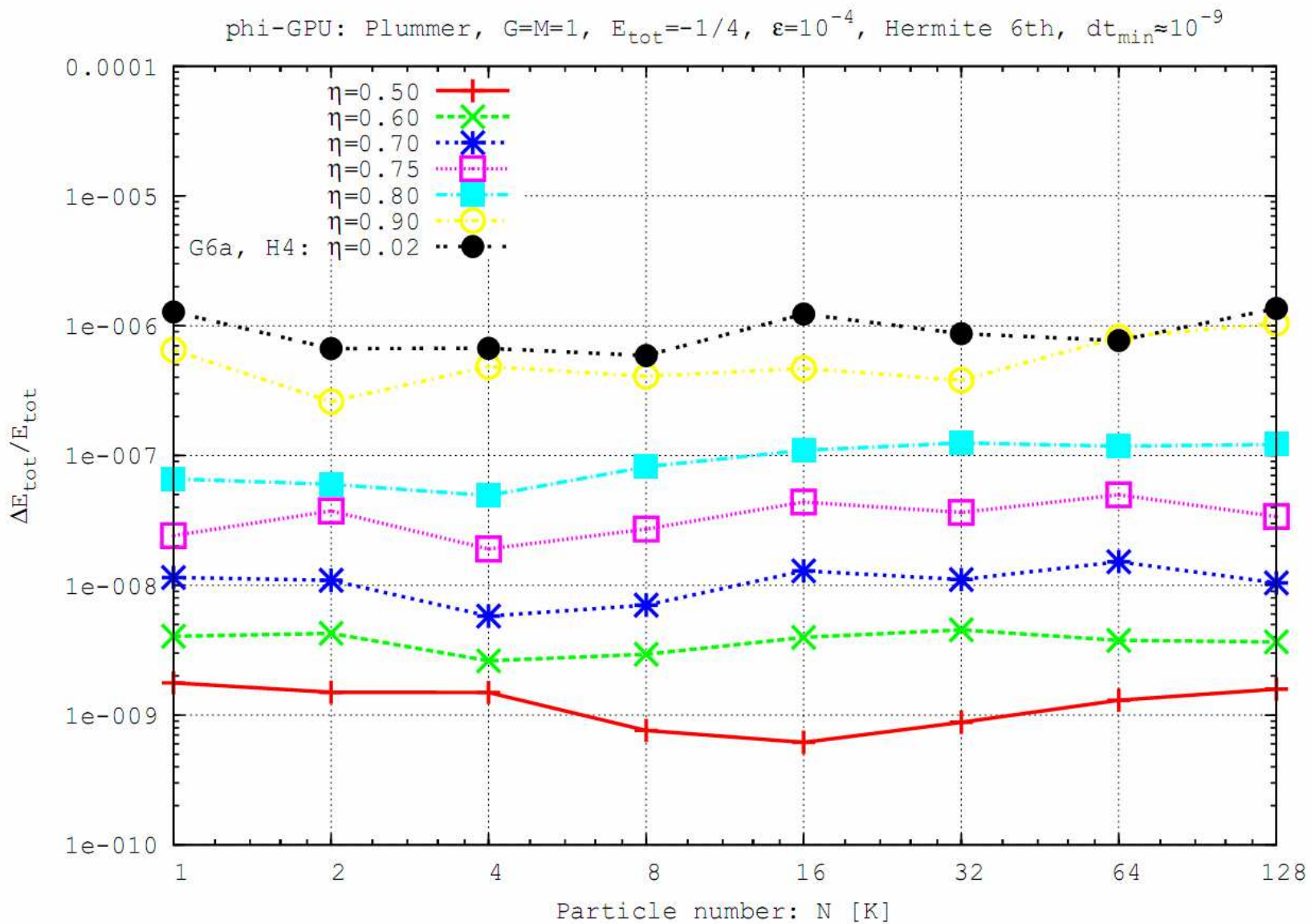
phi-GPU: Plummer, $G=M=1$, $E_{\text{tot}}=-1/4$, $\epsilon=10^{-4}$, Hermite 6th, $dt_{\text{min}}\approx 10^{-9}$



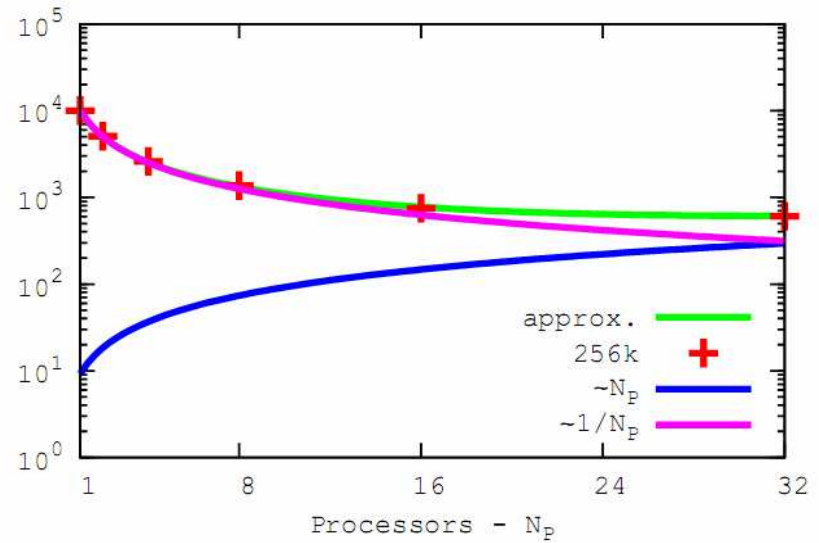
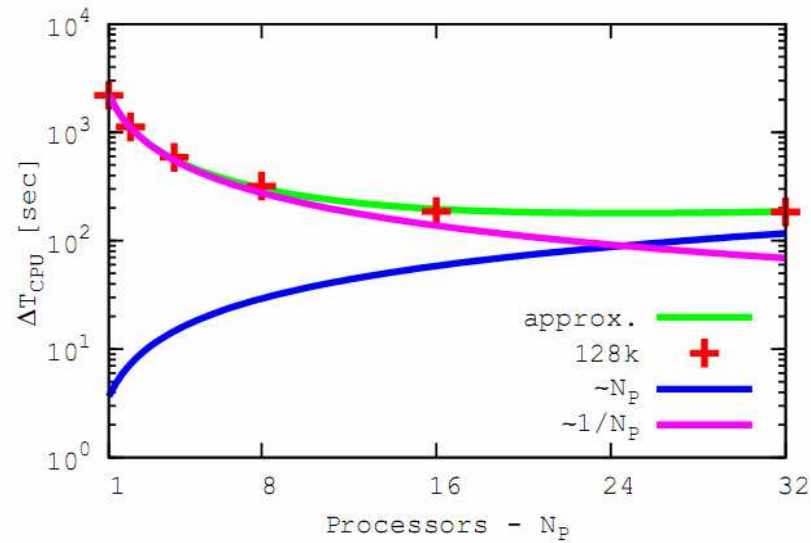
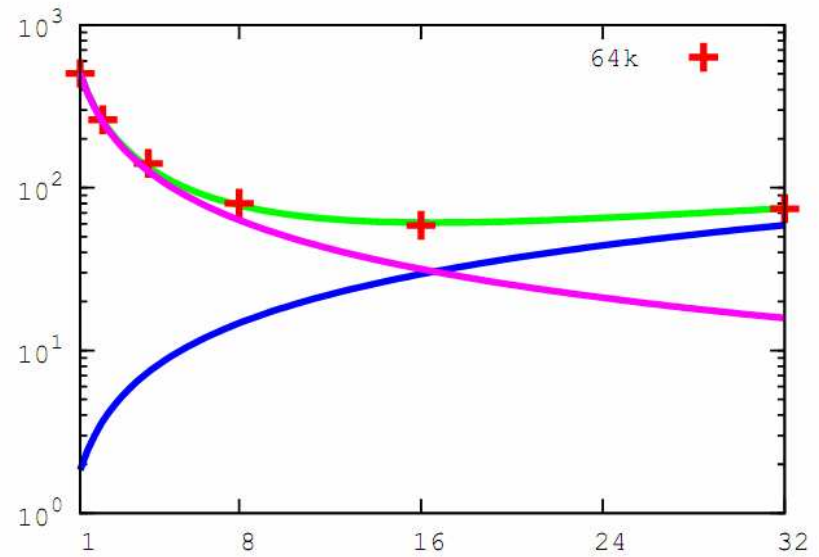
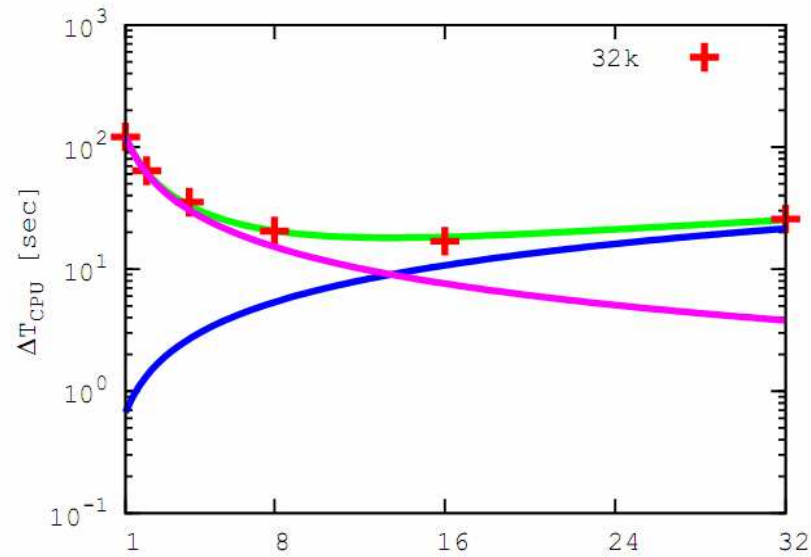
ϕ GPU Hermite 6th results



