

# Radio Interferometry Fundamentals

John Conway

Onsala Space Obs and Nordic  
ALMA ARC-node



## So far discussed only single dish radio/mm obs

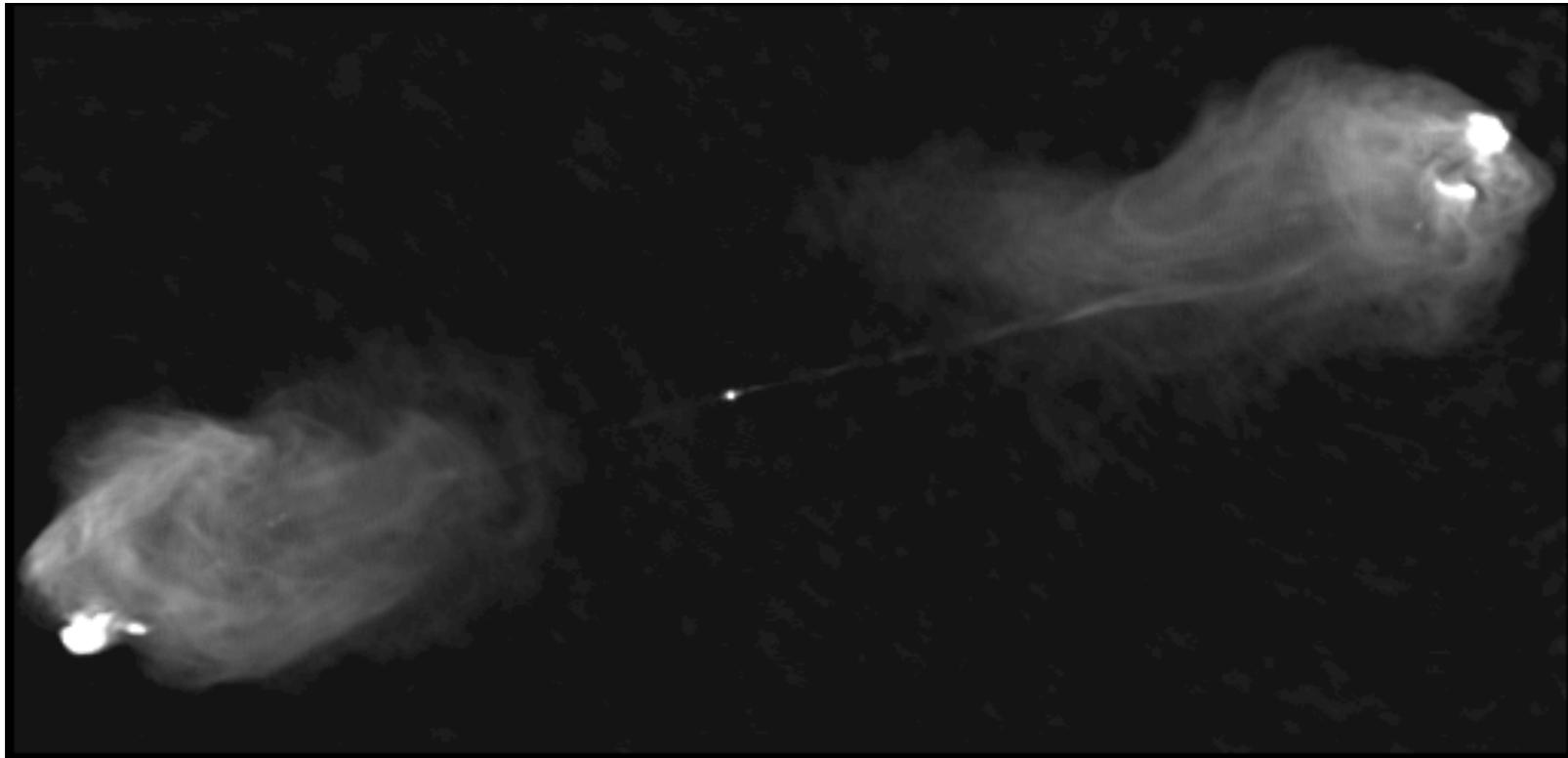
- Resolution  $\lambda/D$ , for  $D=20\text{m}$ , is  $30''$  at mm-wavelengths and  $30'$  (diameter of full moon) at cm (i.e.  $\lambda = 18\text{cm}$ )
- Paradox - In fact despite radio/mm having the longest wavelength it can achieve the highest angular resolution of any wavelength (up to  $1 \text{ mas} = 0.001''$ ).
- Reason- Interferometry is relatively easy to do at radio and mm wavelengths
- Presently at cm 5% science single dish and 95% interferometry, at mm more 50%/50% but with ALMA interferometry will dominate at mm as well.

# Very Large Array(VLA)

Overall View of the VLA



# VLA Image – Radio Galaxy Cygnus A (500 million light years away)



Radio astronomers can make nice  
images too

# ALMA Atacama Large Millimetre Array at 5000m elevation, Atacama Desert, Chile



ALMA at Chajnantor  
(Courtesy NAOJ)

ESO PR Photo 14/01 (6 April 2001)

© European Southern Observatory

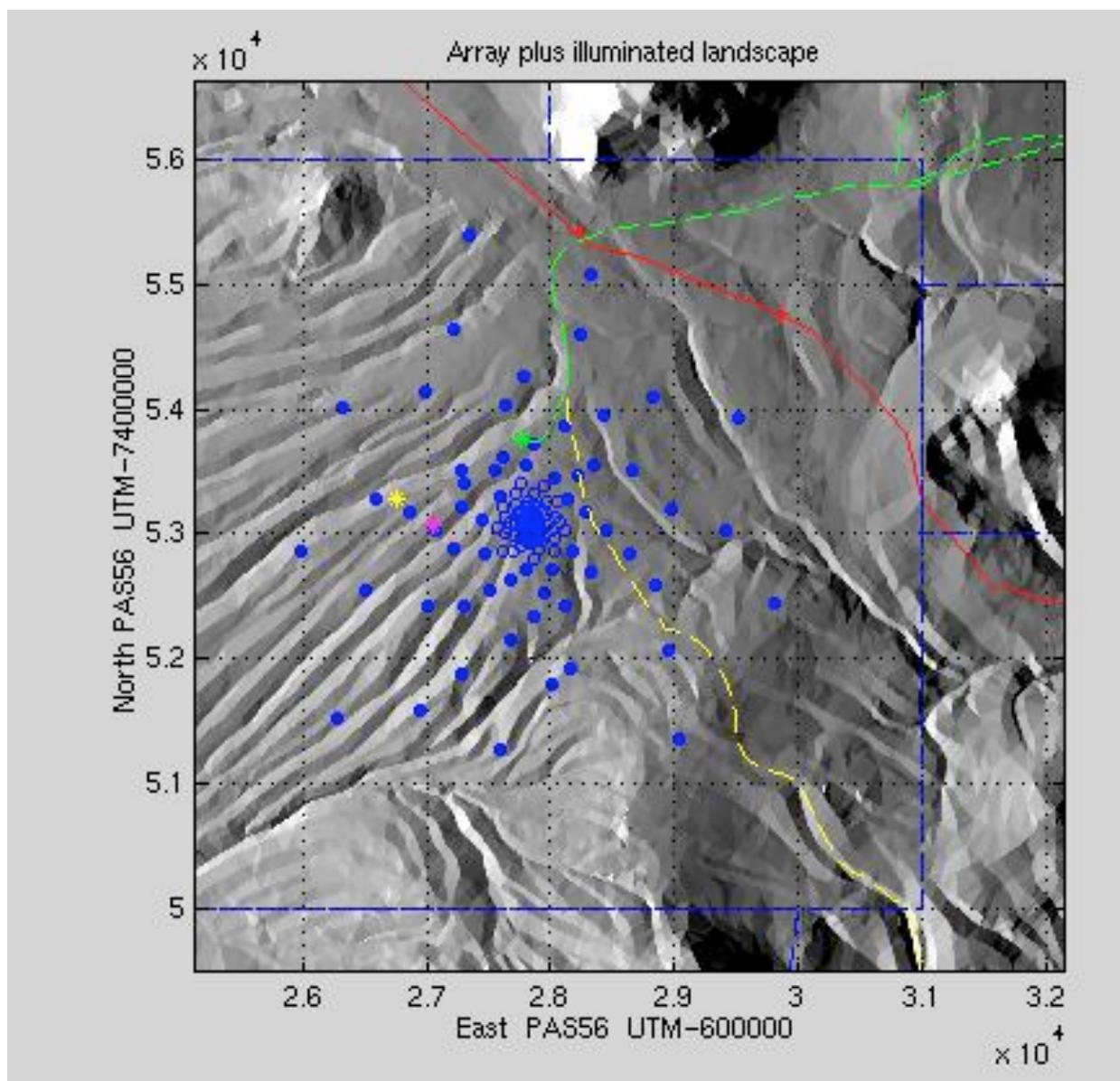


50 times 12m diameter  
dishes, moved around on  
200 pads

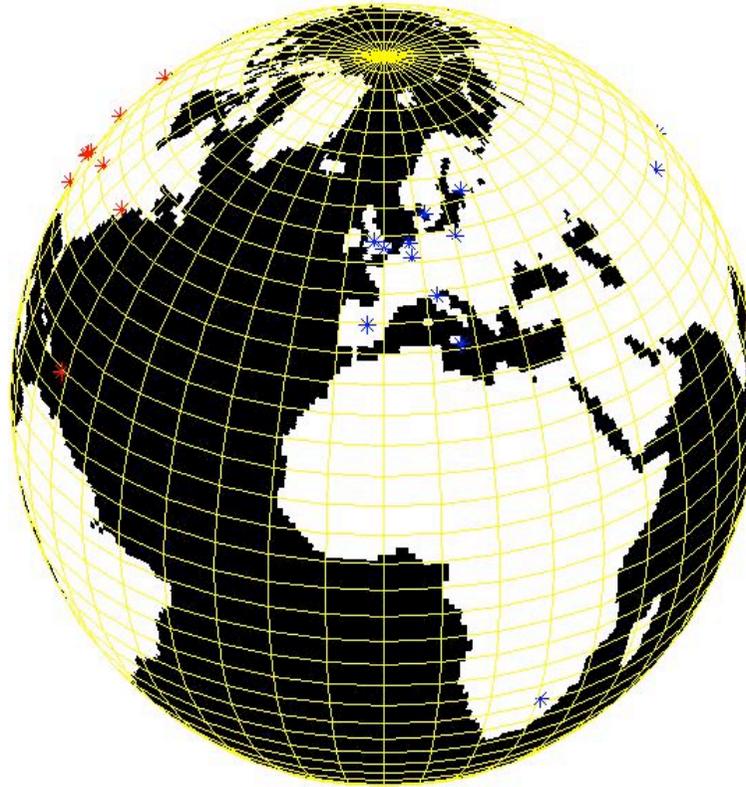
Myself at the site  
in April 2001



## ALMA Configuration layout – John Conway



## Very Long Baseline Interferometry (VLBI)



Make an interferometer the size of the Earth – combine signals from antennas all around Earth – Tape record data and combine later 'off-line' or send in real-time over internet

