Secondary fluctuations Clusters and SZ

Enn Saar

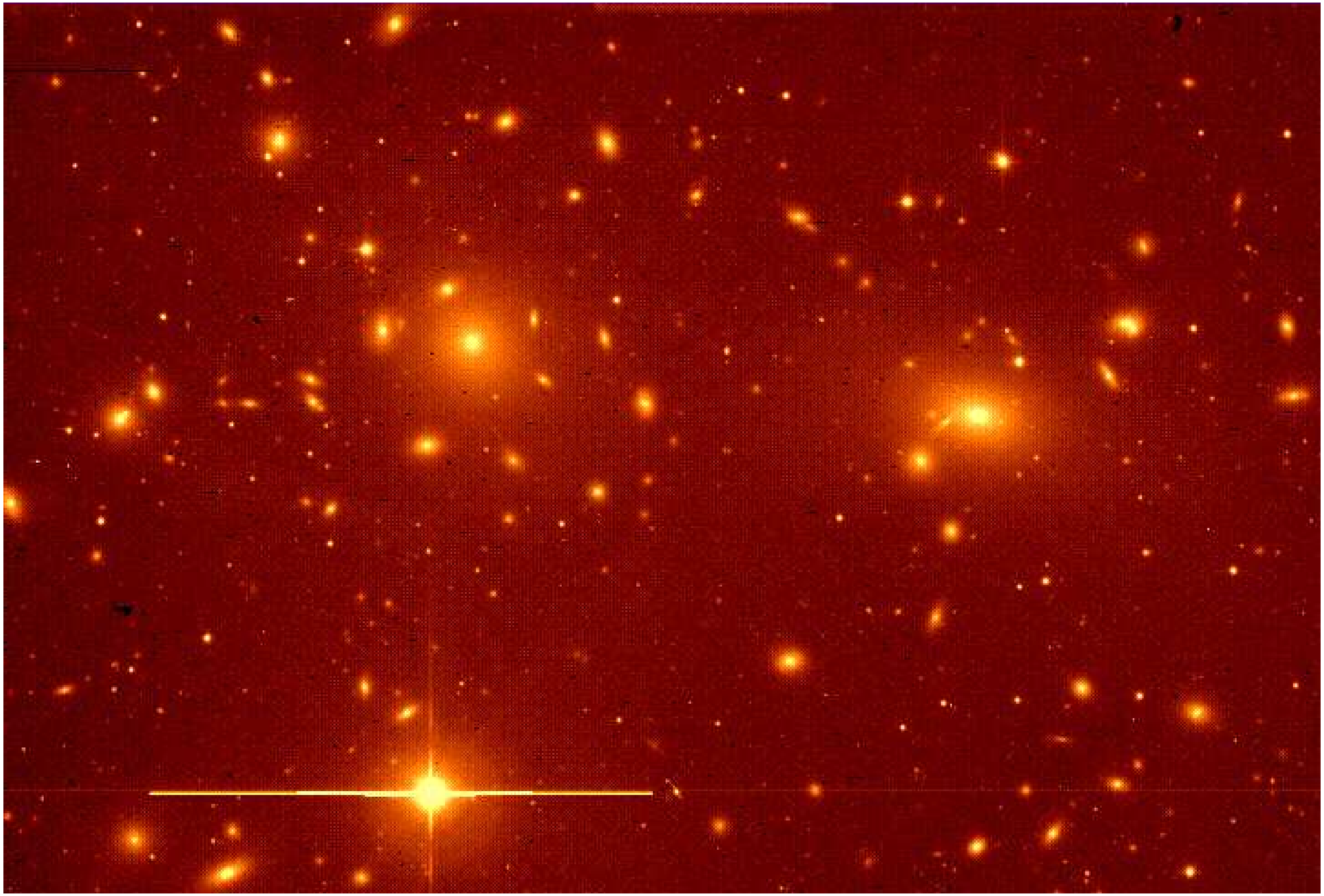
Tartu Observatoorium

Clusters

Property	Value
Mass	$10^{15} M_\odot$
Size	1.5 Mpc
Rms velocity	1000 km/sec
Crossing time	1 Gyr
Temperature	$5 \cdot 10^4$K
$k_B T$	4 keV