ϕ GPU Hermite 6th results

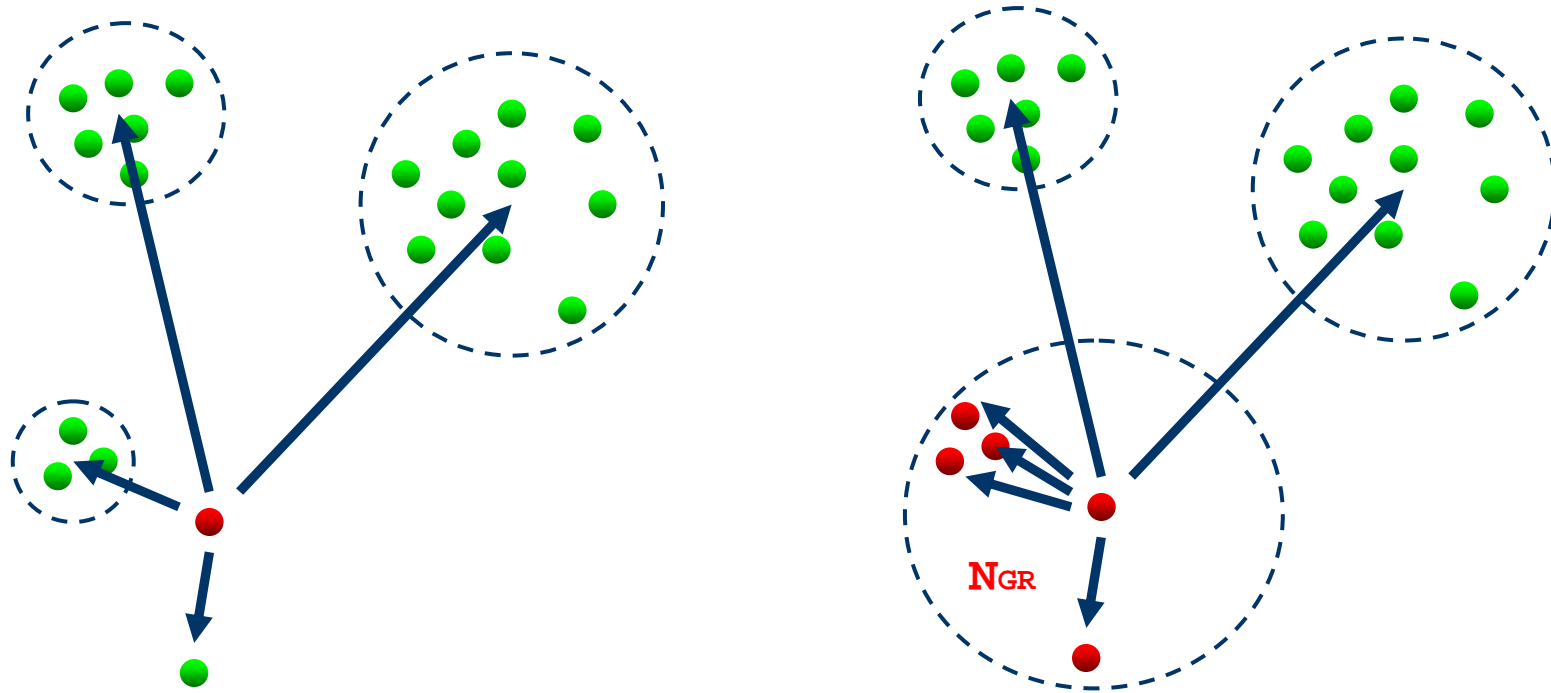


ϕ GPU Hermite 6th results



Parallel TREE GPU gravity

Jun Makino (pC++): TREE+GRAPE/GPU code



Makino, PASJ, 43, 621 (1991)

Inter. list on host $\sim N$
Inter. list length \rightarrow short...

Makino, PASJ, 56, 521 (2004)

Fukushige, Makino & Kawai, PASJ, 57, 1009 (2005)

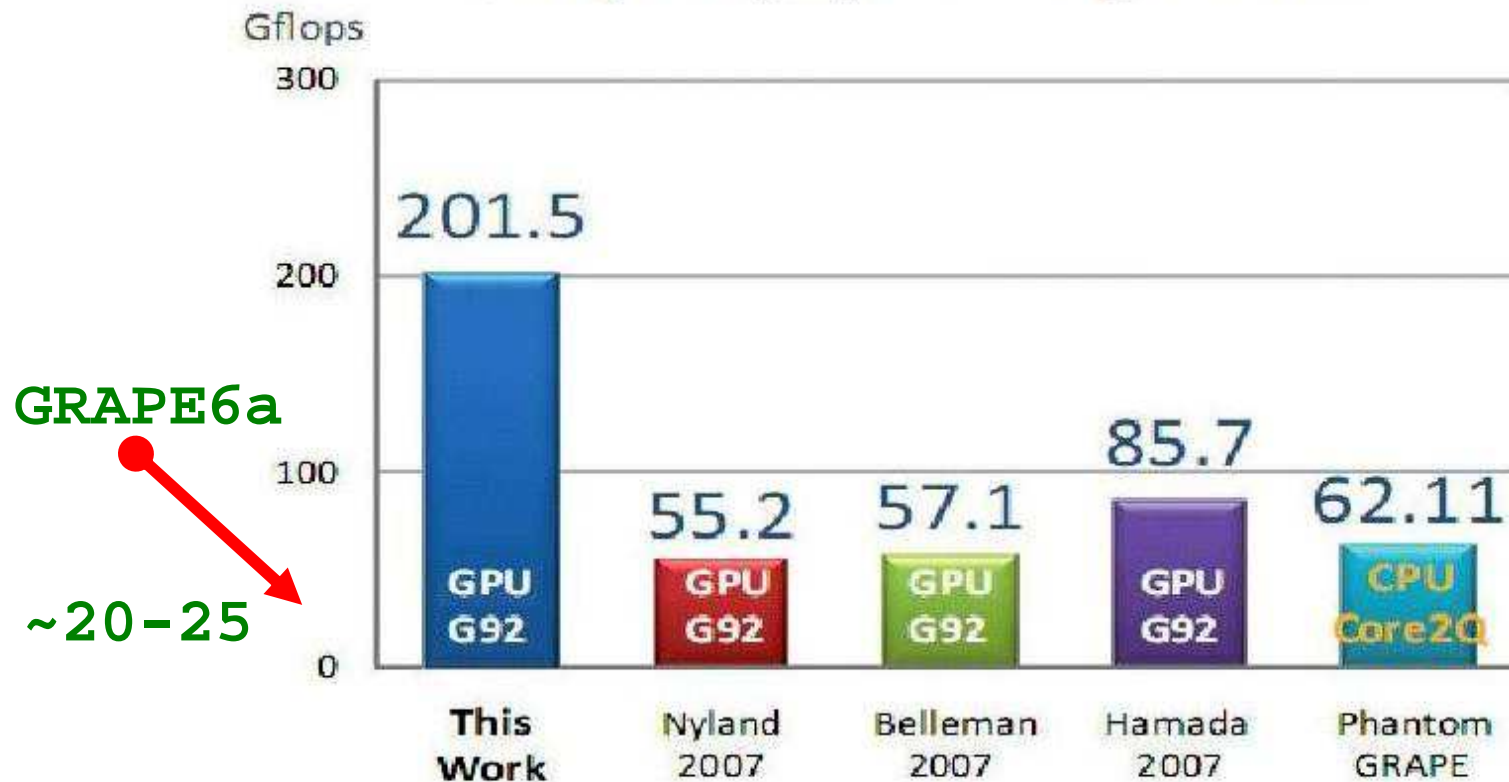
One interaction list is shared among
NGR particles!

Inter. list on host $\sim N/NGR$
Inter. list length \rightarrow larger...