# Square Kilometer Array (SKA)

Future cm wavelength interferometer array

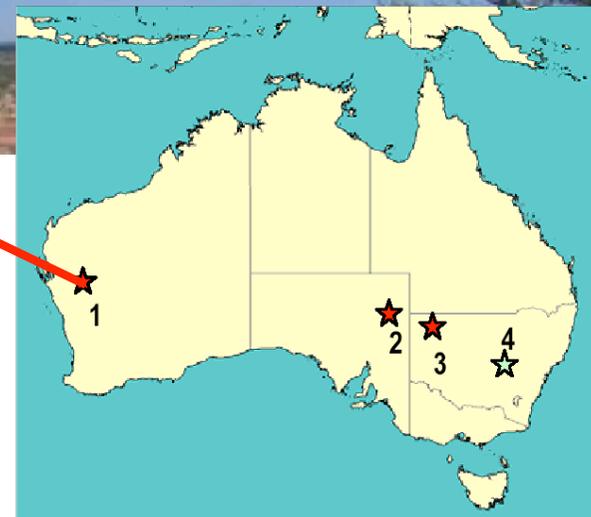


South africa



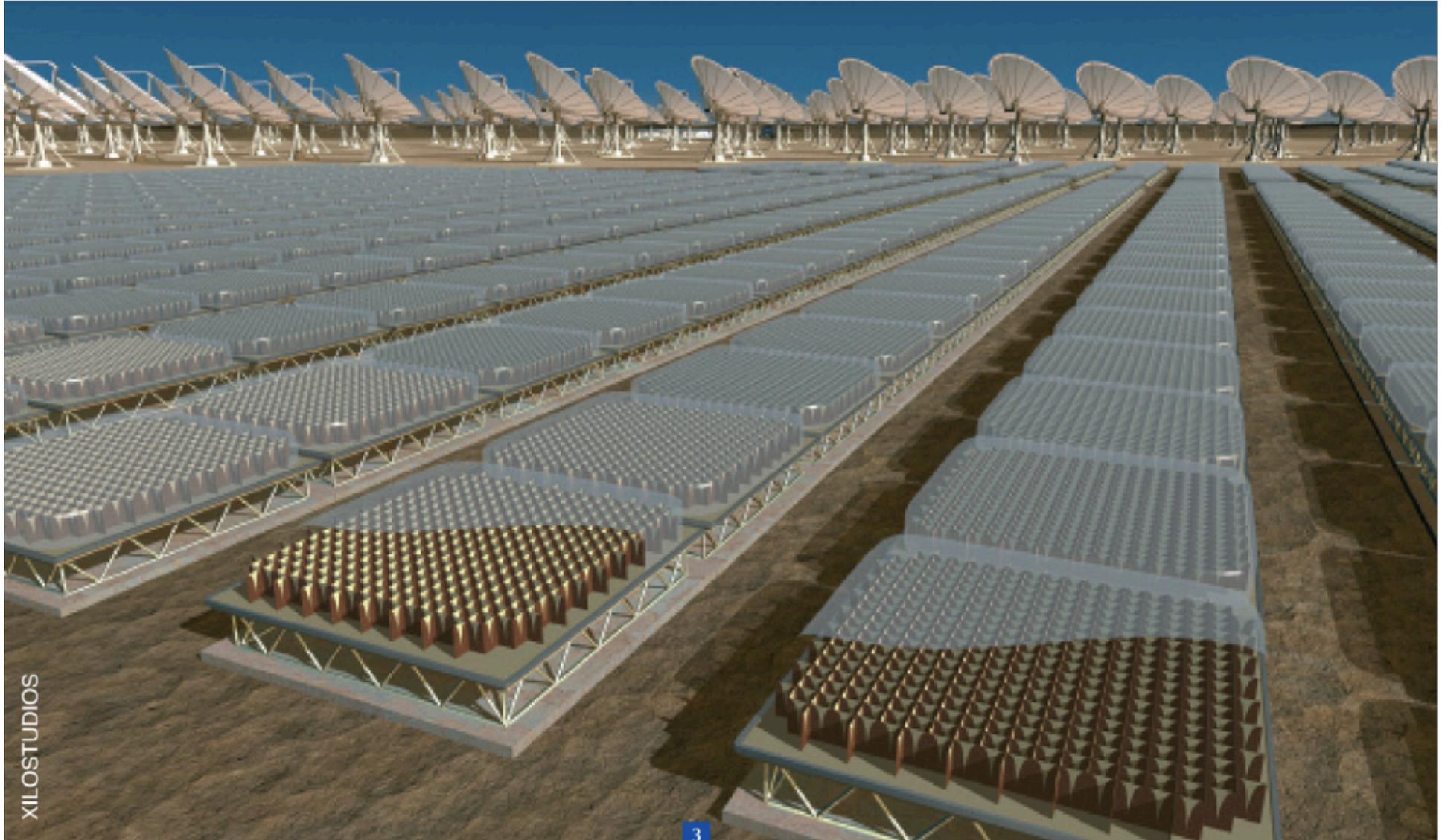
Australia

International project,  
estimated cost 1.5 B€  
phase 2 ready by 2020

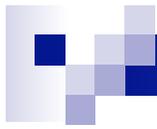


# Inner SKA core

Parabolas + aperture arrays, ...



XILOSTUDIOS



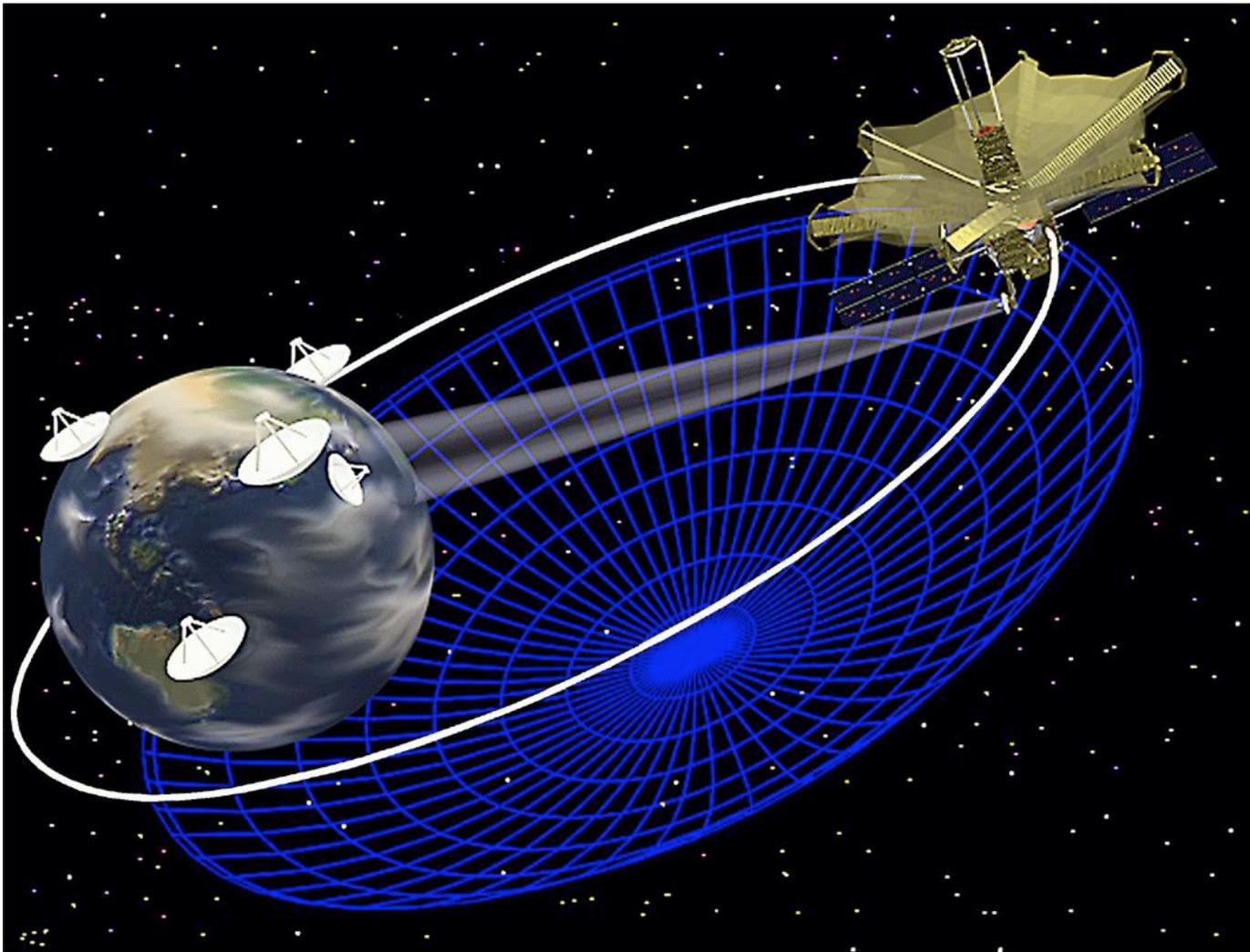
# SN1993J in M81

## VLBI Observations

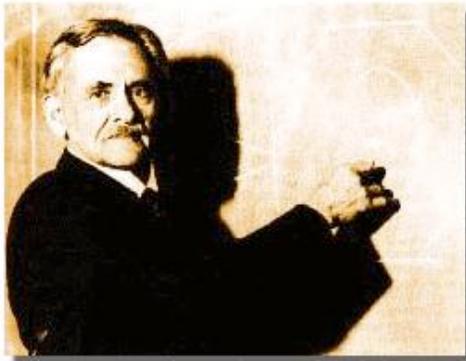
J.M. Marcaide, A. Alberdi, E. Ros,  
et al.

© J.M. Marcaide, Universitat de València, 2000

Japanese 6m antenna on VSOP satellite orbits the Earth out  
to 4 Diameters increasing the resolution of VLBI



# Michelson Stellar Interferometer



Albert Michelson

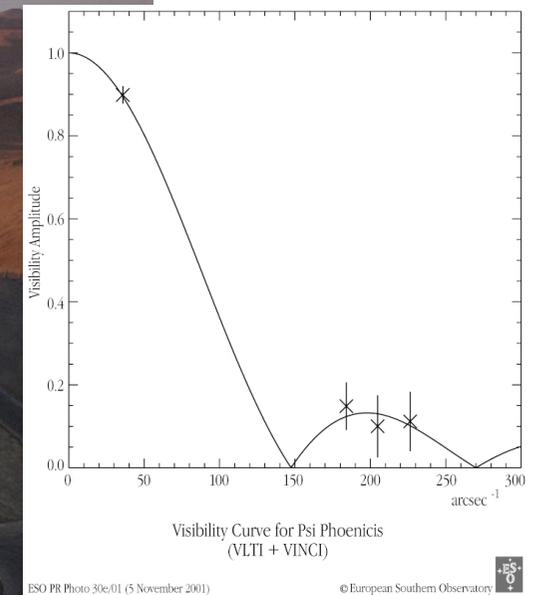
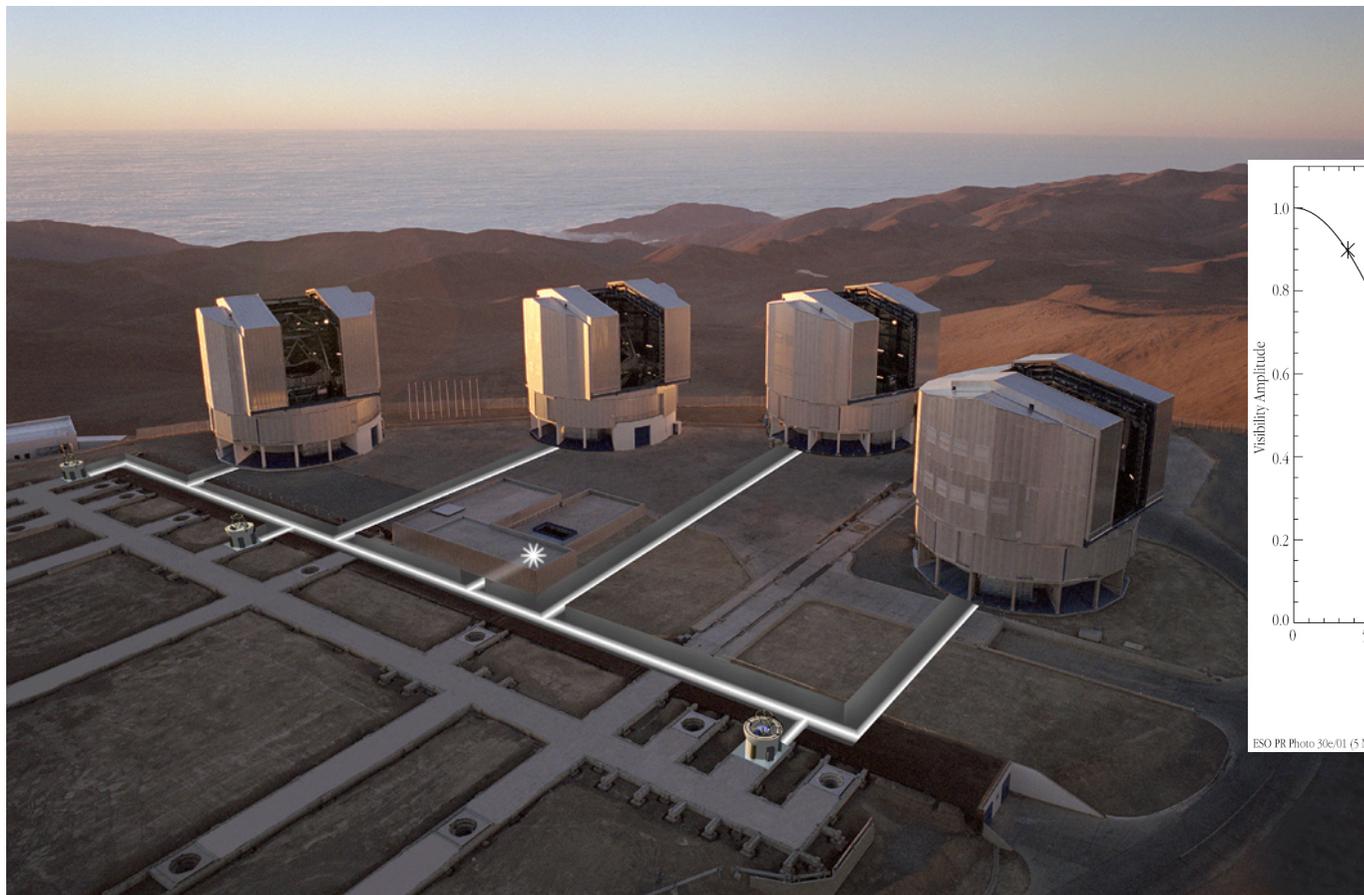


The 20-foot beam on top of the 100-inch Hooker Telescope on Mt. Wilson in Southern California.

