

# N-Body Simulations as Gravitational Lenses

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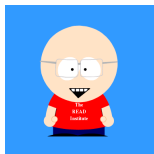
14. August 2008  
NBody2008 Turku, Finland



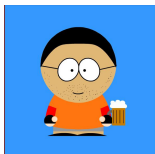
Prasenjit Saha



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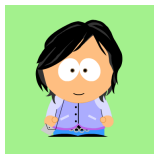
Justin Read



Andrea V. Maccò



Robert Feldmann

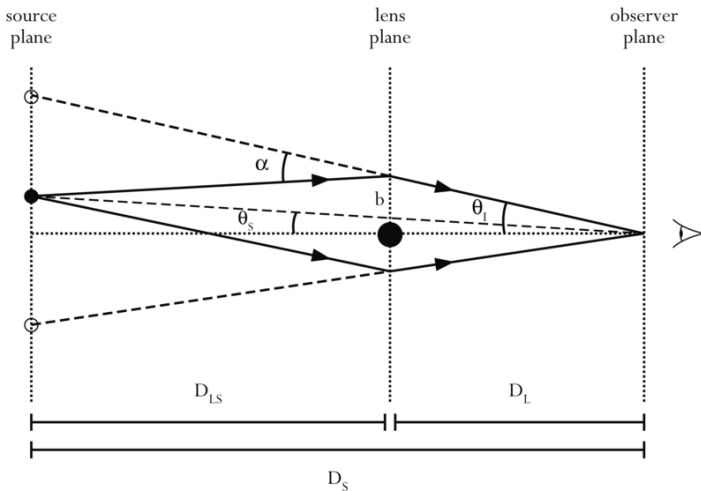


Liliya L.R. Williams

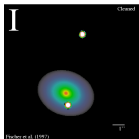
# Outline

- 1 Overview of Gravitational Lenses
- 2 Uses
- 3 Problems
- 4 Solutions
- 5 Testing with N-Body Simulations
- 6 StarSpray

# Schematic View of a Lens



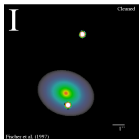
# Lens Examples



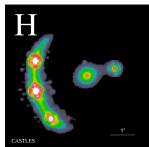
Double Q0957

(Castles Database)

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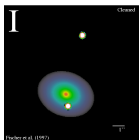
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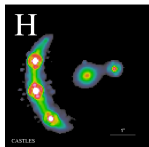
Quad RXJ131

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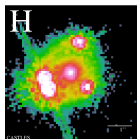


Double Q0957









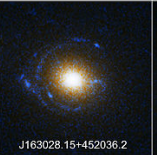

Quad RXJ131

(Castles Database)



Quad PG1115

# Einstein Rings

 <p>J073728.45+321618.5</p>	 <p>J095629.77+510006.6</p>	 <p>J120540.43+491029.3</p>	 <p>J125028.25+052349.0</p>
 <p>J140228.21+632133.5</p>	 <p>J162746.44-005357.5</p>	 <p>J163028.15+452036.2</p>	 <p>J232120.93-093910.2</p>

**Einstein Ring Gravitational Lenses**  
*Hubble Space Telescope • Advanced Camera for Surveys*

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team STScI-PRC05-32



# Abell 1689



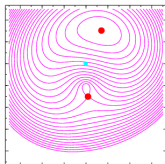
# Lens Equation

The lens equation describes the arrival time of light at the observed position:

$$\tau(\theta) = \frac{1}{2}|\theta|^2 - \theta \cdot \beta - \int \ln |\theta - \theta'| \kappa(\theta') d^2\theta'$$

where  $\theta$  is a position on the sky,  $\beta$  is the (unknown) source position, and  $\kappa$  is the mass at position  $\theta'$ .

Images are at the minimum, maximum, and saddle points of this surface.



# Uses

Knowing the mass distribution  $\kappa$  we can:

- Measure the dark halo profile of clusters and galaxies, particularly the inner profile: cusp or core?
- See dark matter substructure in clusters (Abell 1689)
- Measure the Hubble time (with time delays) – constrains the expansion rate.

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These degeneracies are well-known but not well-accepted.

Many still use parameterized models and find a 'best-fit'.

This is 20th century lensing!

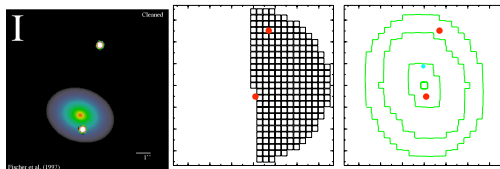


# 21st Century Lensing

We discretize the sky into grid cells and rewrite the equation as

$$\tau(\theta) = \frac{1}{2}|\theta|^2 - \theta \cdot \beta - \sum_n \kappa_n Q_n(\theta) + \text{shear terms}$$

Need to solve for each grid cell mass  $\kappa_n$ .