Clusters: example

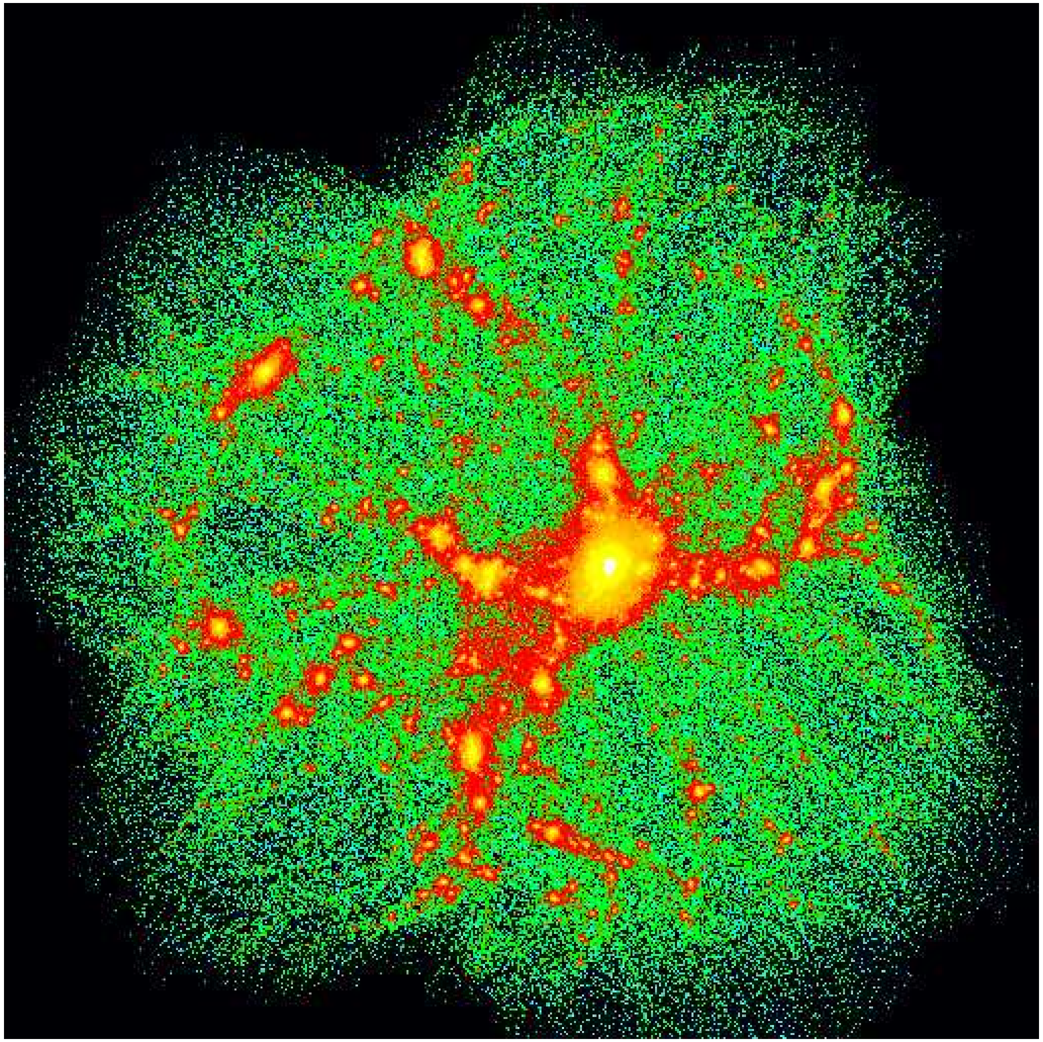
Coma cluster



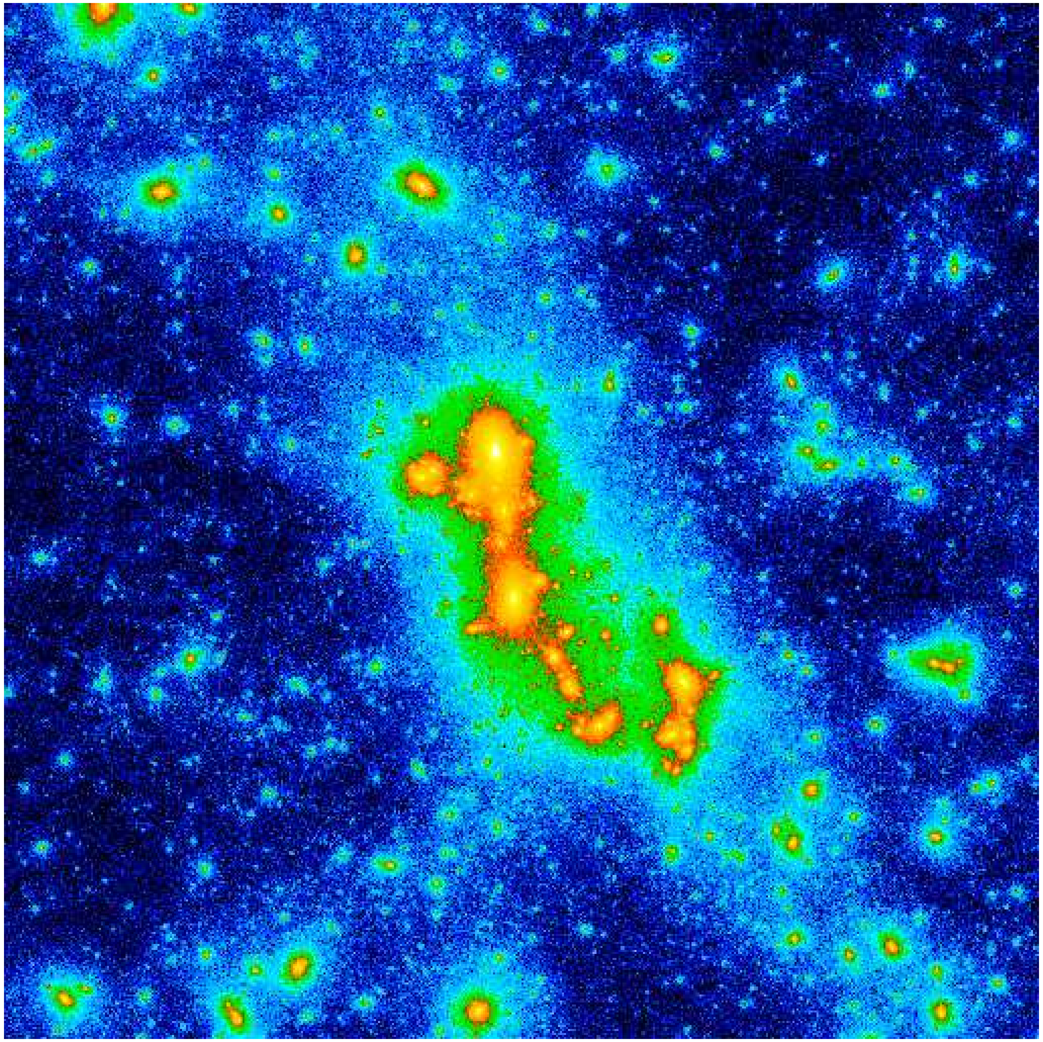