First direct measurement of a stellar diameter.

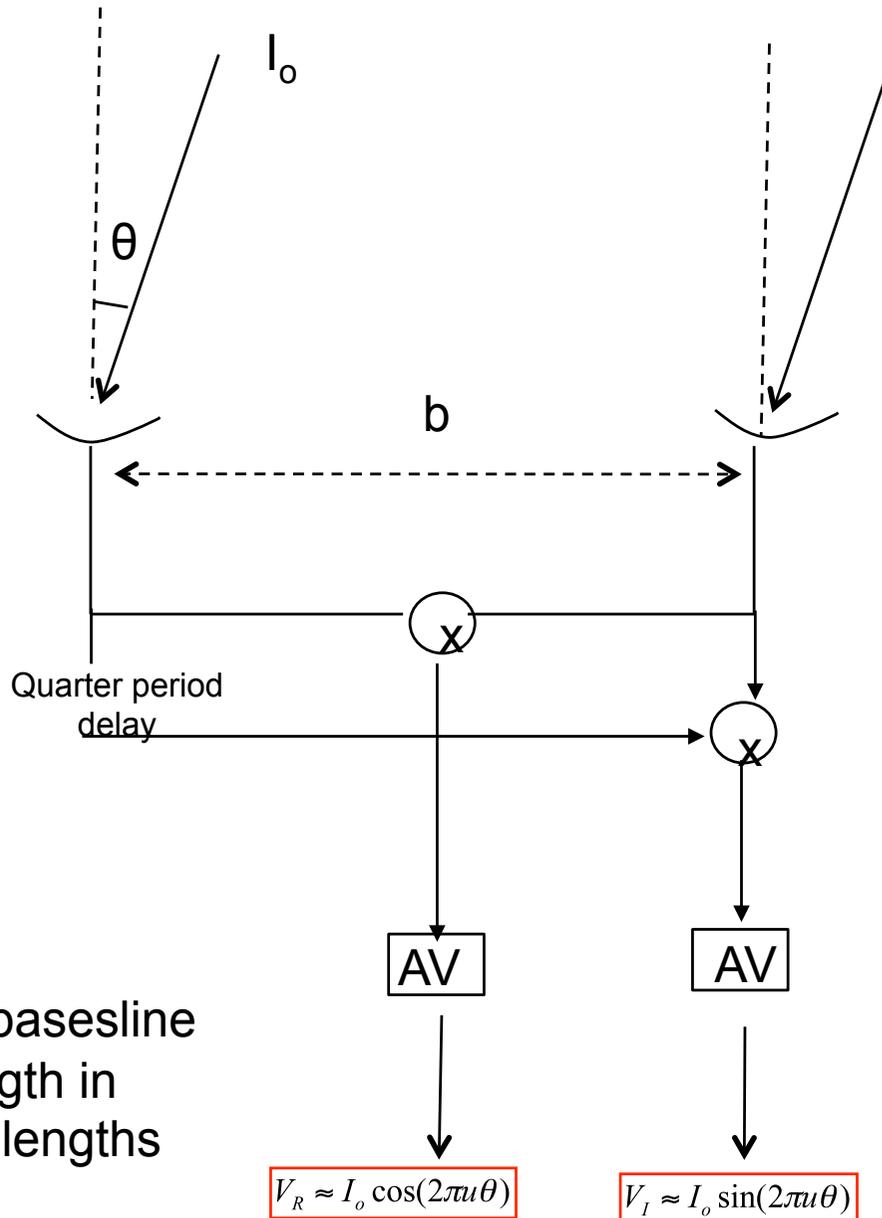
Michelson and Pease (1921)

# Very Large Telescope Interferometer (VLTI) - Paranal, Chile – optical interferometer



Aerial View of Paranal Observing Platform with VLTI Light Paths

# Radio Multiplying interferometer

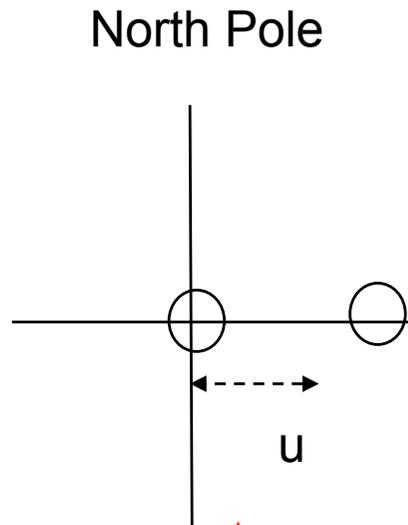


$u = b/\lambda$  baseline length in wavelengths

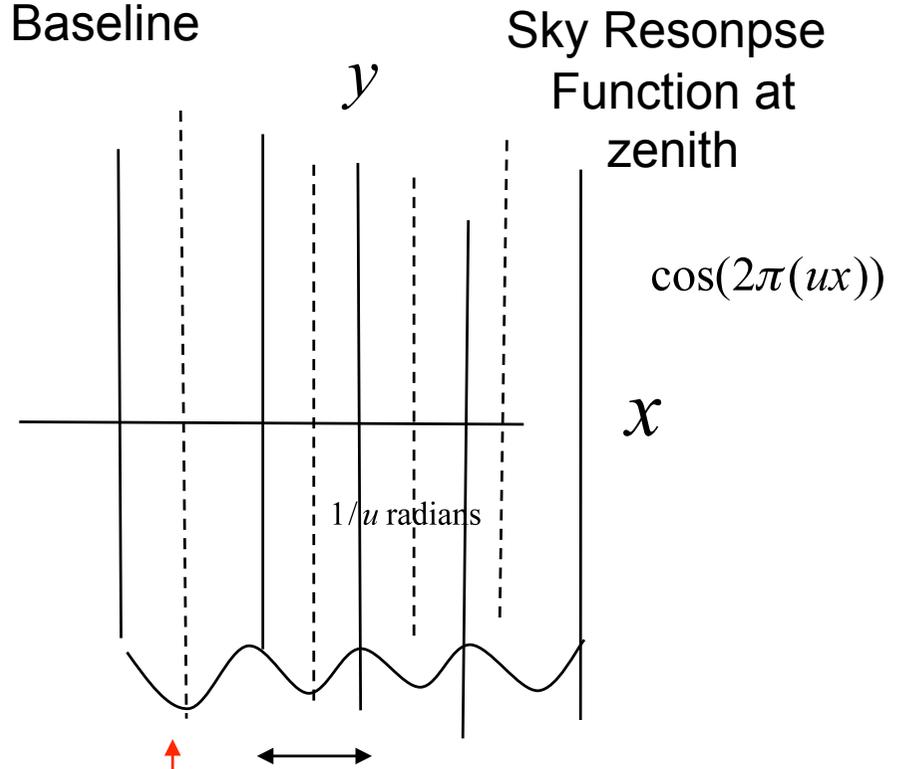
Interferometer response to a point source brightness  $I_0$  an angle  $\theta$  from zenith

Form complex visibility  $V = V_R - i V_I$ . Amplitude of that visibility depends only on point source strength and phase on position

2D Response function of Interferometer Baseline  
close to the zenith is



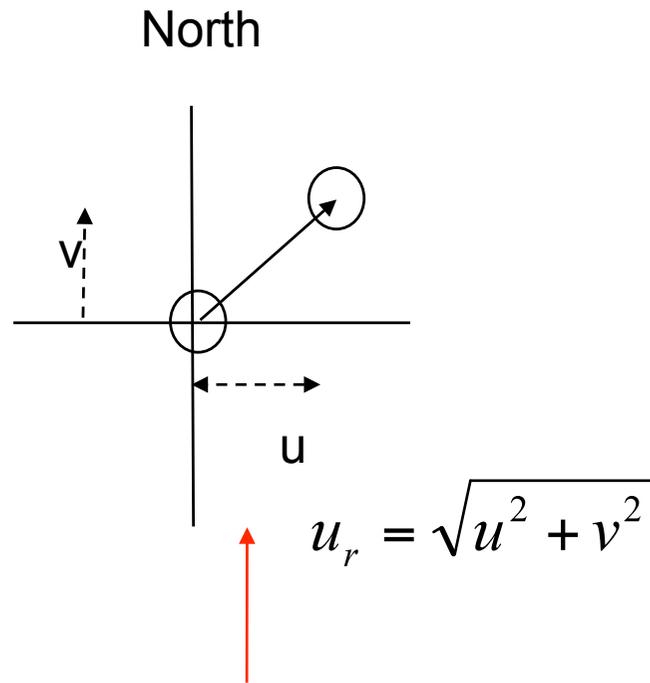
Telescope  
layout



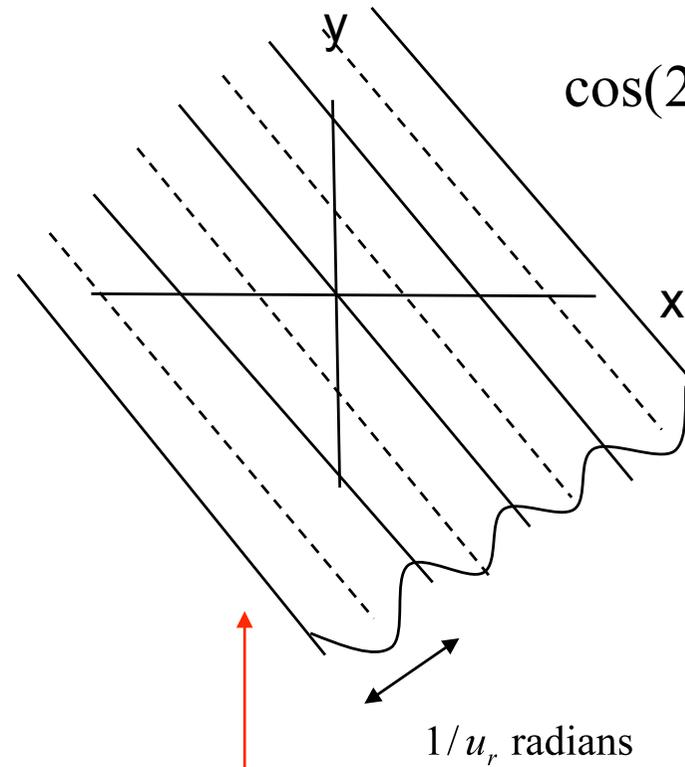
Real channel  
response function at  
zenith. 'Corrugated  
Roof' with spatial  
frequency  $u,v$