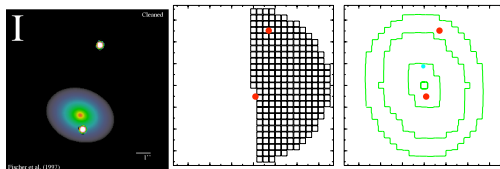
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Encapsulated into the program PixeLens. Easy to use, parallel, freely available, and can even run in a web browser!

# Measuring the Hubble time

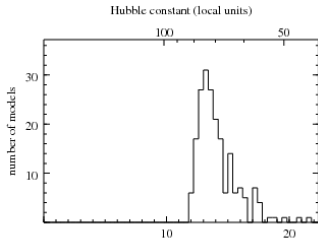
Simultaneous modeling of 11 lenses allowed us to constrain the Hubble Time

## Hubble Time

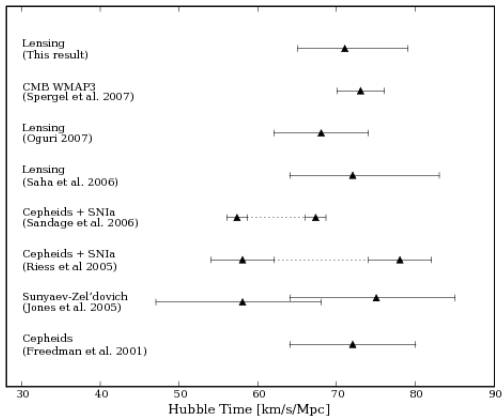
$$H_0^{-1} = 13.7_{-1.0}^{+1.8} \text{Gyr}$$

## Hubble Constant

$$H_0 = 71_{-8}^{+6} \text{km s}^{-1} \text{Mpc}^{-1}$$



# Measuring the Hubble time





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- 1 PKDGRAV – Treecode with Fast Multipole Method
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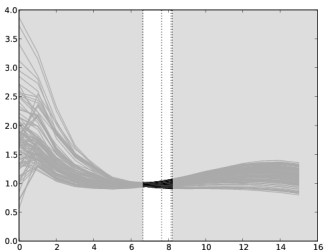
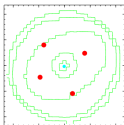
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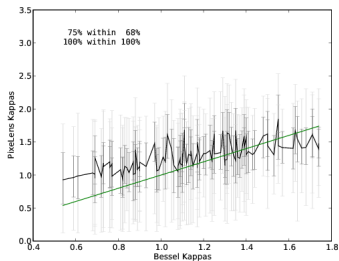
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# Single Quad

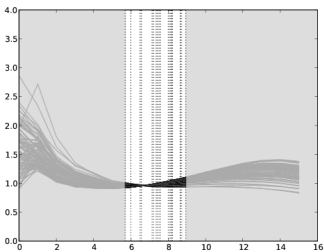
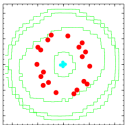


Enclosed mass comparison

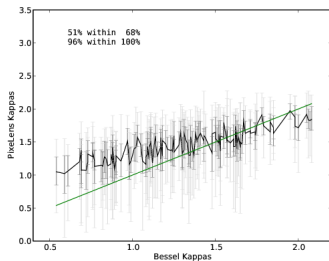


Per pixel recovery

# Einstein Ring

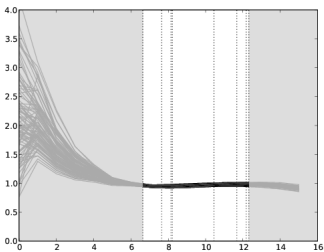
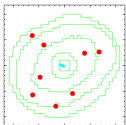


Enclosed mass comparison

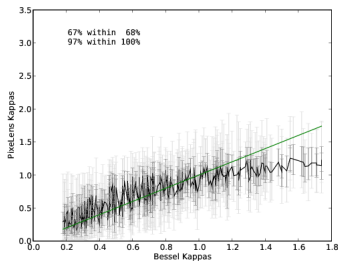


Per pixel recovery

# 2 Quads at different redshifts



Enclosed mass comparison



Per pixel recovery

# Conclusions....But wait! There's more!

- Modern lensing *must* account for degeneracies.
- Studying N-Body simulations allows us to test our mass recovery.
- Lensing tells us nothing about the profile within the inner most image.
- Multiple sources at multiple redshifts break the steepness degeneracy and constrains the mass profile in the image region.



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# StarSpray!