Coma cluster again

Coma cluster again, by Chandra

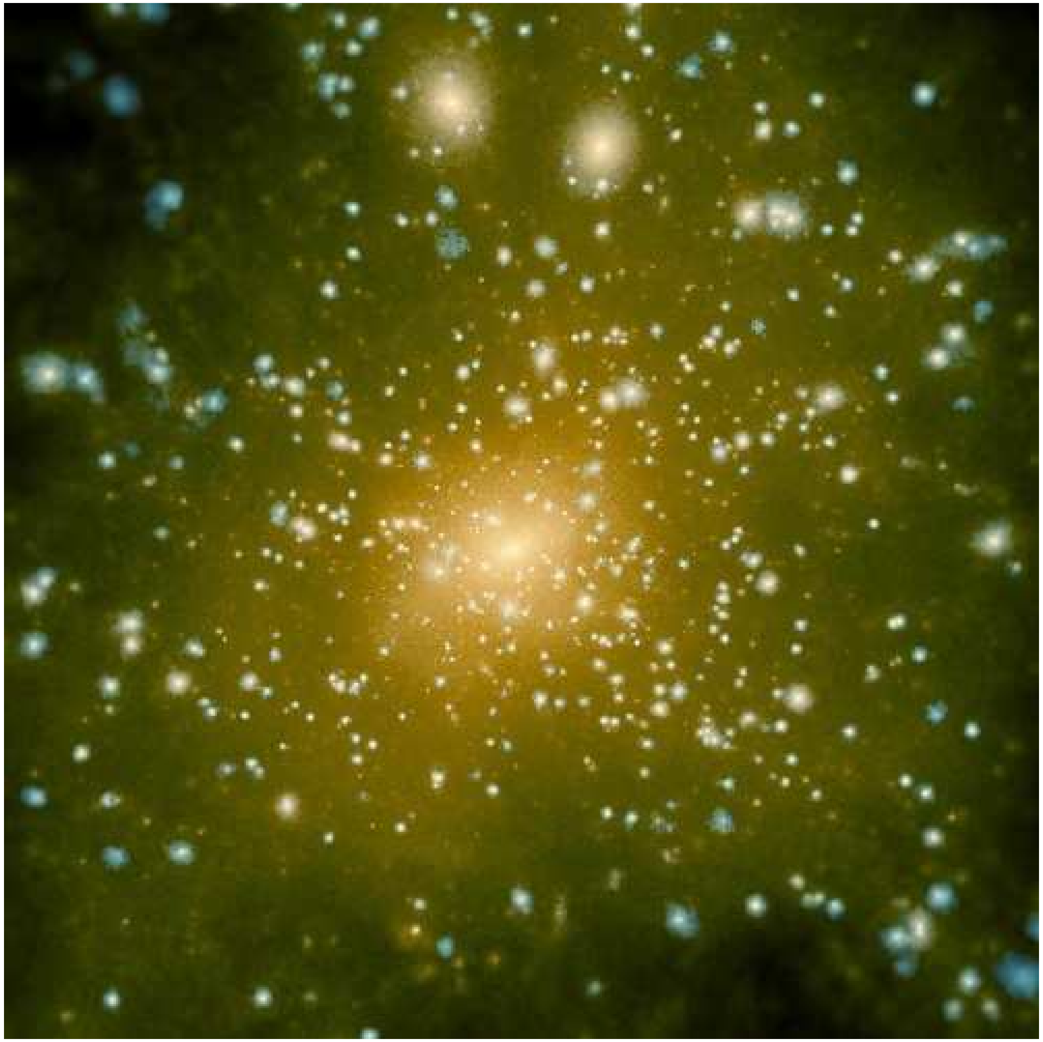
Clusters: simulations