If the two telescope are not East-West

Real Resonse  
Function on  
sky  
 $\cos(2\pi(ux + vy))$



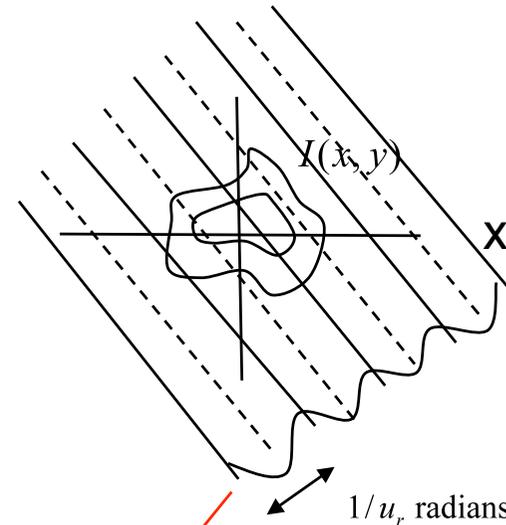
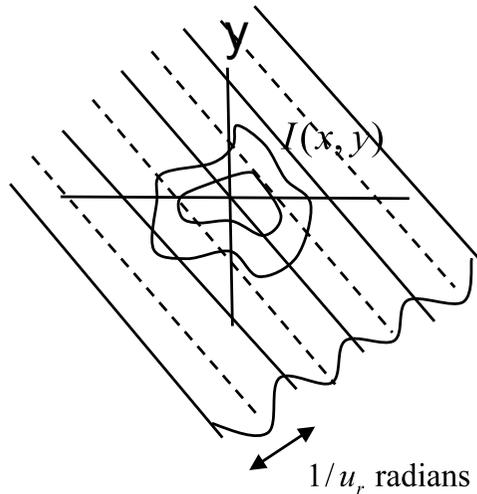
Telescope  
layout



Real channel  
response function at  
zenith. 'Corrugated  
Roof' with spatial  
frequency  $u, v$

$$V_R = \iint I(x, y) \cos(2\pi(ux + vy)) dx dy$$

$$V_I = \iint I(x, y) \sin(2\pi(ux + vy)) dx dy$$



Real  
part

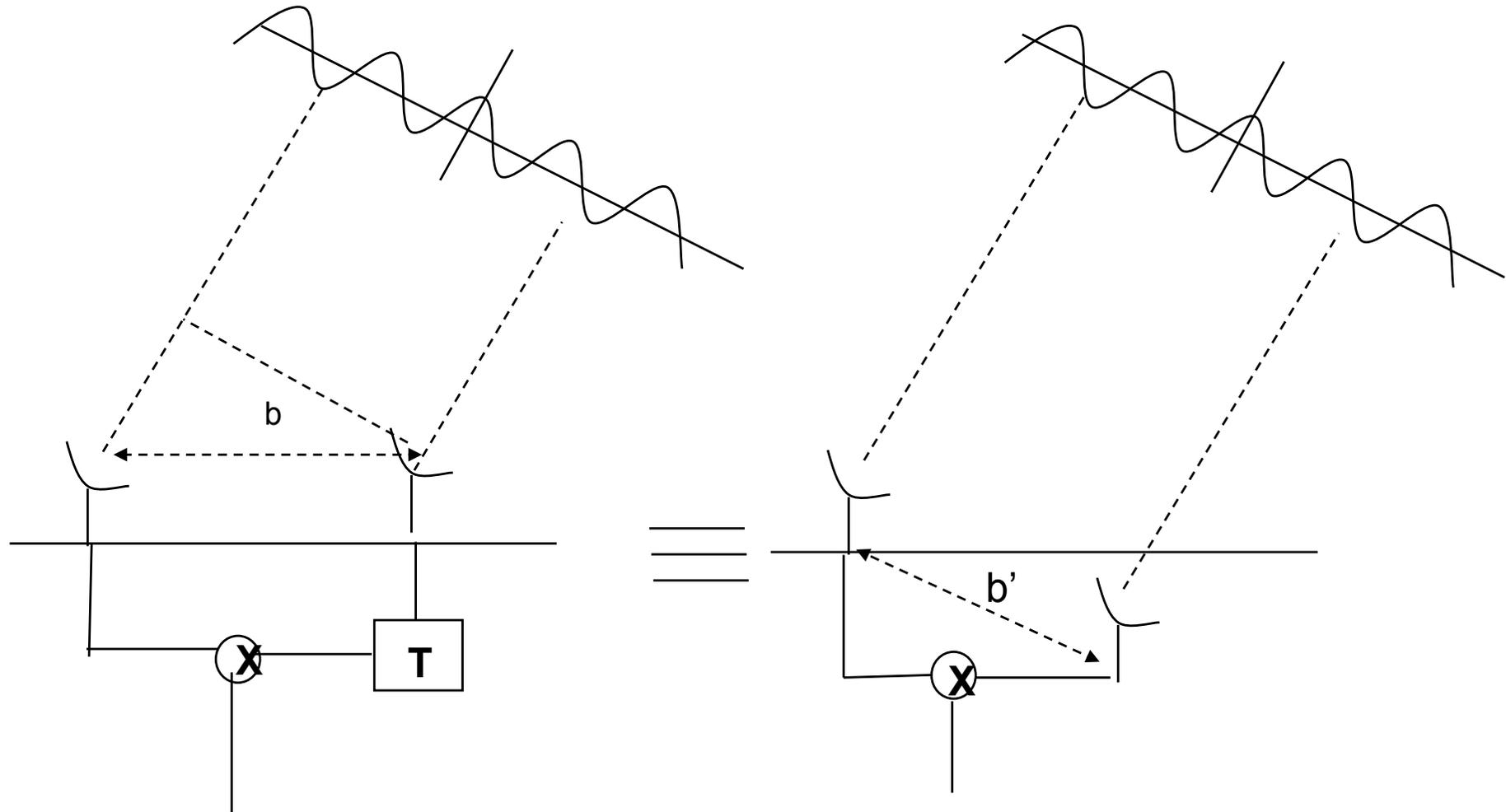
Imaginary  
part

$$V(u, v) = \iint I(x, y) \exp(-2\pi i(ux + vy)) dx dy$$

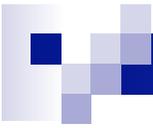
Van Cittert – Zernicke theorem

Interferometer outputs 2 numbers, signal in real channel ( $V_r$ ) and in imaginary ( $V_i$ ), together give complex visibility  $V(u, v)$  – this equals the 2D FT of  $I(x, y)$  evaluated at a  $u, v$  given by the projected baseline coordinates EW and NS measured in wavelengths. Have many baselines can collect lots of information about the 2D FT of the source.

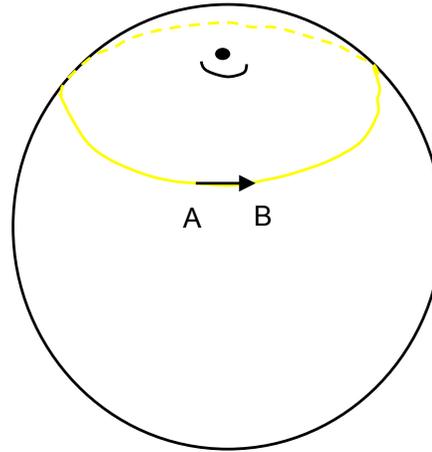
To observe not at zenith and track source across sky add delay T to compensate extra geometrical path, equivalent to moving an antenna so its in the plane perpendicular to ray



Electronic delay moves centre of fringe pattern so its always centred on the source, and hence 'tracks' its position. Fringe spacing on the sky depends on **PROJECTED BASELINE** measured in wavelengths  $u = b'/\lambda$  as seen from source. .Rotation of Earth helps us collect more data.

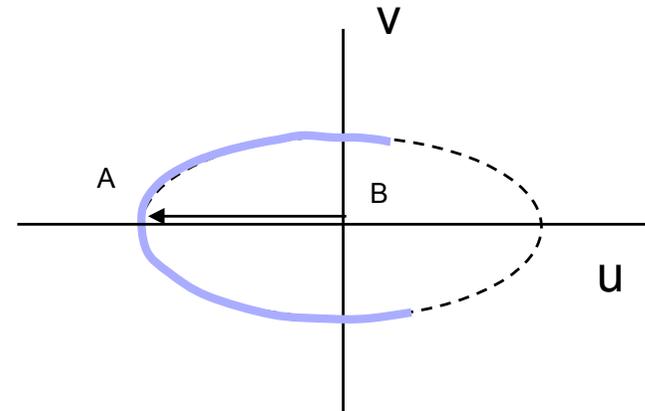
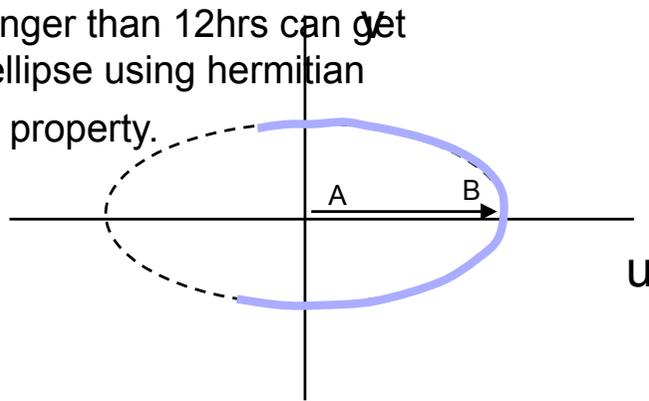


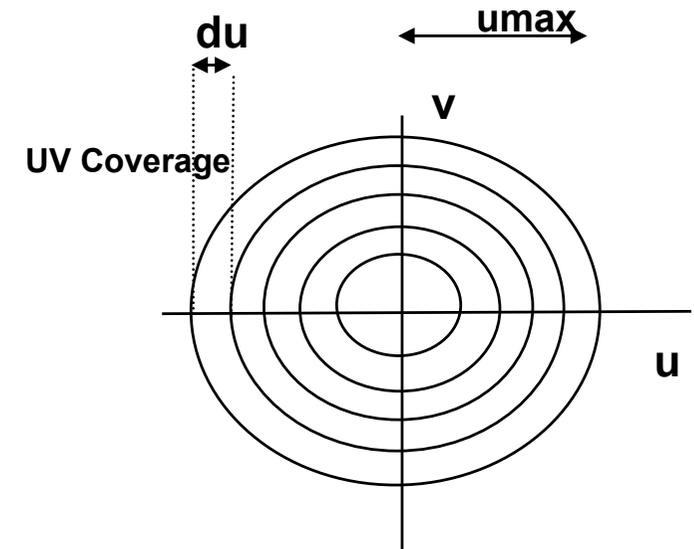
Earth Rotation  
East-West baselines  
Source at declination  
 $\Delta = +30$



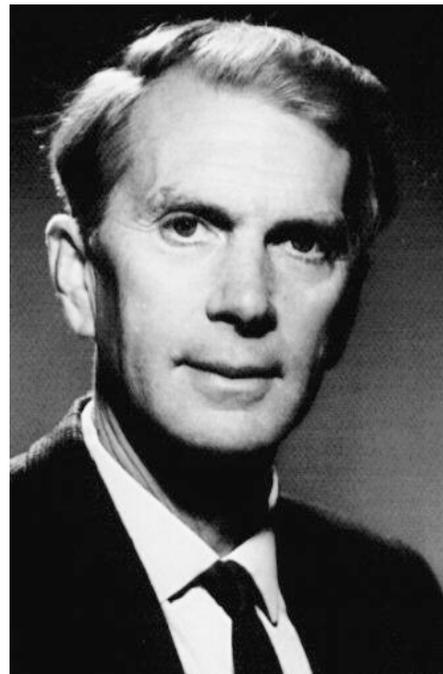
View of East  
-West baseline as  
Earth rotates

uv tracks are ellipses – squashed by  $\cos(\Delta)$ , cannot usually observe for 24hrs because most sources not above horizon that long, but if above horizon for longer than 12hrs can get complete ellipse using hermitian property.