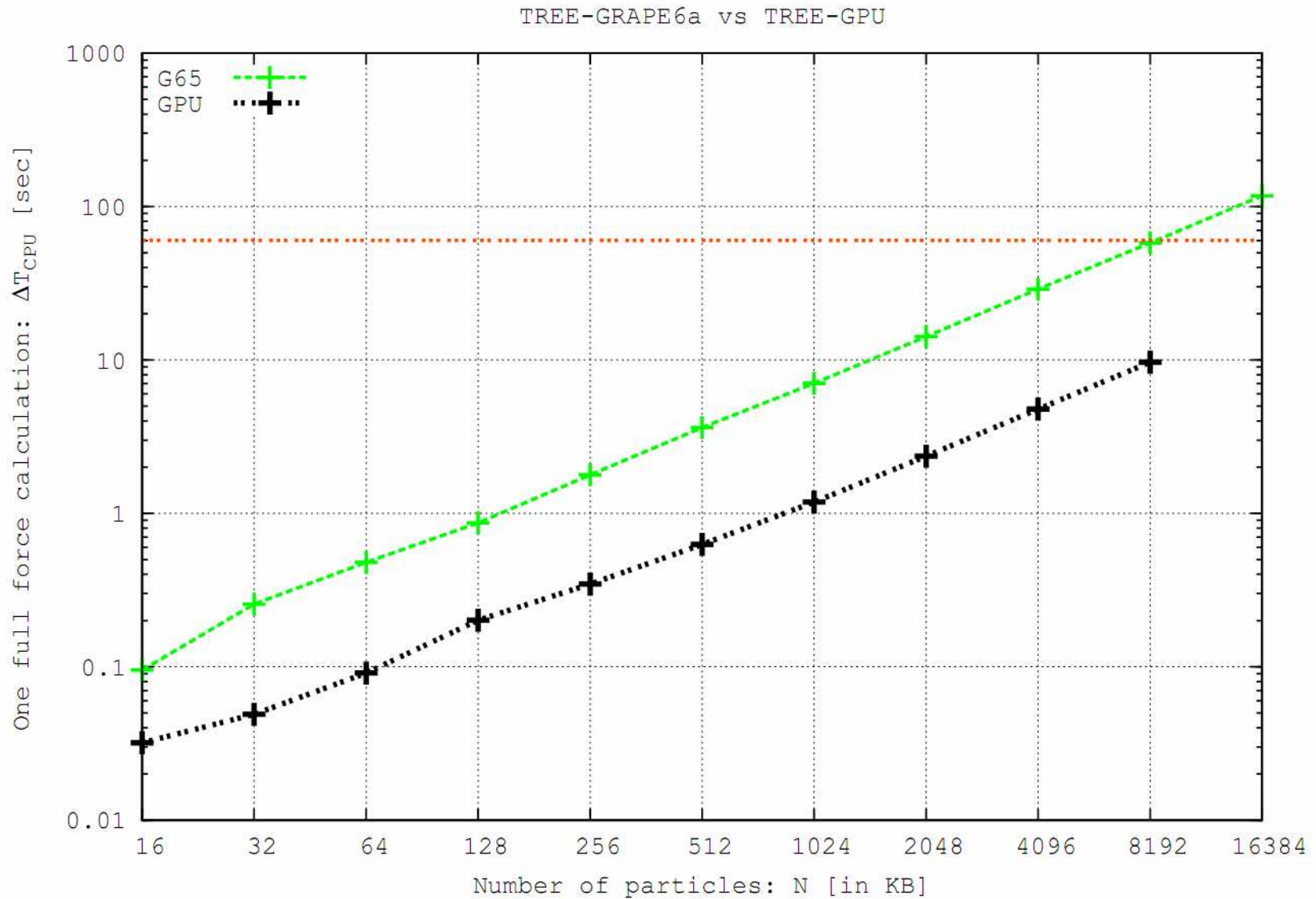
Parallel TREE GPU gravity

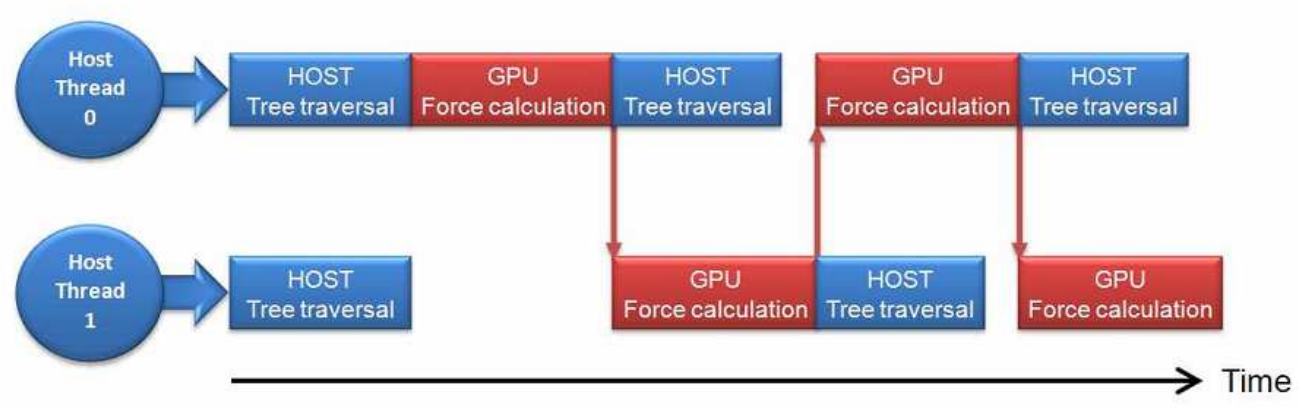
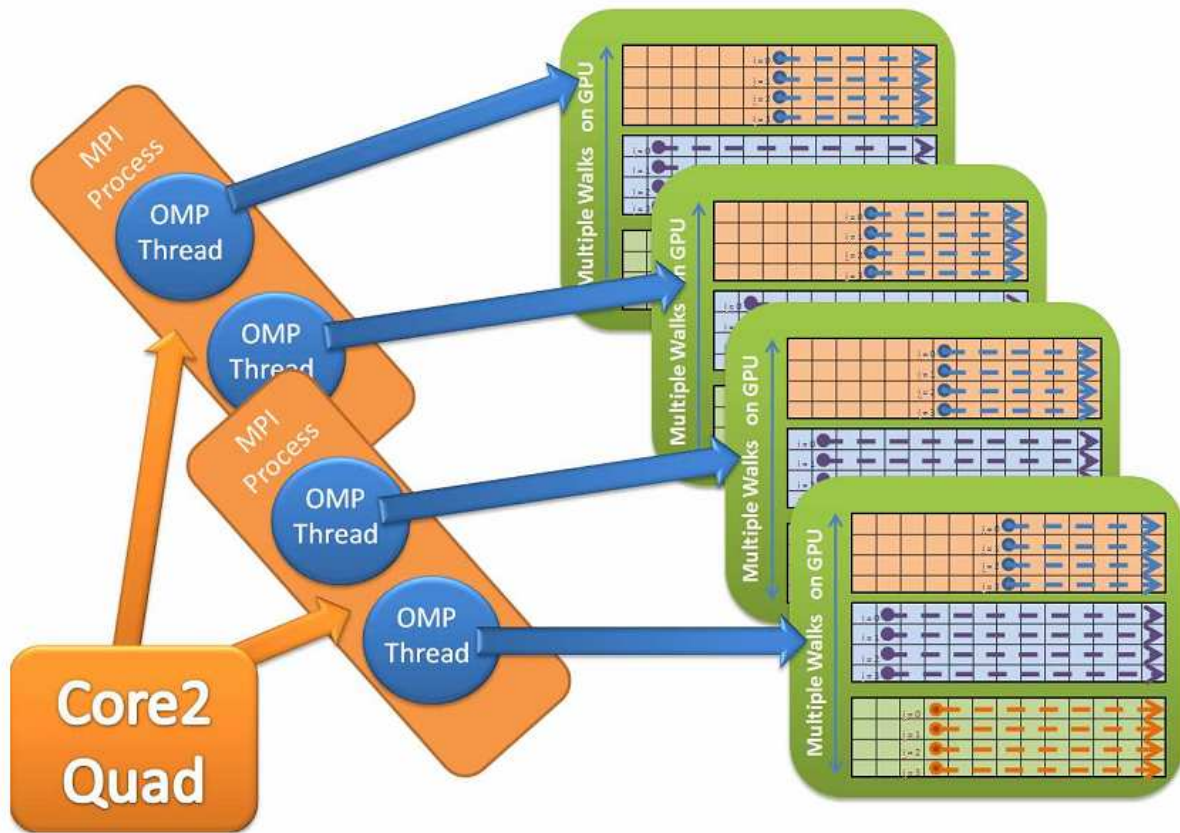
Hamada et al. 2008: TREE+GRAPE/GPU code

$O(N \log N)$ tree algorithm

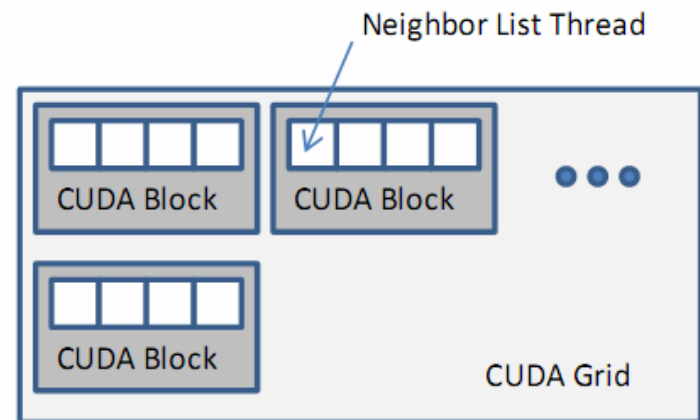
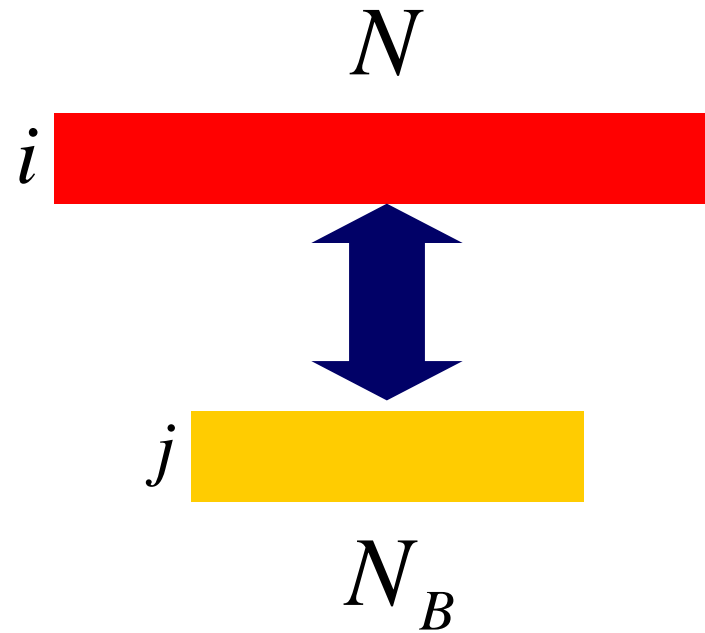
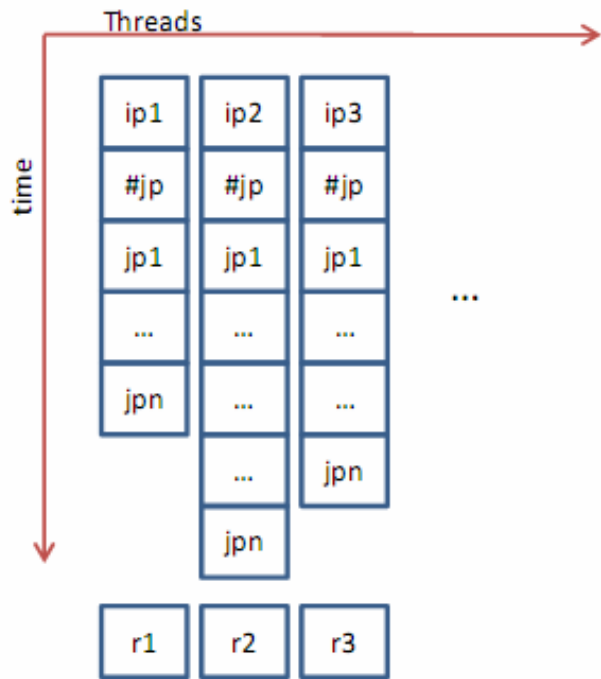
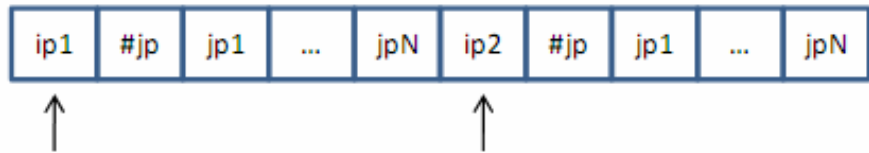