A dark matter cluster

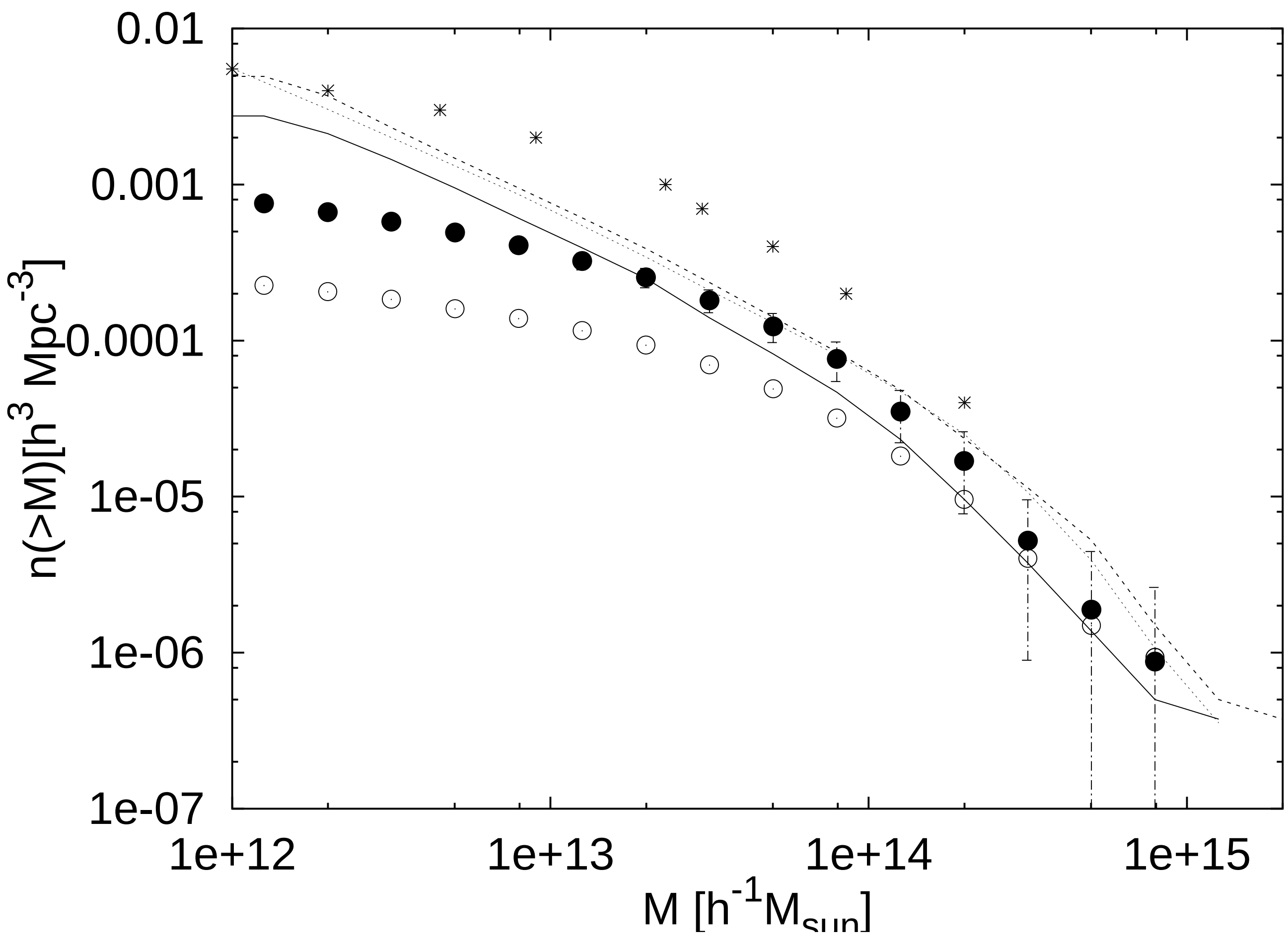


A better dark matter cluster