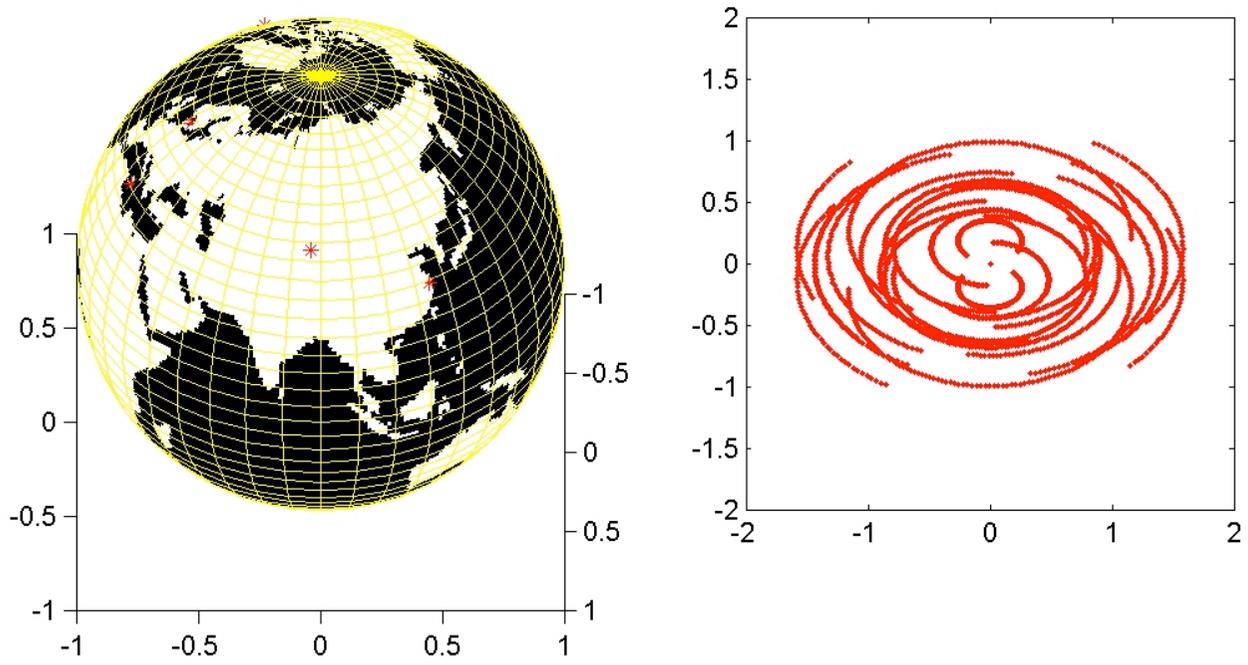


uv coverage and imaging simplest for E-W arrays, but also have some disadvantages



**1974 Nobel Prize in Physics to Martin Ryle (Cambridge) for development of Aperture Synthesis**

Example – Small VLBI array, gives an irregular uv coverage. Each baseline gives an elliptical track, but not centred on origin.





## Interferometry Difficulty 1 - The Inverse Problem

We know how visibility is related to source structure

$$V(u,v) = \bar{I}(u,v) = \int \int I(x,y) \exp(-2\pi i(ux + vy)) dx dy$$

If we had complete information on  $V(u,v)$  at all  $u,v$  then inverting the problem is easy

$$I(x,y) = \int \int V(u,v) \exp(2\pi i(ux + vy)) du dv$$

Unfortunately we don't know value of  $V(u,v)$  everywhere.



## 'Dirty Imaging'

$$V'(u,v) = S(u,v)V(u,v)$$

Where  $V(u,v)$  is the true visibility  $S(u,v)$  the sampling function and  $V'(u,v)$  the sampled visibility

The visibility is related to the image via

$$V(u,v) = FT(I(x,y))$$

If we take the IFT of  $V'(u,v)$  then using the convolution theorem we get

$$IFT(V') = IFT(S) * FT(V)$$

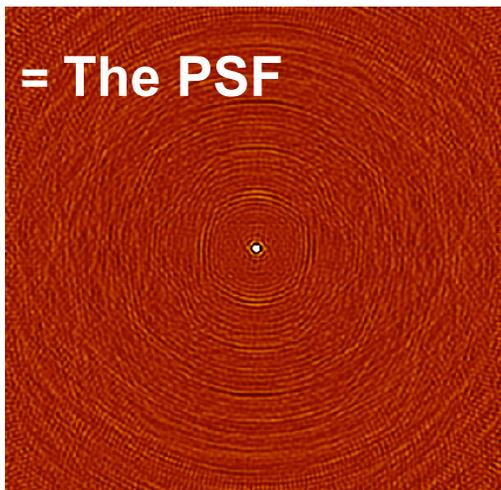
where  $*$  indicates convolution

$$IFT(V') = B(x,y) * I(x,y)$$

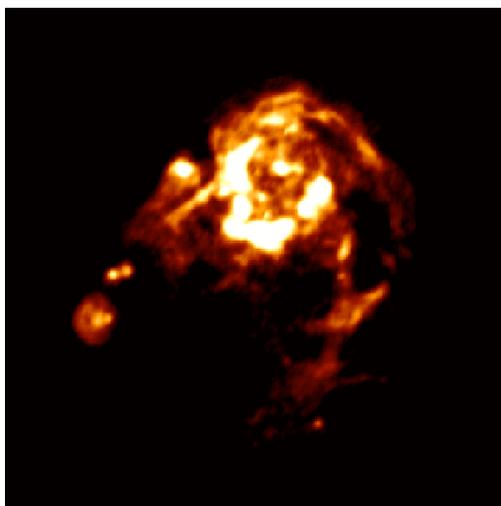
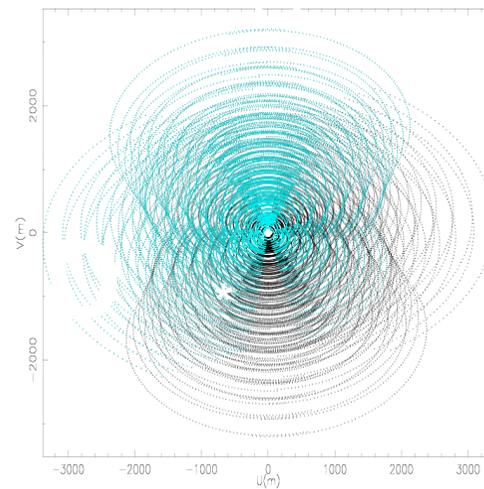
where  $B(x,y) = IFT(S(u,v))$  is the 'dirty beam' and  $*$  is convolution (copying of the beam, at every point on the source).

# The Dirty Image

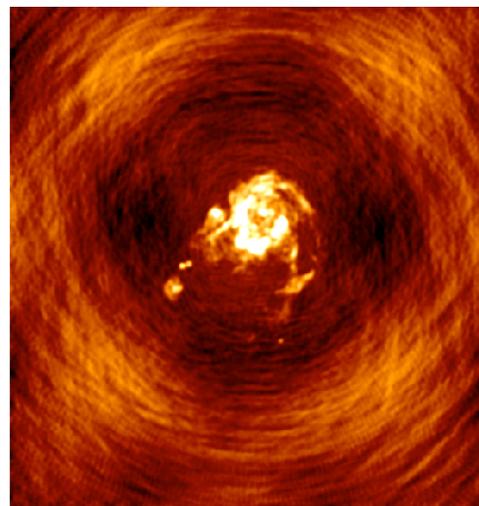
Dirty beam



FT

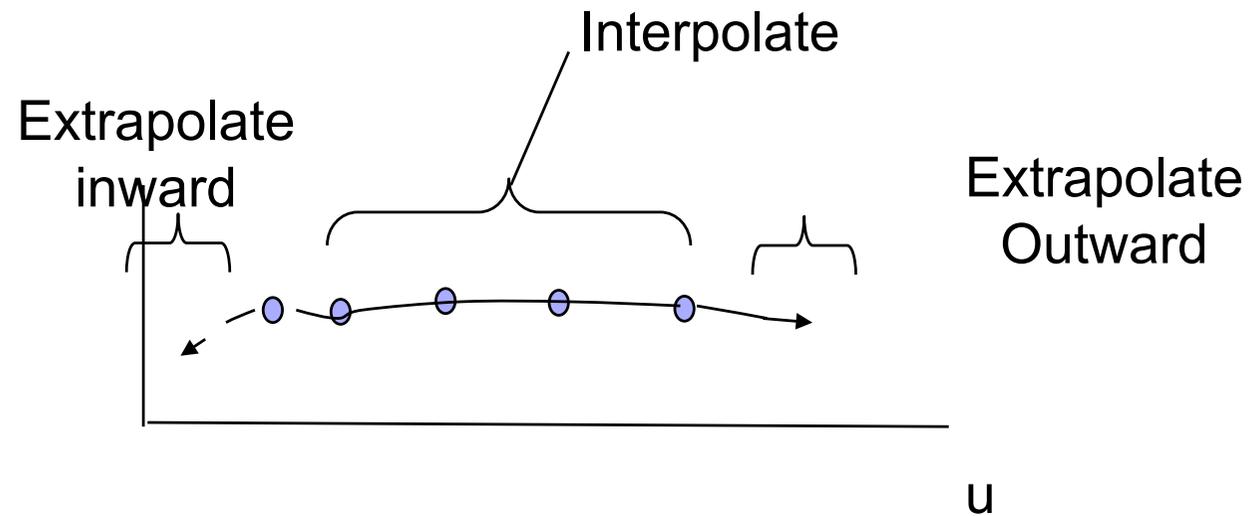


True image



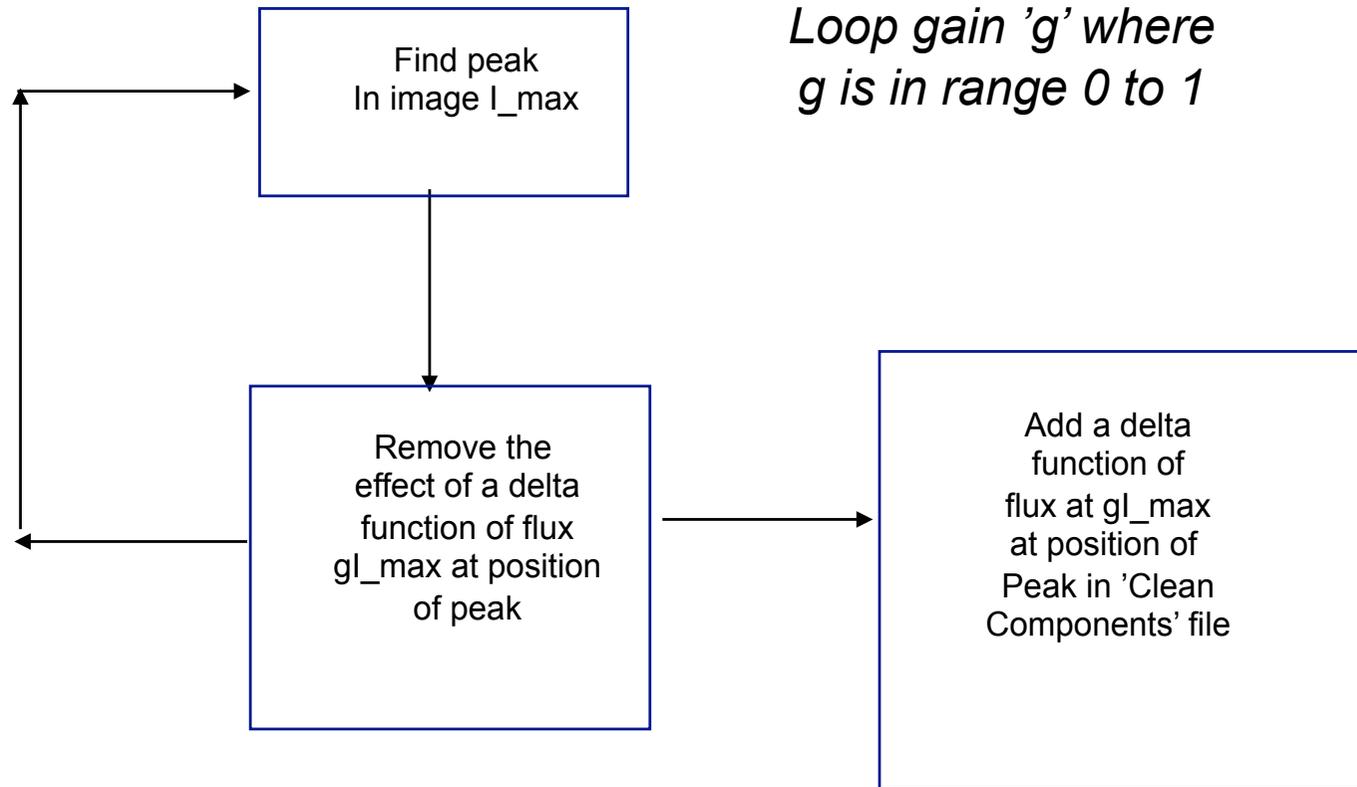
Dirty Image = True Image  
convolved with dirty beam