Parallel TREE GPU gravity

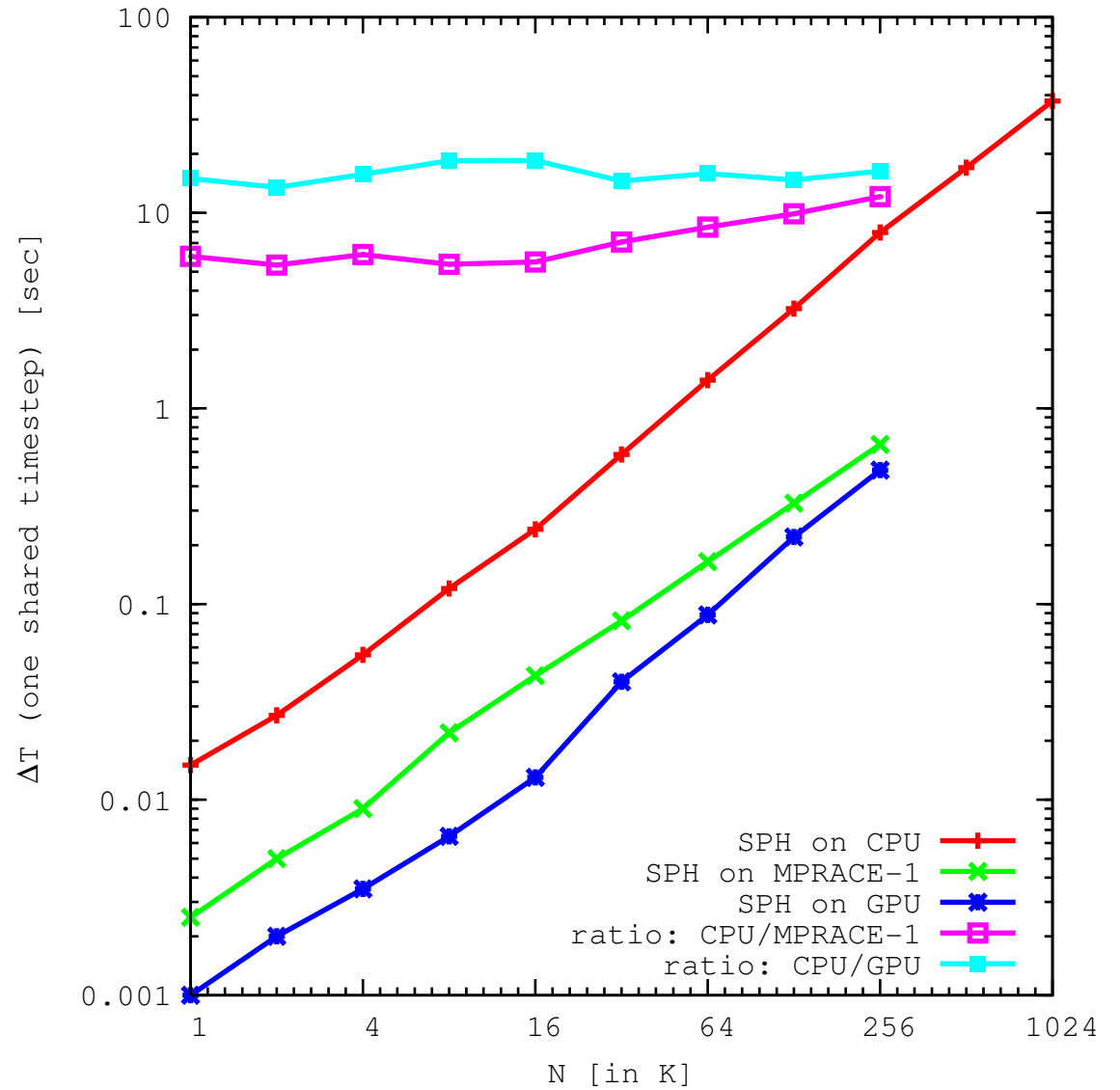
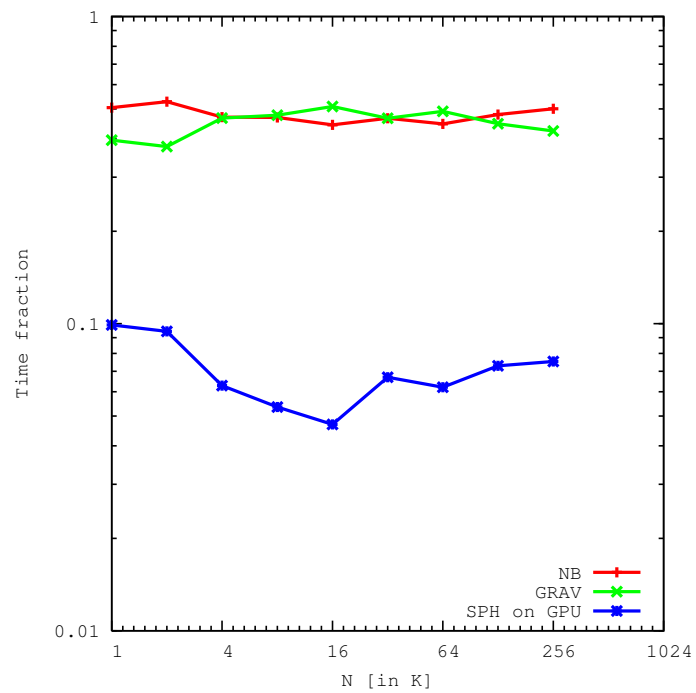
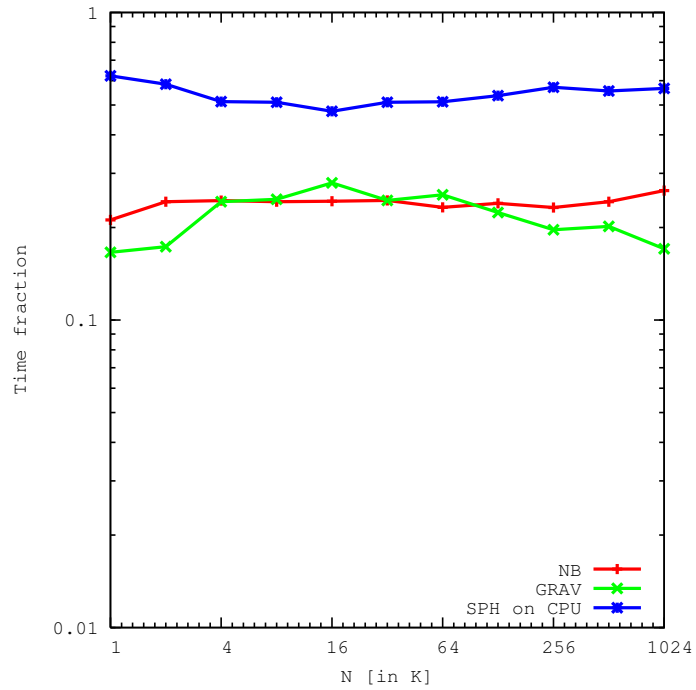




Simple GPU SPH code



SPH speedup with GPU



SPH astrophysical results

TREE-GRAPE SPH (on 4 nodes)

M = 2000 M_⊙ R = 3 pc fully -> H₂

Isothermal evolution.

Initial density distr. $\sim 1/r$

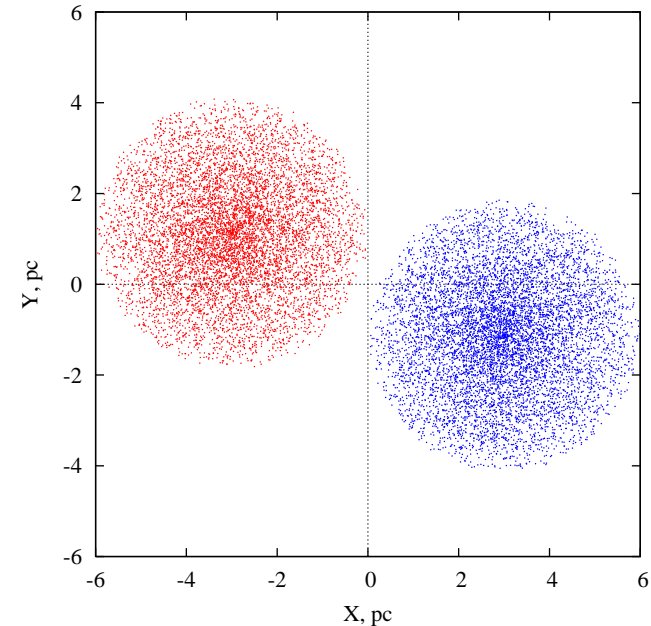
T = 20 K (c_{sound} = 0.3 km/sec)