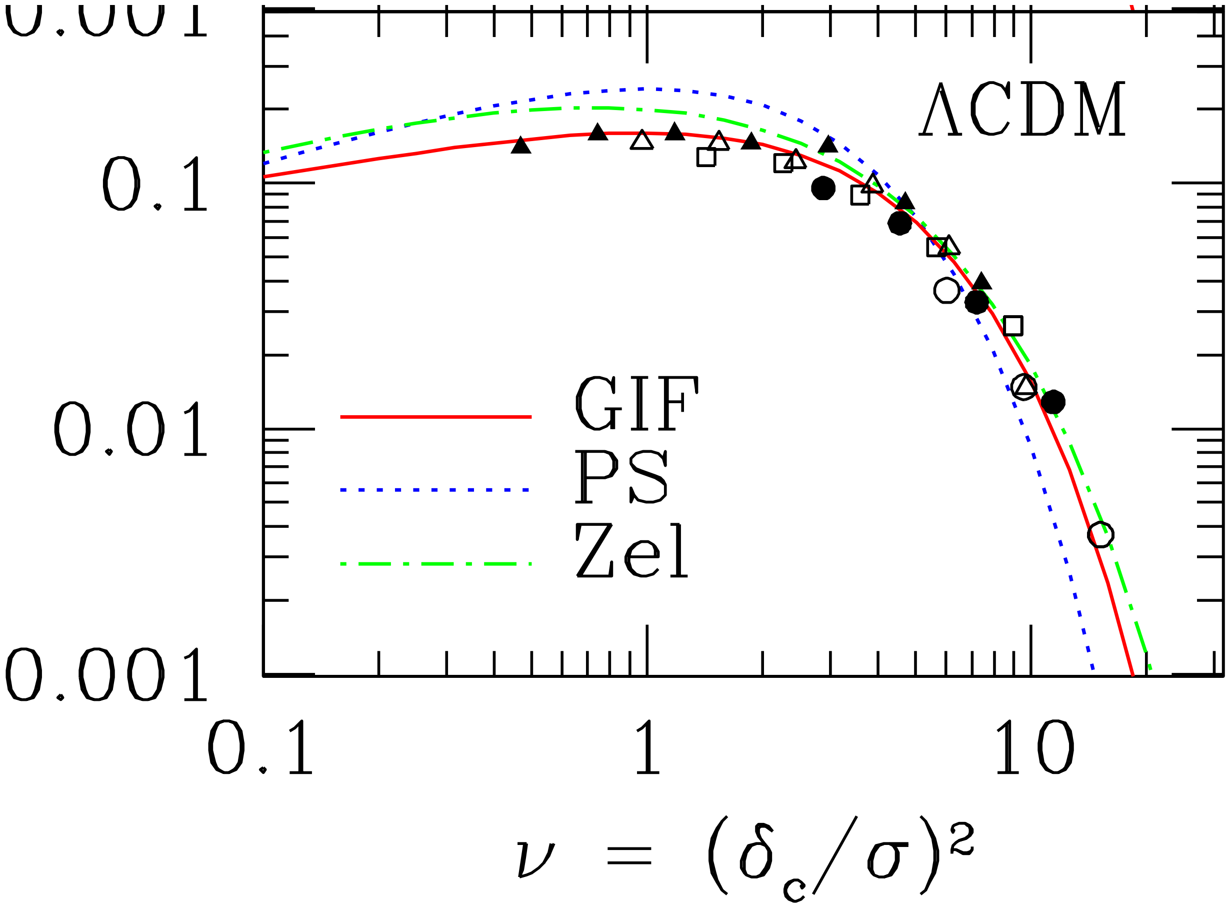


Zooming in

Clusters: Mass function



Observational mass functions: LCRS, LMED.