## Need for Deconvolution



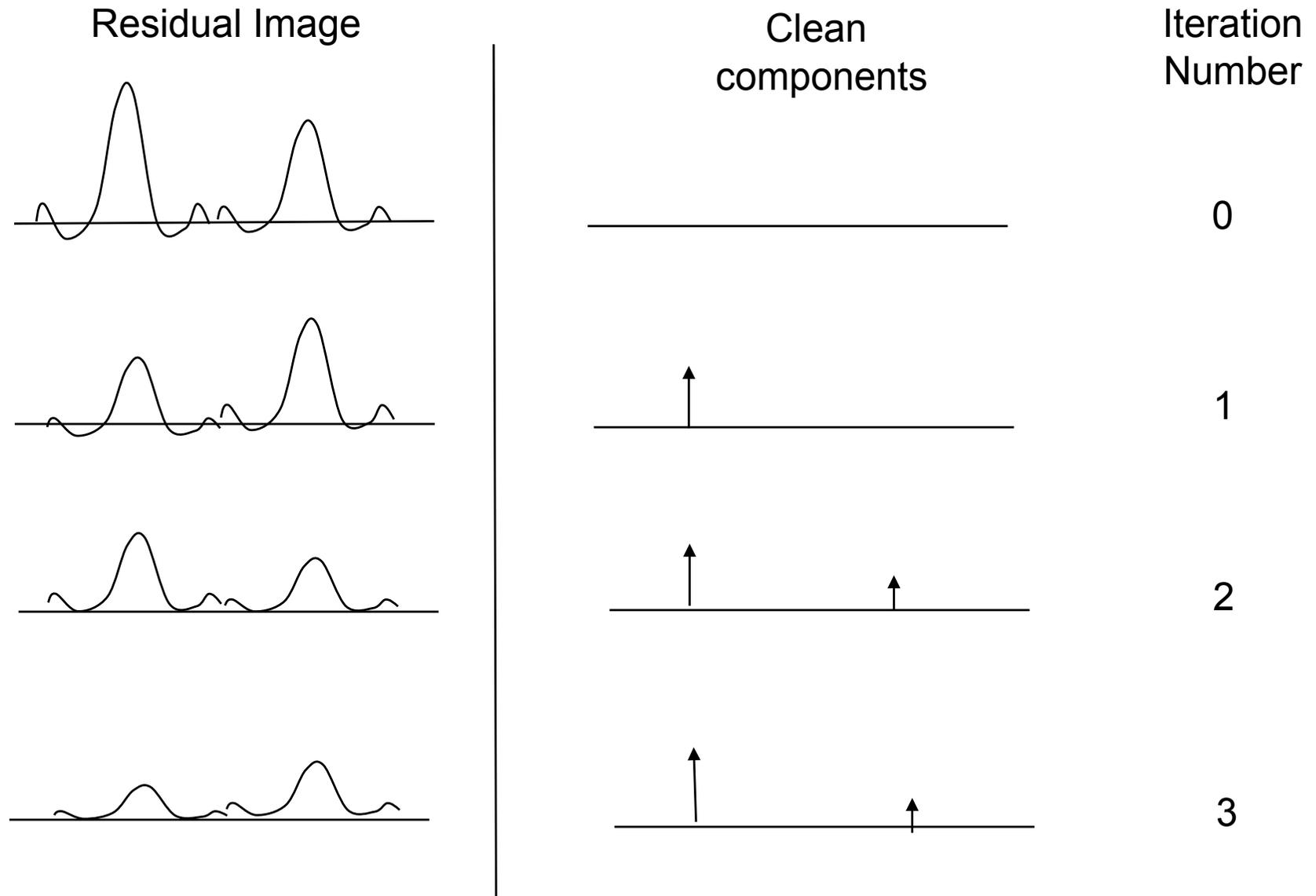
To improve image further must both 'interpolate' between measured visibilities and 'extrapolate' beyond outer edge and into inner hole. 'Interpolation' removes effects of far sidelobes and the 'Extrapolation' that of Inner sidelobes and central bowl.

CLEAN Algorithm (Nordic invention Högbom 1974 – one of top 10 cited AA papers)



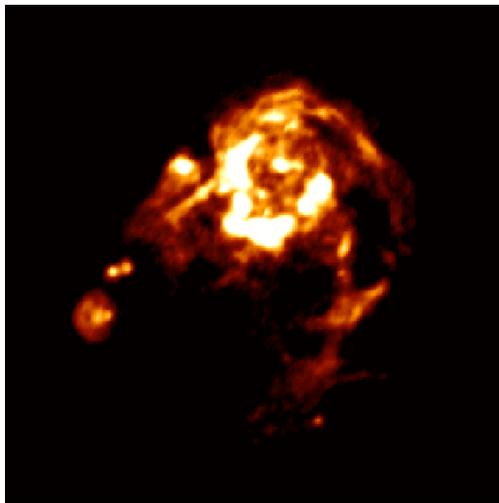
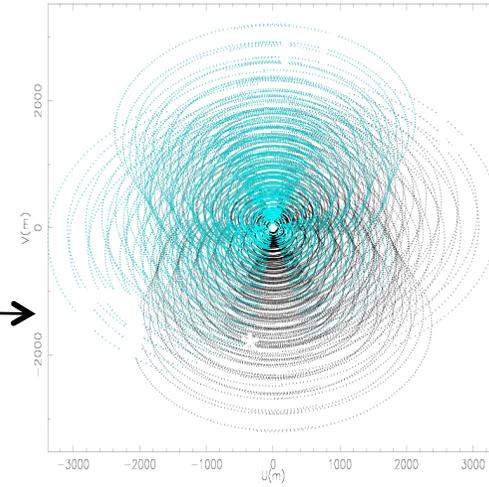
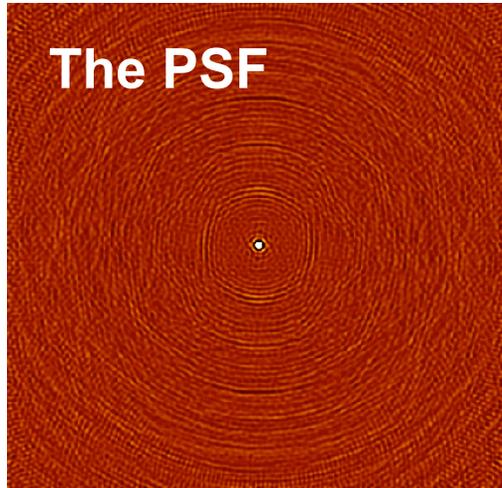
Iterate until nothing but noise is left in 'residual'  
Image- usually convolve 'clean components'  
by a gaussian 'restoring' beam

Example – in ID, Gain = 0.5

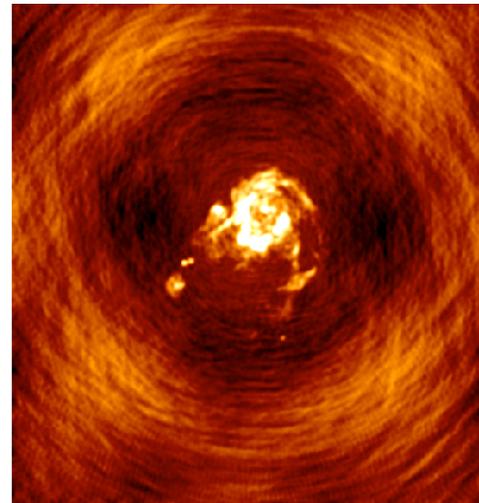


# The Dirty Image

Dirty beam



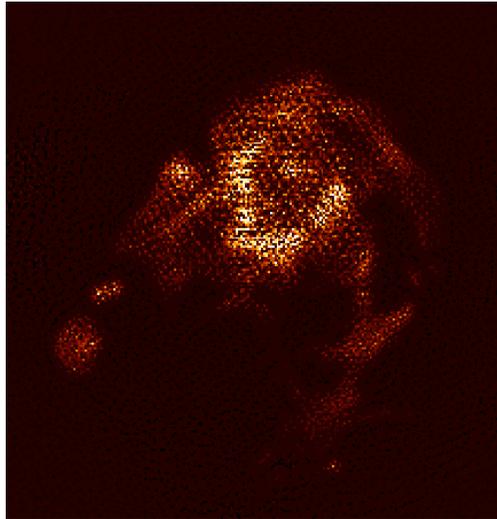
True image



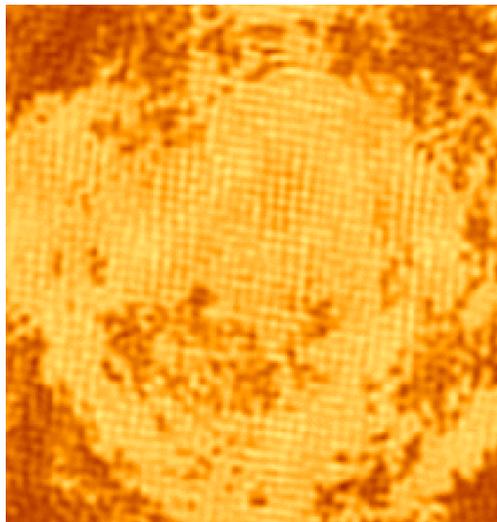
Dirty Image

# Clean: Example

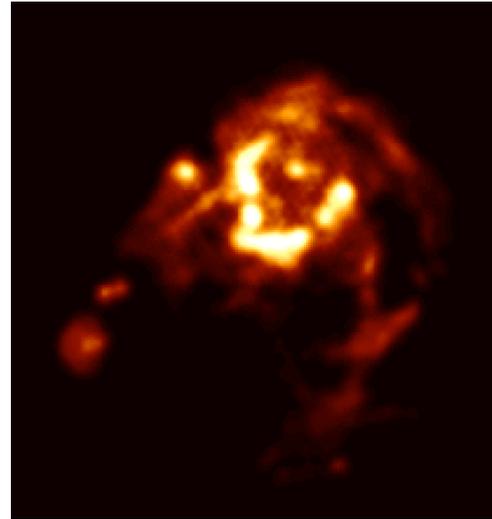
Clean components found



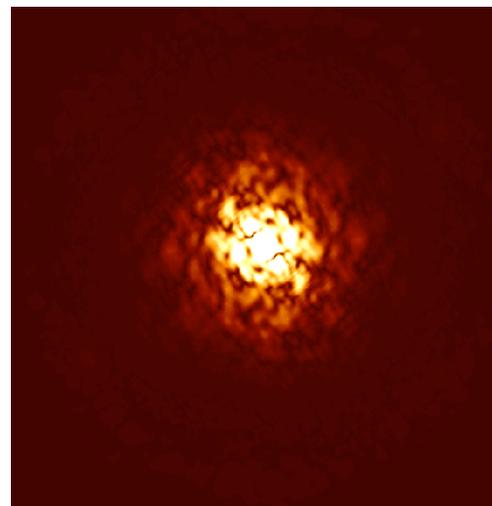
FT of Clean components, model visibility



Clean components convolved with Gaussian 'restoring beam' ( $X=FWHM$ ), our estimated source structure at resolution  $X$