V_{merge} = 5 km/sec

Calculation time 3*t_{ff} = 6 Myr

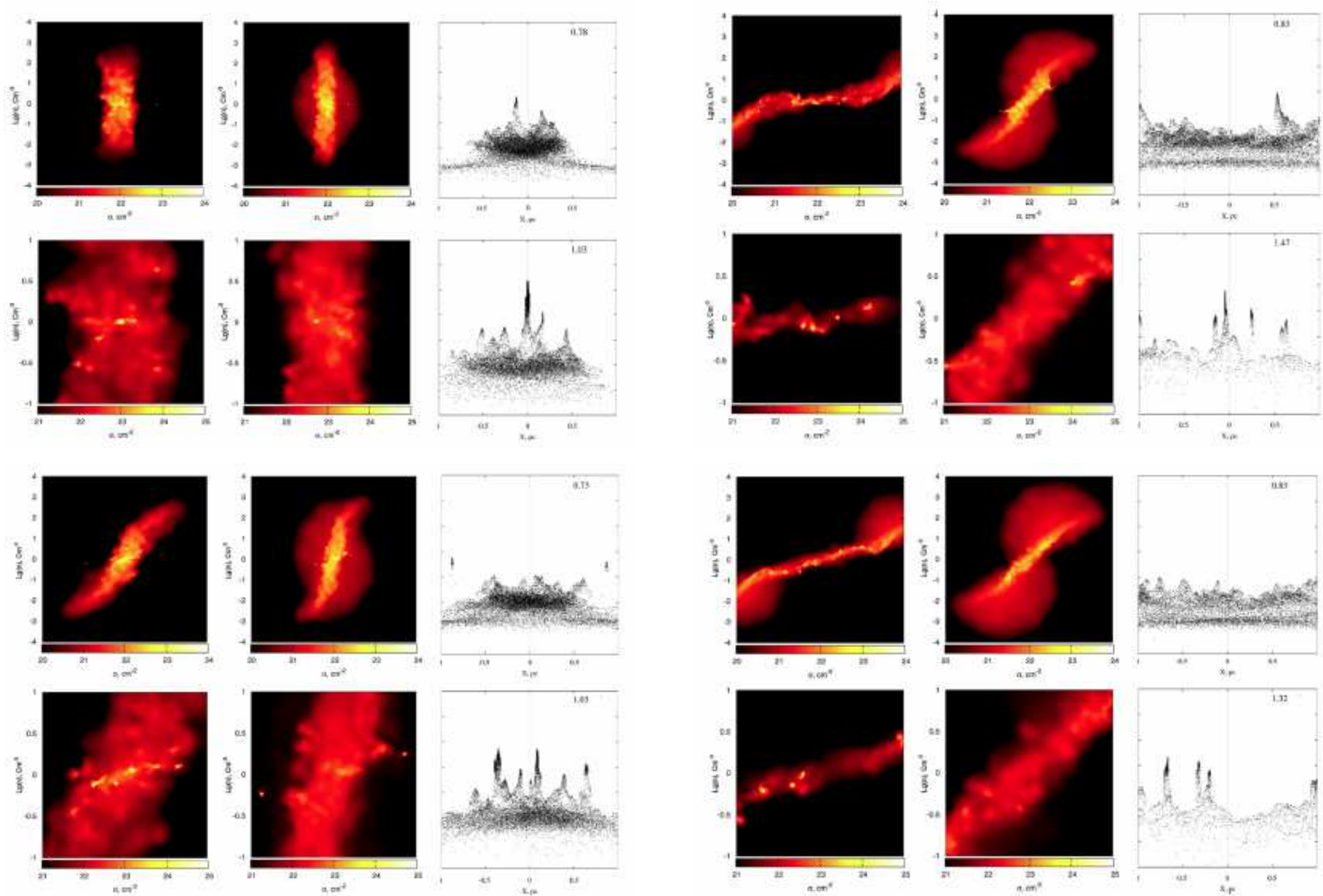
Resolution is h_{min} = 1e-4 pc

SPH: GPU/CPU speedup ~20

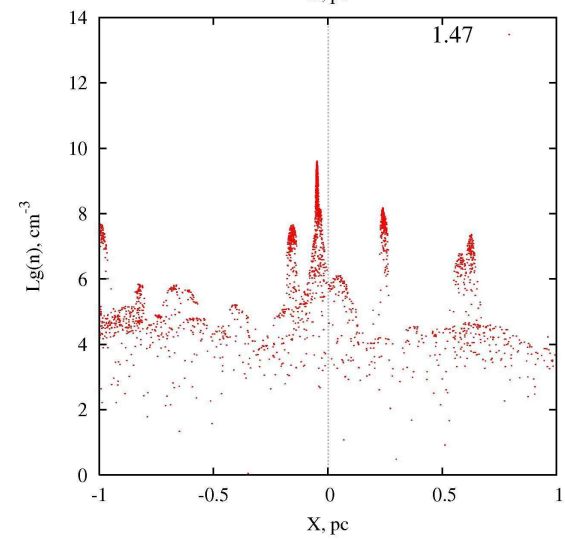
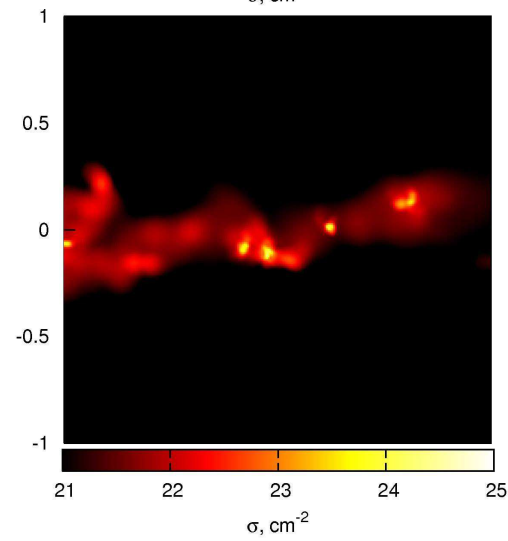
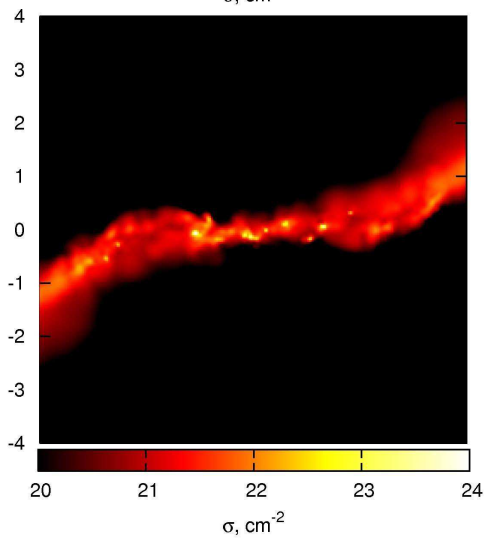
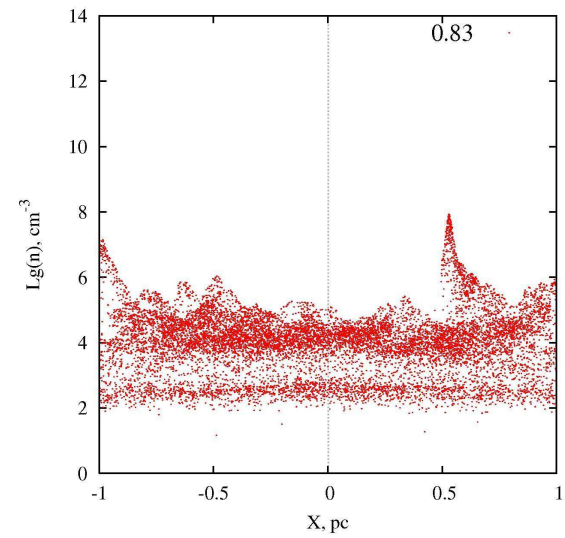
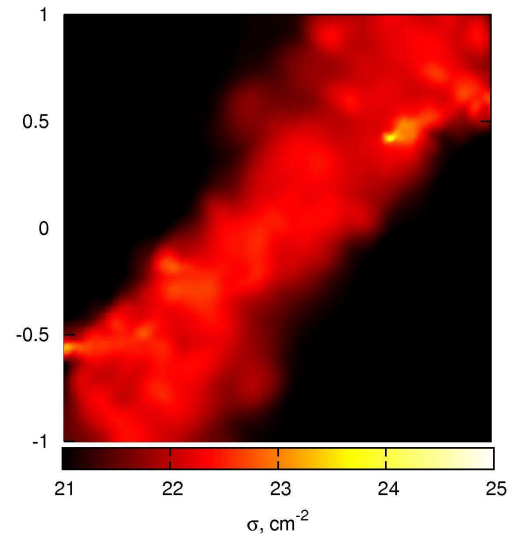
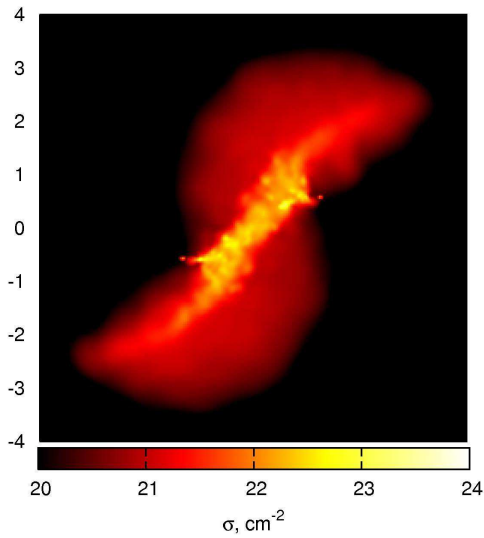


N = 2x4k	DT_CPU = 52 min
2x8k	1.74 hours
2x16k	3.5 hours
2x32k	6.9 hours
2x64k	14 hours
* <u>2x128k</u>	<u>28 hours</u>
2x256k	55 hours
2x512k	111 hours