Theoretical mass functions: PS, Sheth-Tormen.

Clusters: Press-Schechter

- Take a filter $W(r; M)$ with a typical $R(M)$; $M = \bar{\rho} V_W(R)$.
- Take $\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}} > \delta_c$; e.g., $\delta_c = 1.686$ (normalized to the present).
- Assume Gaussianity;

$$P_G(\delta \leq \delta_c | R) = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\delta_c}{\sqrt{2}\sigma(R)}\right)\right]$$

- Introduce the fudge factor, write

$$F(M) = 1 - \text{erf}(\nu/\sqrt{2}), \qquad \nu = \delta_c/\sigma(M),$$

- Define the differential mass function $n(M)$:

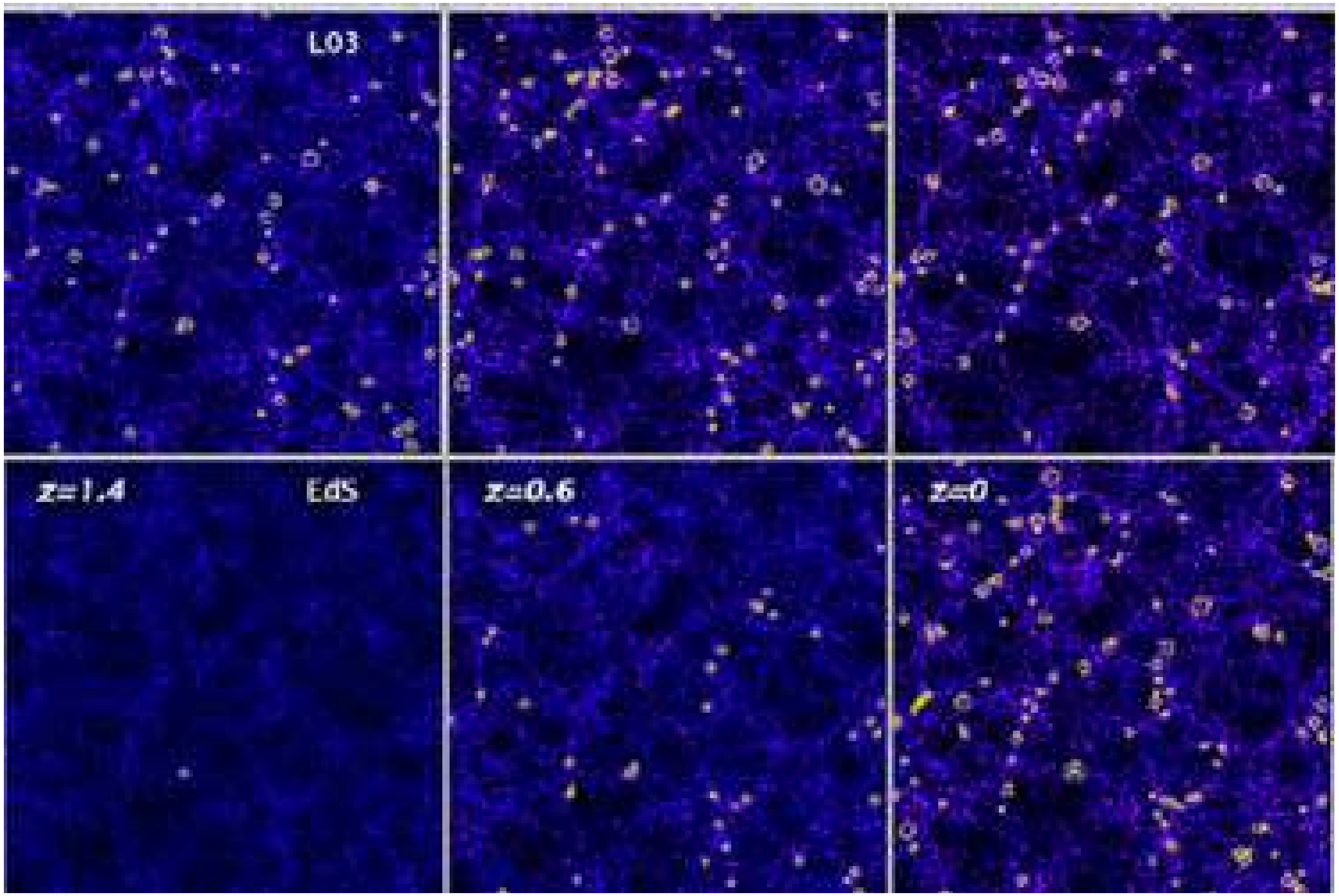
$$Mn(M) = \bar{\rho}\left|\frac{dF}{dM}\right|.$$