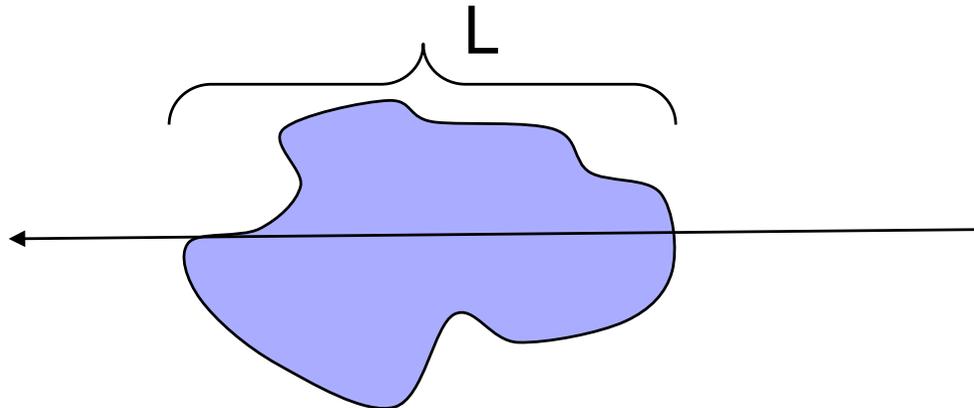


FT of restored image



## Interferometry Difficulty 2 – Phase errors

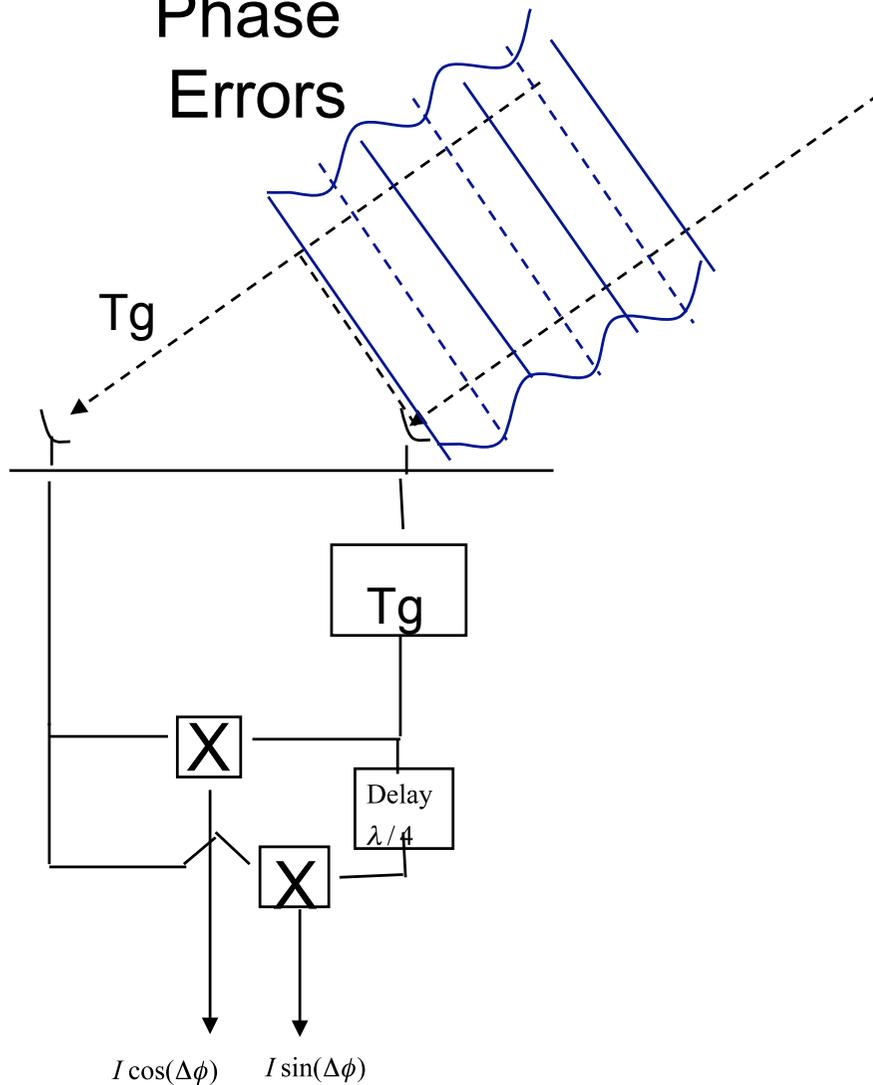
Clouds of water vapour in atmosphere or ionised gas in ionosphere cause the 'refractive index' ( $n$ ) for radio waves to be different from 1, hence radio waves travel slightly slower than the speed of light by a factor  $1/n$ .



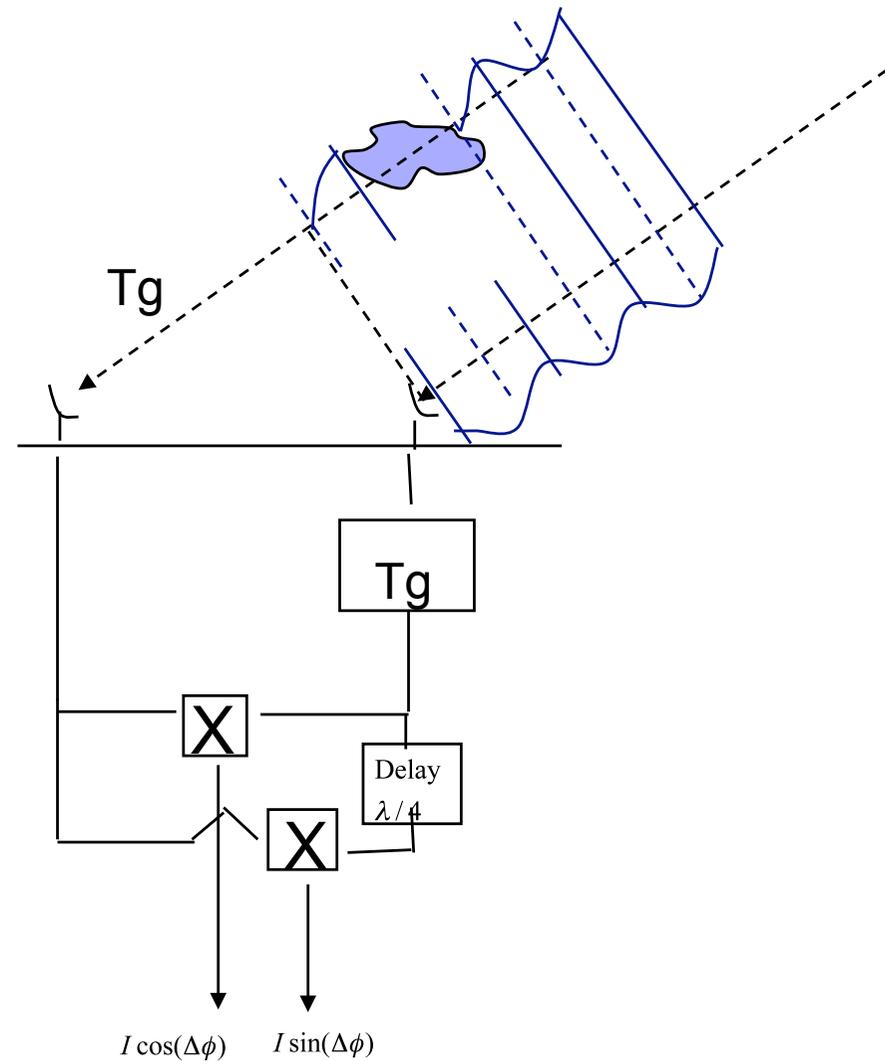
A ray passing a length  $L$  through a cloud causes a change in phase compared to free space propagation of

$$\Delta\phi = 2\pi L(n - 1) / \lambda$$

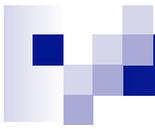
# Phase Errors



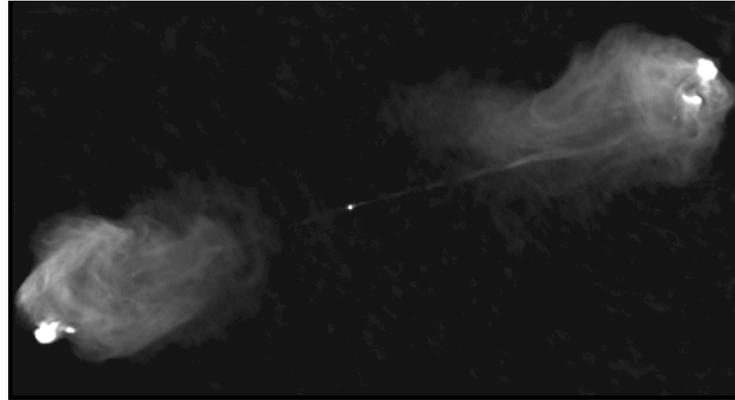
Visibility output depends on relative Phase of E-field on wavefront, in this case zero.



If there is cloud above one antenna which delays signal then relative phase of E-field on wavefront and hence visibility phase effected

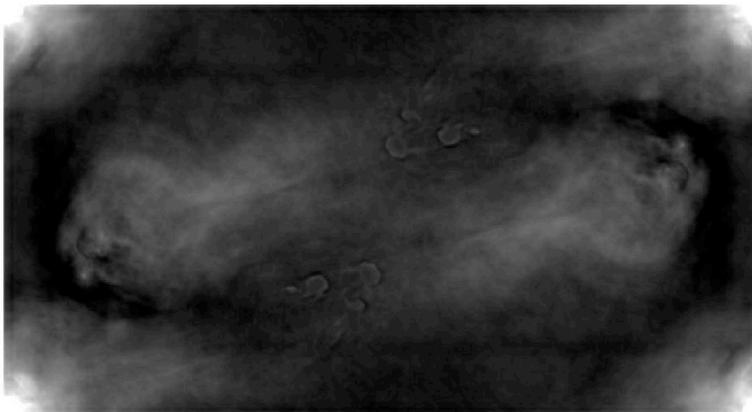


True Image



Fourier Phase  
Is more important than  
Amplitude for  
finding source  
structure

Reconstruction from FT  
amplitude



Reconstruction from FT  
phase



# Closure Phase

Consider the measured phase of the visibility  $\theta'$  and the true phase of the visibility  $\theta$  on each baseline

$$\theta'_{12} = \theta_{12} + \Delta\phi_1 - \Delta\phi_2$$

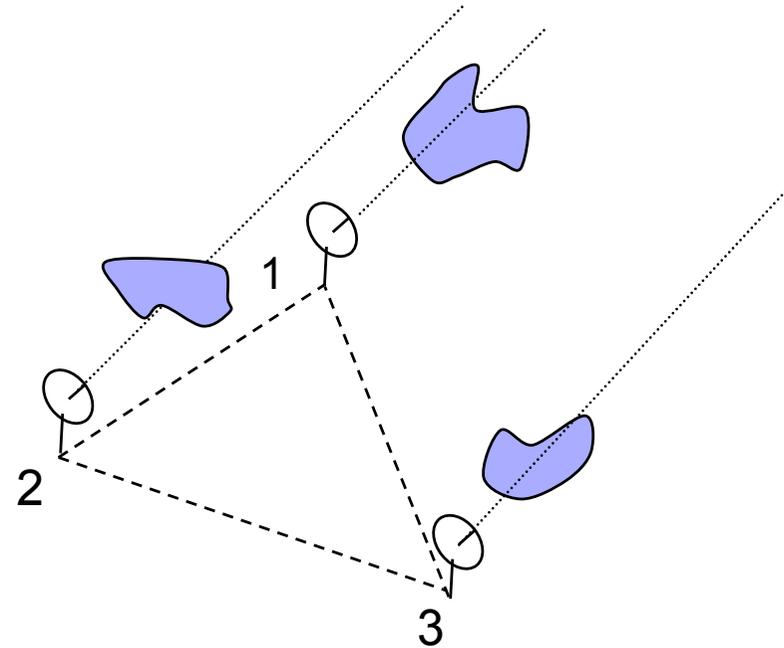
$$\theta'_{23} = \theta_{23} + \Delta\phi_2 - \Delta\phi_3$$

$$\theta'_{31} = \theta_{31} + \Delta\phi_3 - \Delta\phi_1$$

We can form a 'closure phase' by adding the phase on a closed loop of baselines

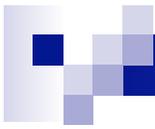
$$C_{123} = \theta'_{12} + \theta'_{23} + \theta'_{31} = \theta_{12} + \theta_{23} + \theta_{31}$$

This 'Closure phase' depends only on the source structure and is independent of the atmosphere!