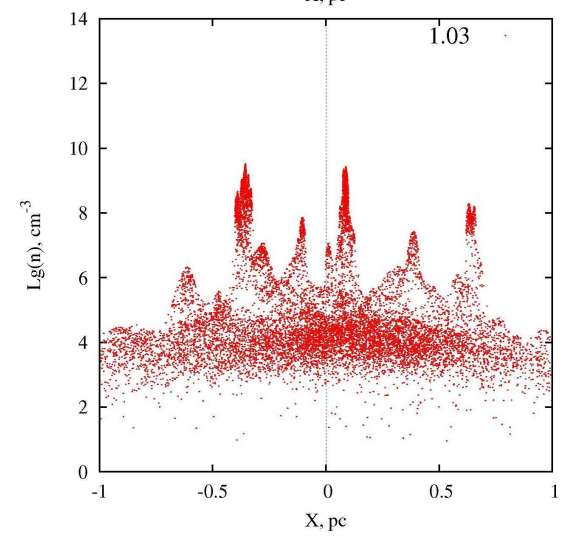
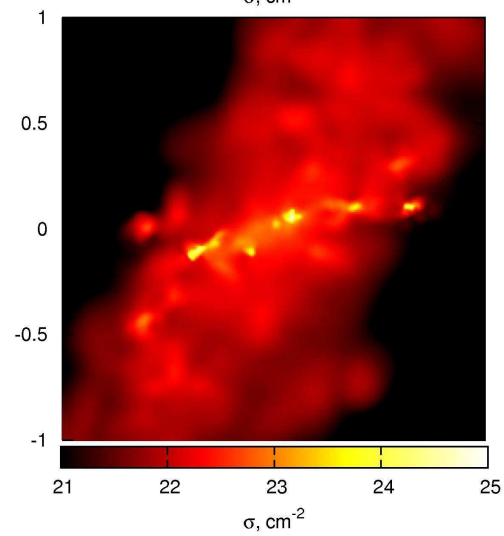
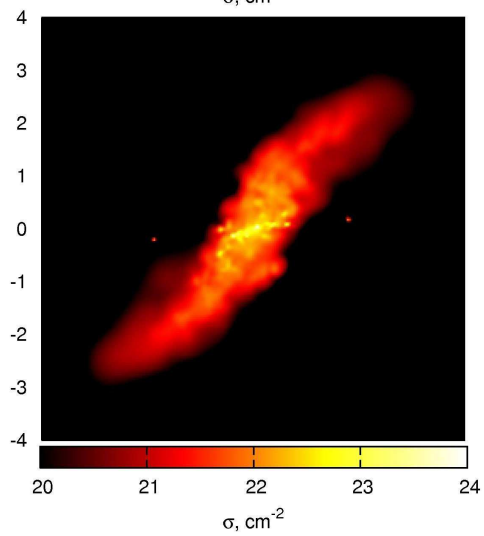
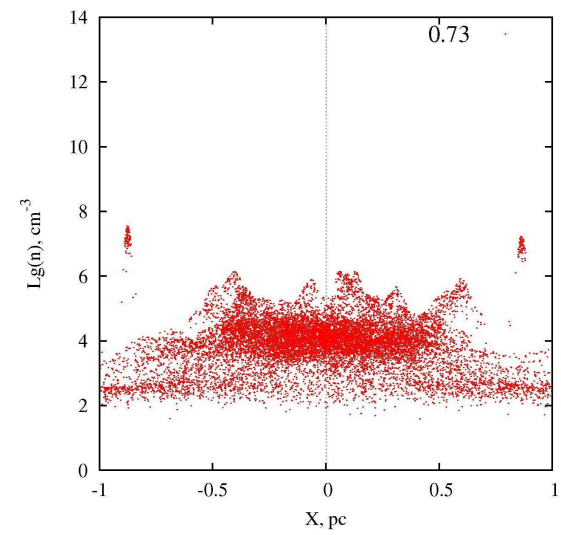
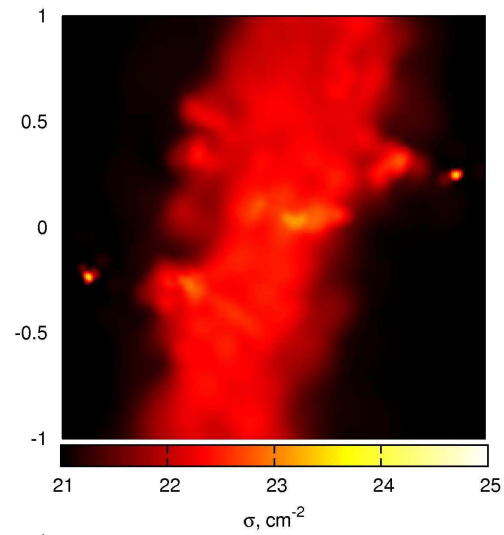
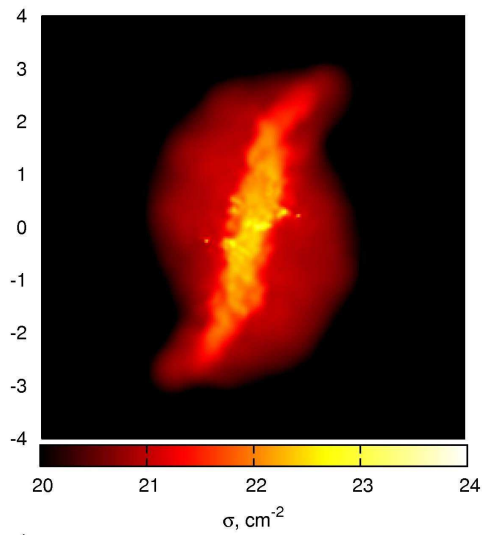
SPH astrophysical results



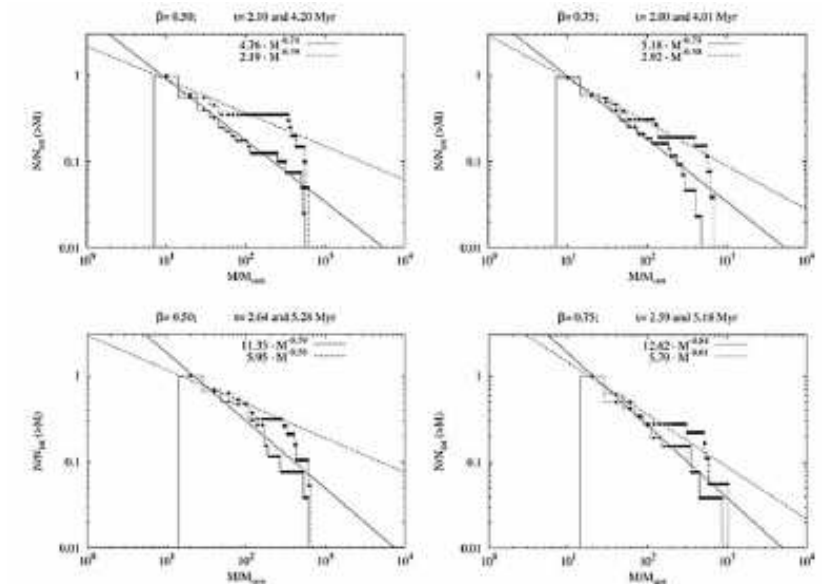
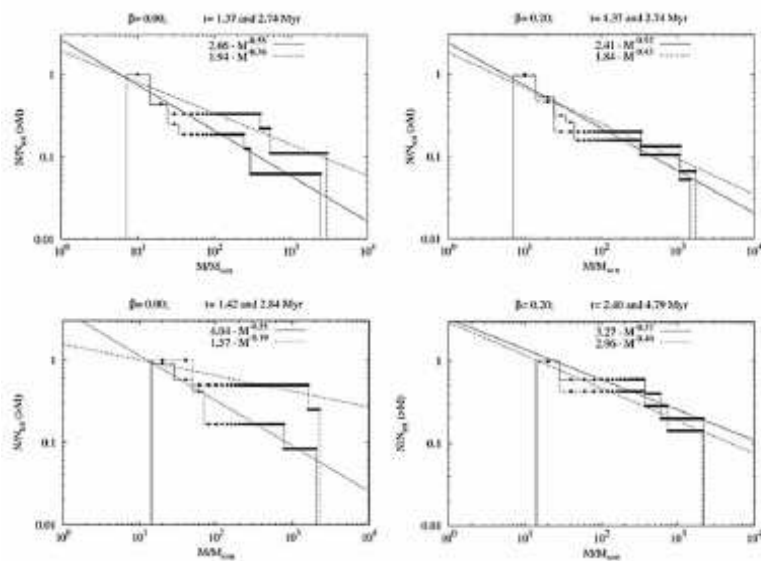
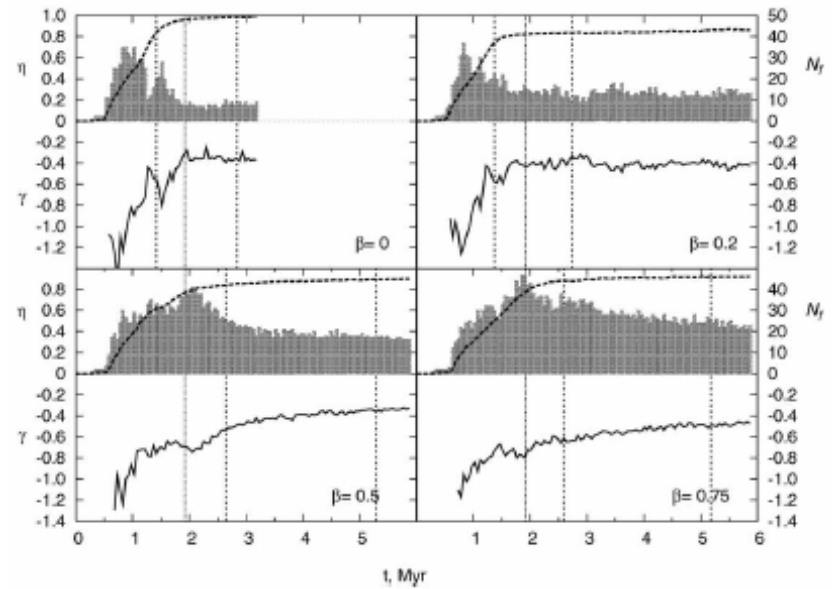
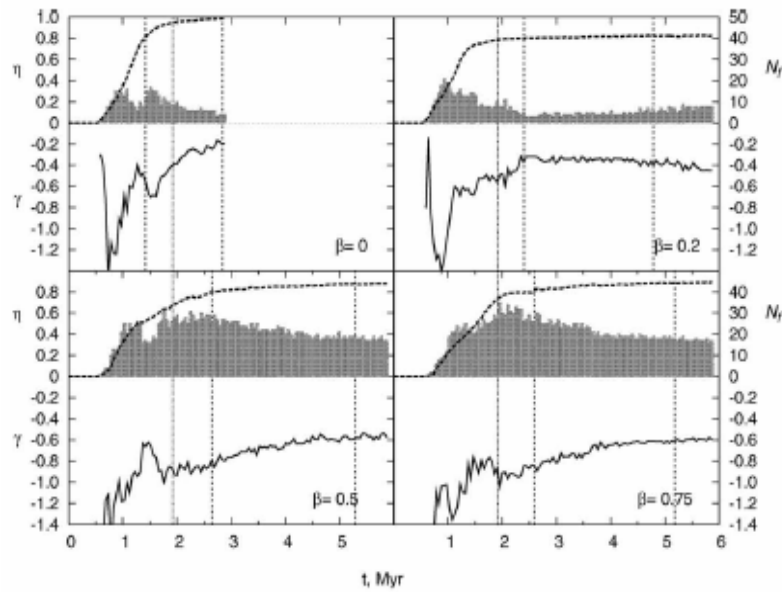
SPH astrophysical results