$$n(M) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\nu\left|\frac{d\ln\sigma(M)}{d\ln M}\right|e^{-\nu^2/2}.$$

- Model dependence: $\bar{\rho} = 3H_0^2\Omega_M/(8\pi G)$,
- Power normalization:

$$\sigma^2(M) = 4\pi\int_0^\infty |\widetilde{W}(k;M)|^2 P(k)\frac{k^2\,dk}{(2\pi)^3}.$$

Clusters: evolution



Evolution of the cluster number density; upper panel – a ΛCDM-cosmology, lower panel – the SCDM cosmology.

- Evolution:

$$\delta_c(t) = \frac{D_1(0)}{D_1(t)}\delta_c;$$

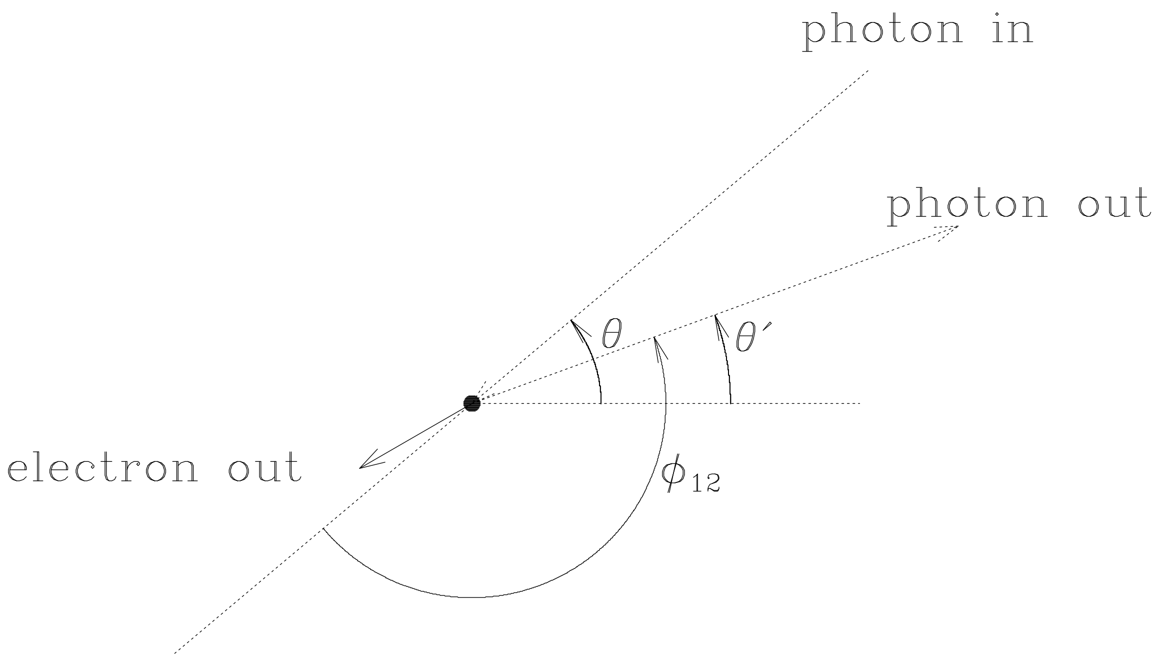
$D_1(t)$ – growing mode:

- $\Omega_M = 1, \qquad D_1(t) \sim t^{2/3};$
- $\Omega_M = 0, \qquad D_1(t) =$ const;

- Cluster properties check:
  - cosmological parameters
  - perturbation spectrum
  - Gaussianity
- References:
  - Press W.H. & Schechter P., Ap. J. 187, 425, 1974.
  - Sheth R.K. & Tormen G., MNRAS 308, 119, 1999 (astro-ph/9901122).

Sunyaev-Zeldovich effect

Scattering of CMB photons off high energy electrons (inverse Compton scattering).