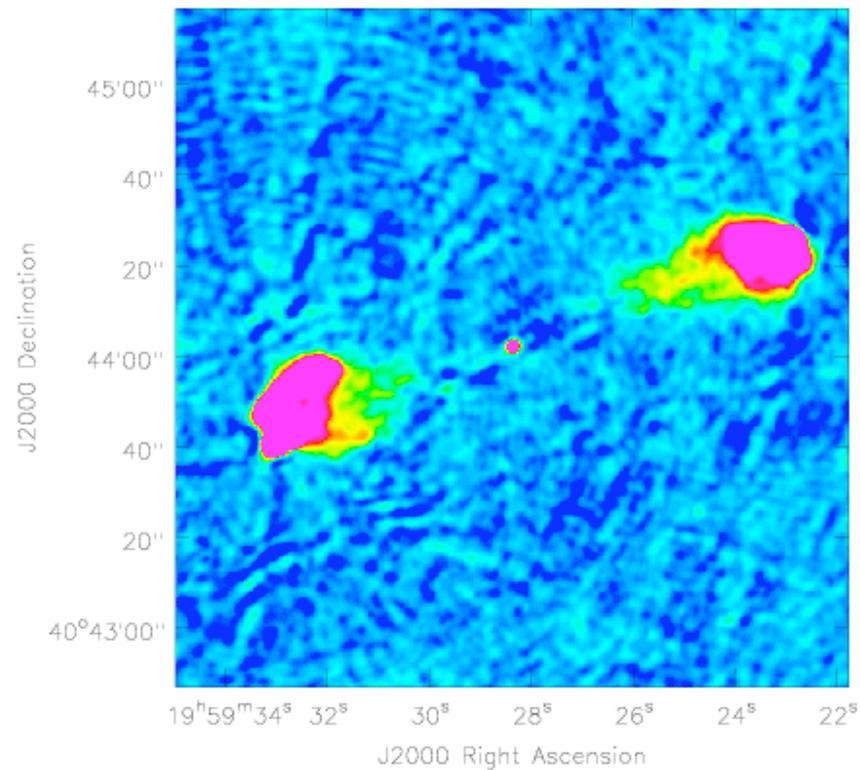
(Re)discovery of closure phase idea made imaging on baselines >5km possible.

Two approaches – closure phase, combine measurements to make quantity independent of atmosphere, or solve explicitly for antenna-based phase errors and correct – must be done at same time we deconvolve, needs iterative methods

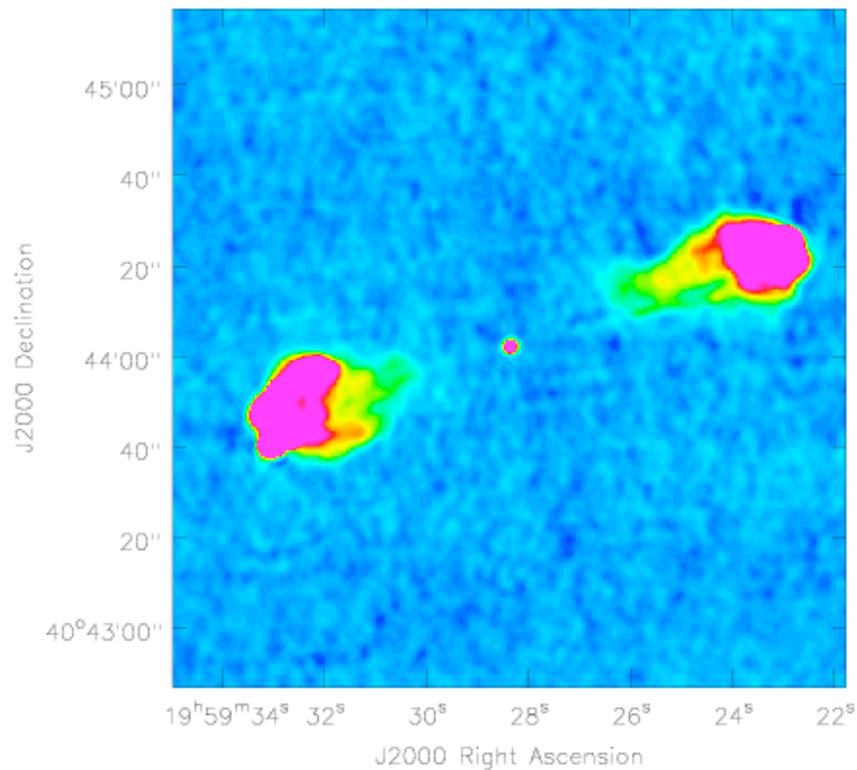


# ■ Image without self-calibration

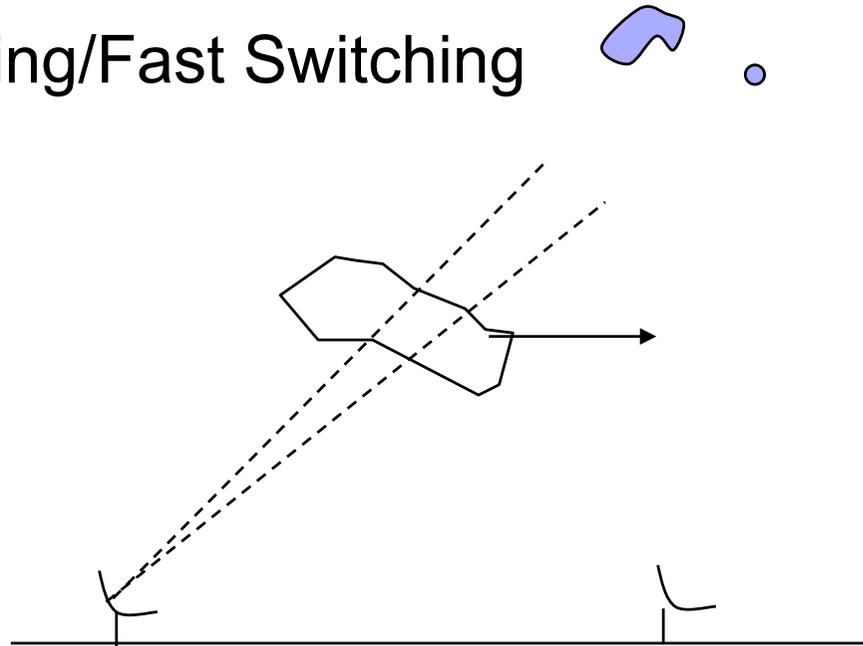
- Phase calibration using nearby source observed every 20 minutes
- Peak  $\sim 22\text{Jy}$
- Display shows  $-0.05\text{Jy}$  to  $0.5\text{Jy}$



# After 4 amplitude and phase calibrations



# Phase Referencing/Fast Switching



To work need a bright target, if our target weak wwitch back an forth every few minutes between the target source and nearby (few degrees away) bright, compact calibrator. Use self-cal methods to estimate atmospheric phase/amplitude toward calibrator source (which is bright and often simple) – interpolate in time and apply these corrections to data for a target source (which is maybe weak and complicated).

# A differentially rotating disc in a high-mass protostellar system

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<sup>1</sup> Dept of Physics, University of Gothenburg, S-412 96, Göteborg, Sweden  
e-mail: michele.pestalozzi@physics.gu.se

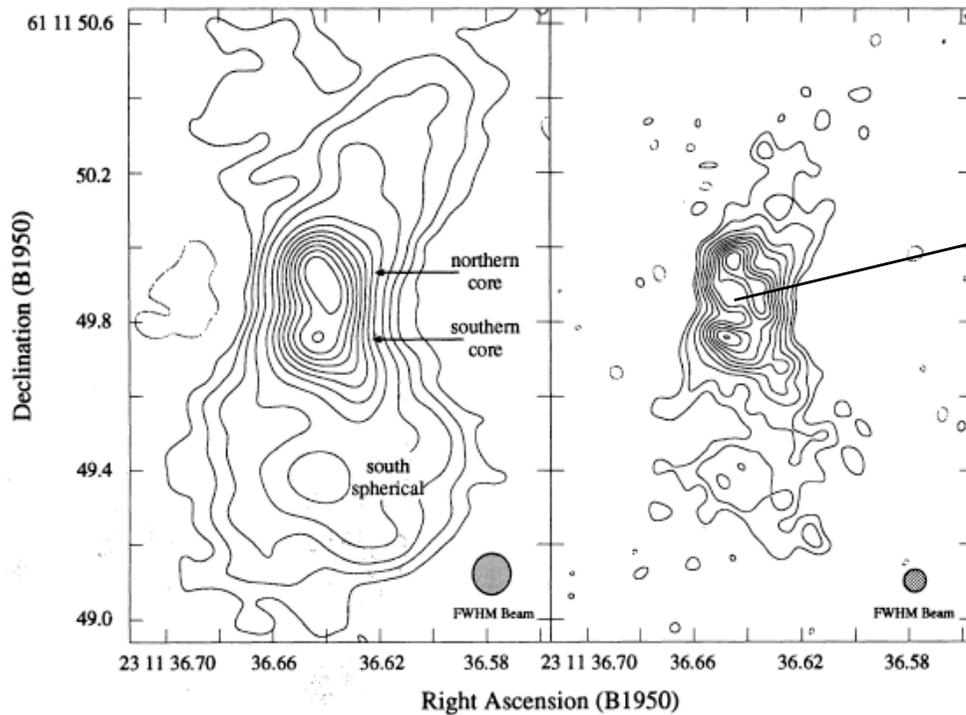
<sup>2</sup> Dept of Physics and Astronomy, Univ. of Kentucky, Lexington, KY 40506-0055, USA  
e-mail: moshe@pa.uky.edu

<sup>3</sup> Onsala Space Observatory, S-439 92 Onsala, Sweden  
e-mail: john.conway@chalmers.se

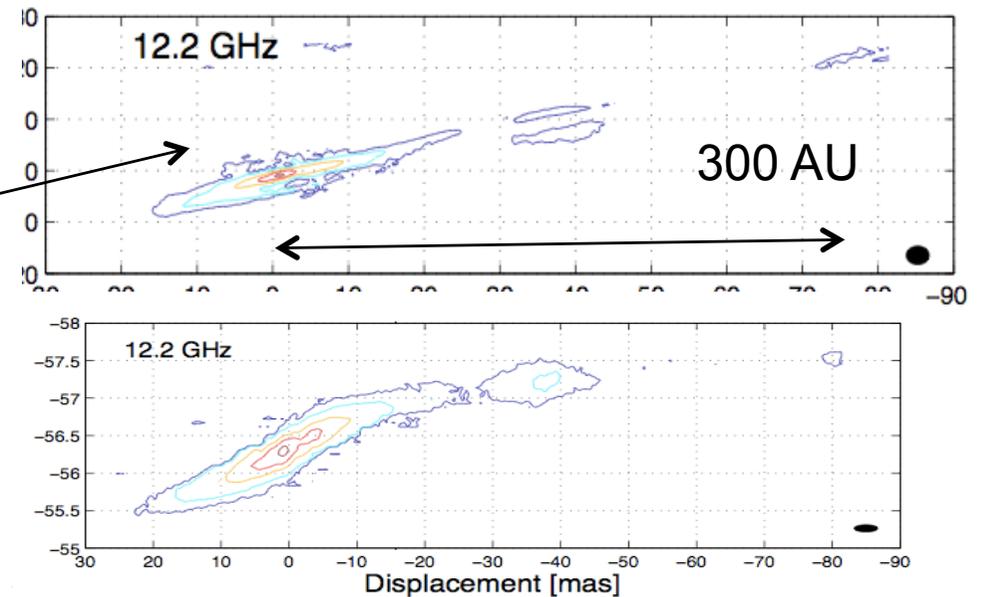
Preprint online version: March 25, 2009

Pestalozzi, Elitzur, Conway, 2009 arXiv:0904.3722

Model linear methanol feature  
as Keplerian rotating disk  
around a 30Msol protostar-  
Massive stars form in same  
was as low mass stars by  
accretion disks.



NGC7538-IRS1N VLA obs



VLBI methanol maser  
observations "1mas resolution