SPH astrophysical results



SPH astrophysical results



Plummer Initial Condition

$$M(r) = M_o \cdot \frac{1}{(1 + (r_o/r)^2)^{3/2}}$$

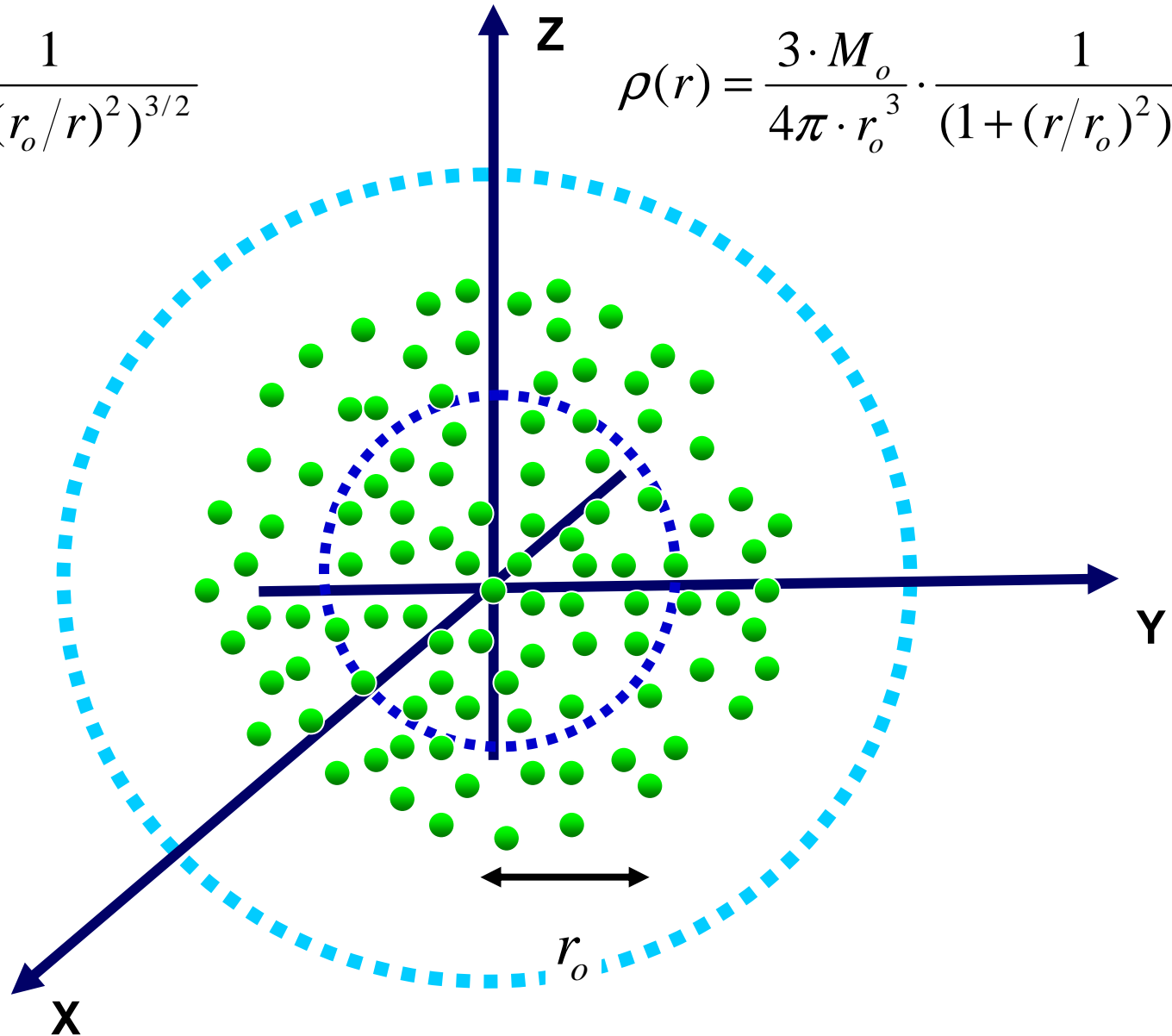
$$\rho(r) = \frac{3 \cdot M_o}{4\pi \cdot r_o^3} \cdot \frac{1}{(1 + (r/r_o)^2)^{5/2}}$$

