# Numerical differentiation

Sensitive to errors!

E.g. a Newton–Gregory interpolation polynomial:

$$f(x_s) \approx P_n(x_s) = f_0 + s\Delta f_0 + \binom{s}{2}\Delta^2 f_0 + \ldots + \binom{s}{n}\Delta^n f_0.$$

This gives the derivative

$$\begin{aligned} f'(x_s) \approx P_n'(x_s) &= \frac{d}{ds}P_n(x_s)\frac{ds}{dx} = \frac{d}{ds}P_n(x_s)\frac{1}{h} \\ &= \frac{1}{h}\left(\Delta f_0 + \frac{2s-1}{2}\Delta^2 f_0 + \ldots\right) \end{aligned}$$

When $s = 0$, we have

$$f'(x_0) = \frac{1}{h}\left(\Delta f_0 - \frac{1}{2}\Delta^2 f_0 + +\frac{1}{3}\Delta^3 f_0 - \ldots \pm \frac{1}{n}\Delta^n f_0\right)$$

The error of this is of the order

$$\frac{1}{n+1} h^n f^{(n+1)}(\xi).$$

More symmetric versions:

$$f'(x_0) = \frac{1}{2h}(f_1 - f_{-1}).$$

$$f'(x_0) = \frac{1}{12h}(f_{-2} - 8f_{-1} + 8f_1 - f_2).$$

In the case of noisy measurement data it has to be first smoothed by fitting e.g. a least squares function.