Numerical methods and F90, fall 2015

Exercise 5

Return the solutions by Oct 28.

1. Evaluate the integral
$$\int_0^{\pi/2} \sin^2 x \, dx$$
using the trapezoid rule. Try different step lengths. How many steps are needed to obtain a relative error that is less than 10^{-5}?

2. Evaluate the previous integral using the Gaussian quadrature and five points. The file `gauss.dat` contains weights and coordinates needed in the method.

3. Modify the program for Simpson's method so that it will evaluate the double integral
$$\int_0^1 \int_0^1 e^{xy} \, dx \, dy.$$
(It is easiest to make two copies of the integrator; one will integrate the original function over x using a fixed value of y, and the other one will call this integrator with suitable values of y.) The result is about 1.3179, but there is no elementary way to find it analytically.

4. Make a program to find the volume of an n dimensional sphere by Monte Carlo integration. Investigate how the accuracy improves when more points are used. Test the program with $n = 3$ and 5. The volume of an n dimensional sphere is
$$V_n = \frac{\pi^{n/2}}{(n/2)!} R^n,$$
where R is the radius of the sphere. If n is odd, the factorial has the value
$$\left(m + \frac{1}{2}\right)! = \frac{(2m+2)!}{(m+1)!\, 4^{m+1}} \sqrt{\pi}.$$