$$r_{HM} \approx 0.769$$

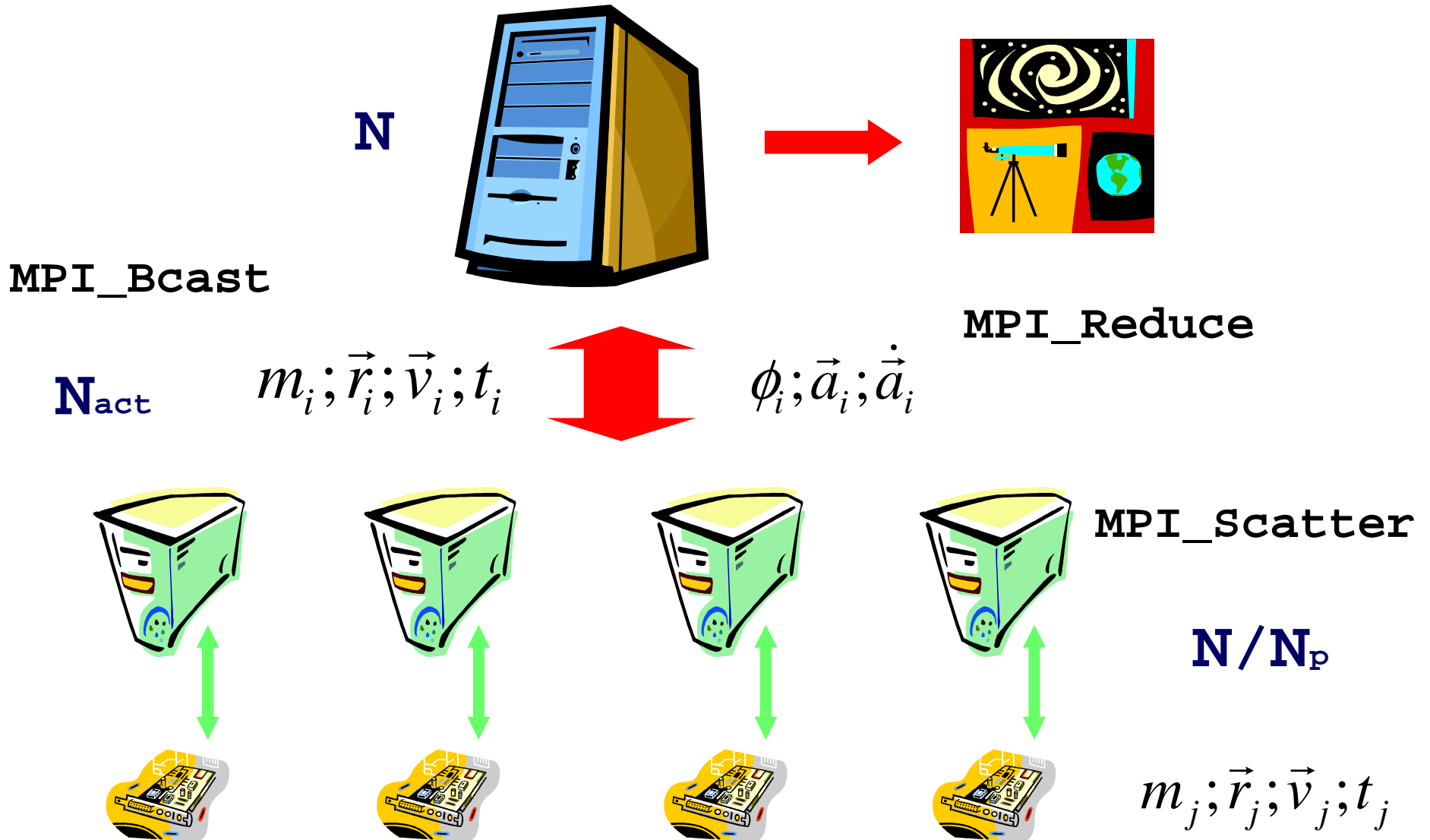
$$G = M = 1$$

$$E_{TOT} = -\frac{1}{4}$$

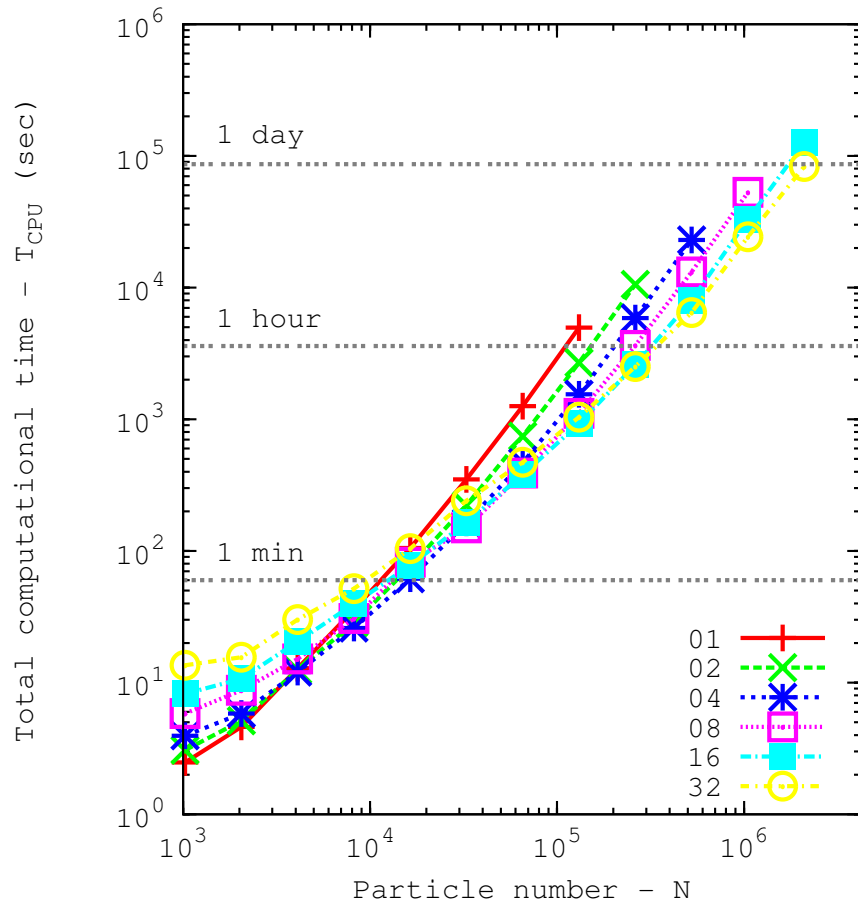
$$r_o = \frac{3\pi}{16} \approx 0.6$$



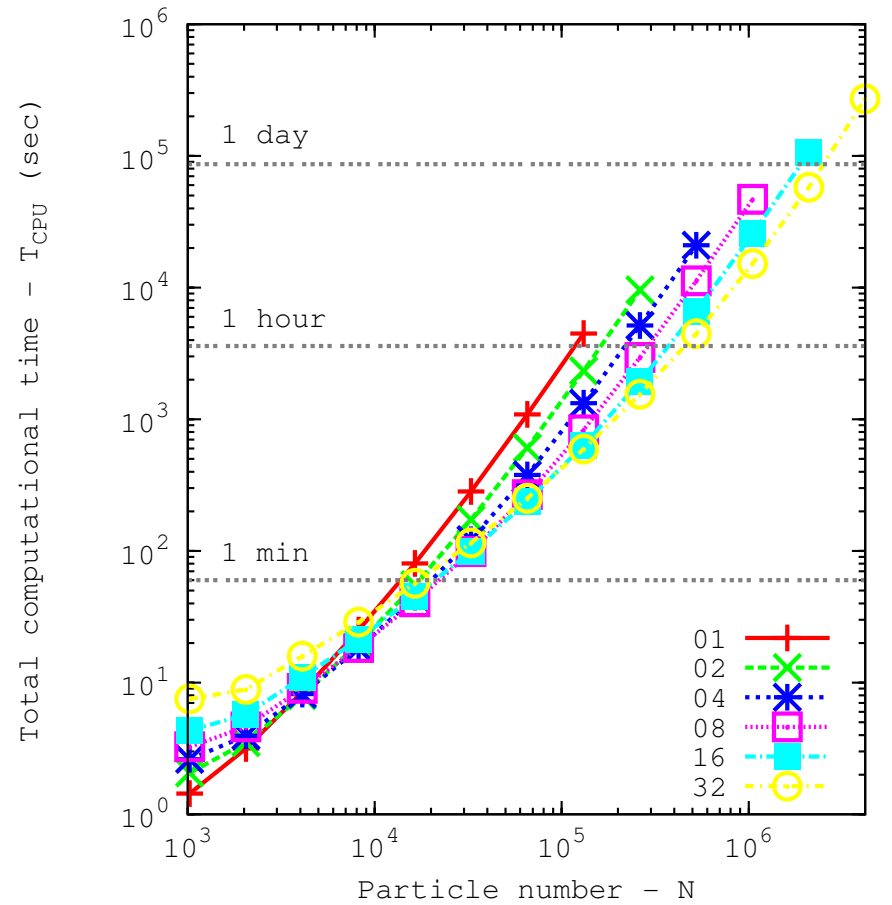
Parallel code on GRAPE cluster



Parallel PP on GRAPE clusters

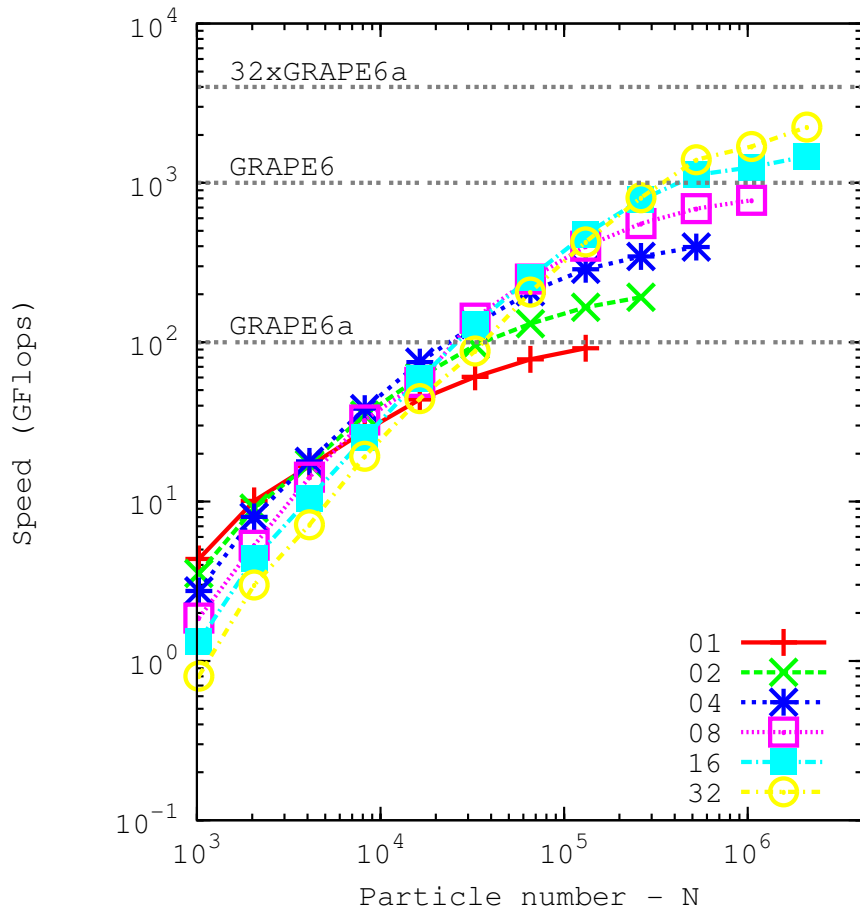


RIT

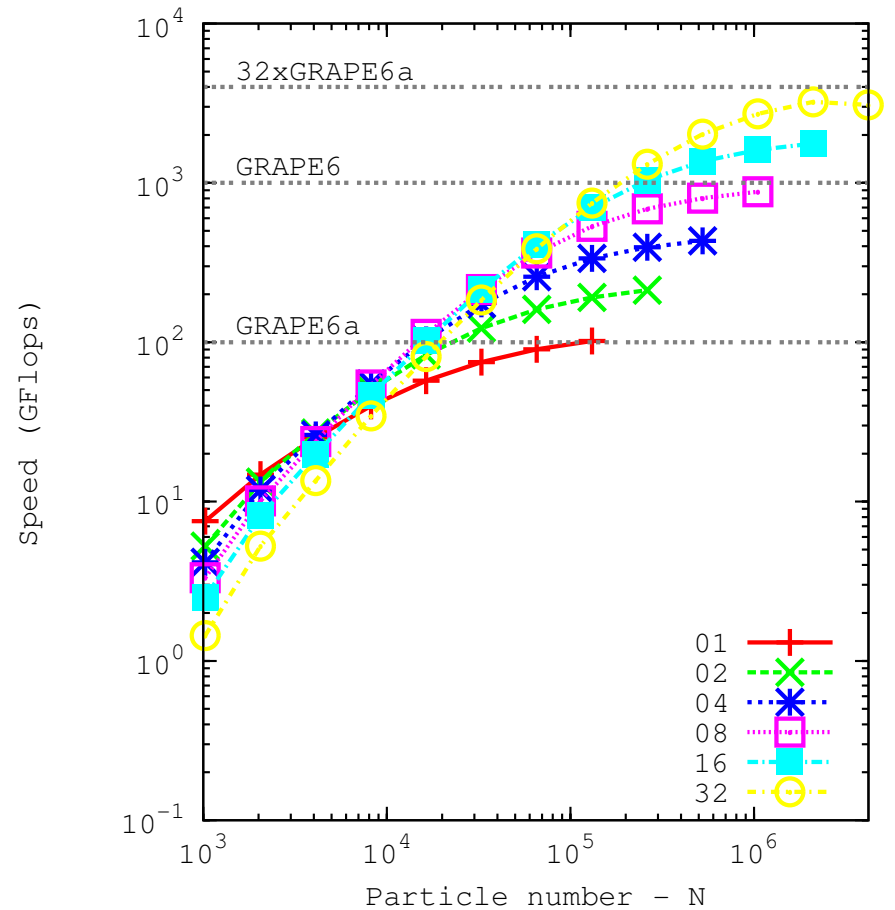


ARI

Parallel PP on GRAPE clusters



RIT: ~2.2 TFlops



ARI: ~3.2 TFlops

Parallel code on GRAPE cluster

$$\Delta T_{total} = \Delta T_{comm} + \Delta T_{calc}$$

$$\Delta T_{comm} \propto N_{act} \cdot N_{proc} \quad \Delta T_{calc} \propto N_{act} \cdot \frac{N_{tot}}{N_{proc}}$$

$$N_{act} \propto N_{tot}^{2/3} \quad \varepsilon = \frac{T_1}{N_{proc} \cdot T_{N_{proc}}}$$

Parallel code on GRAPE cluster