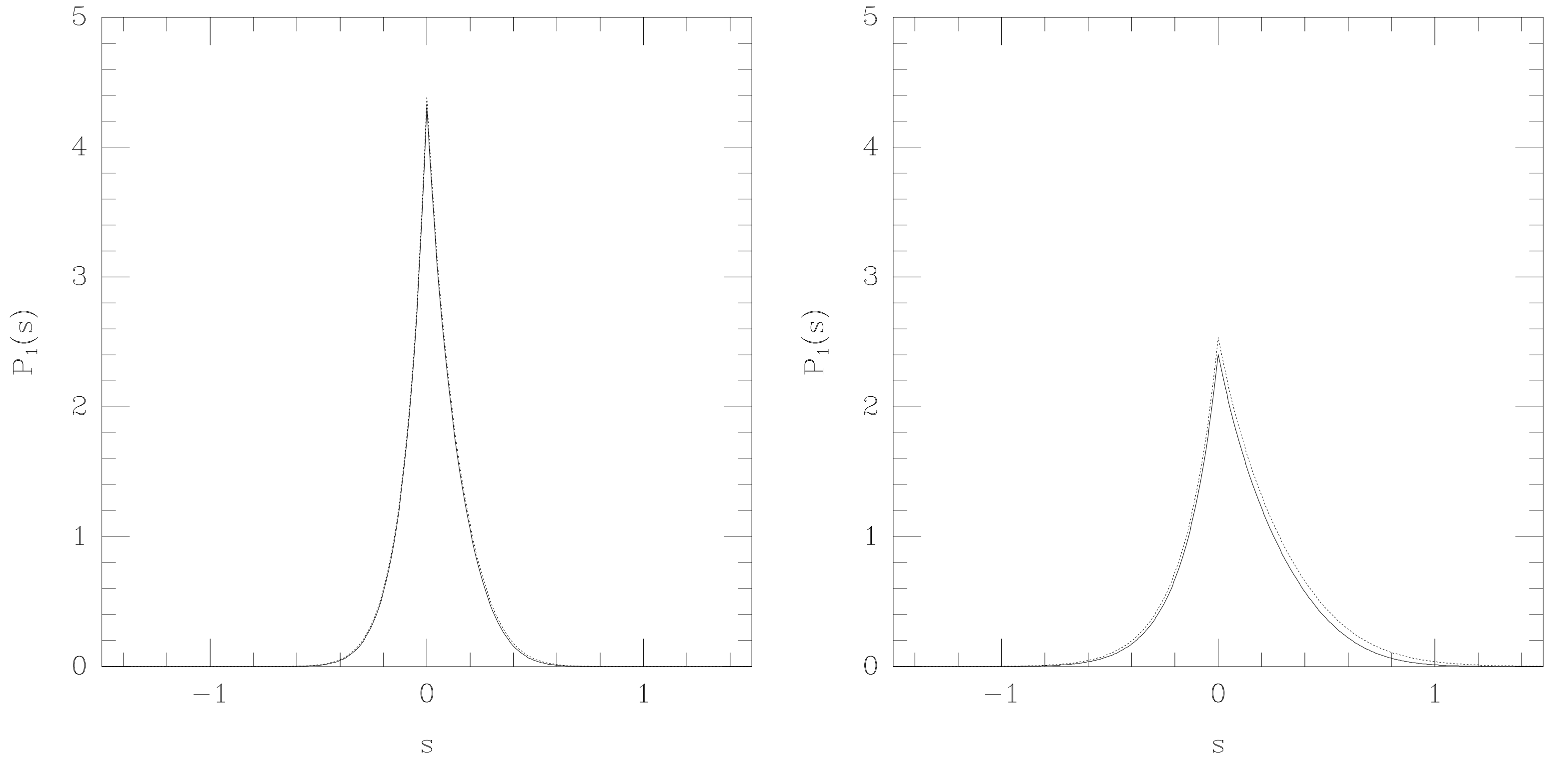


Scattering geometry.

$$\epsilon' = \frac{\epsilon}{1 + (\epsilon/m_e c^2)(1 - \cos\phi_{12})}$$

Derive:

1. Probability of scattering from $\theta \Rightarrow p(\theta)d\theta$;
2. Probability of scattering to $\theta' \Rightarrow \phi(\theta';\theta)d\theta'$;
3. Probability that a single scattering causes a frequency shift $s = \log(\nu'/\nu) \Rightarrow P(s,\beta)$ ($v = \beta c$; integrate over distributions 1 and 2)
4. Average over electron distribution $\beta \Rightarrow P_1(s)$ (single scattering).



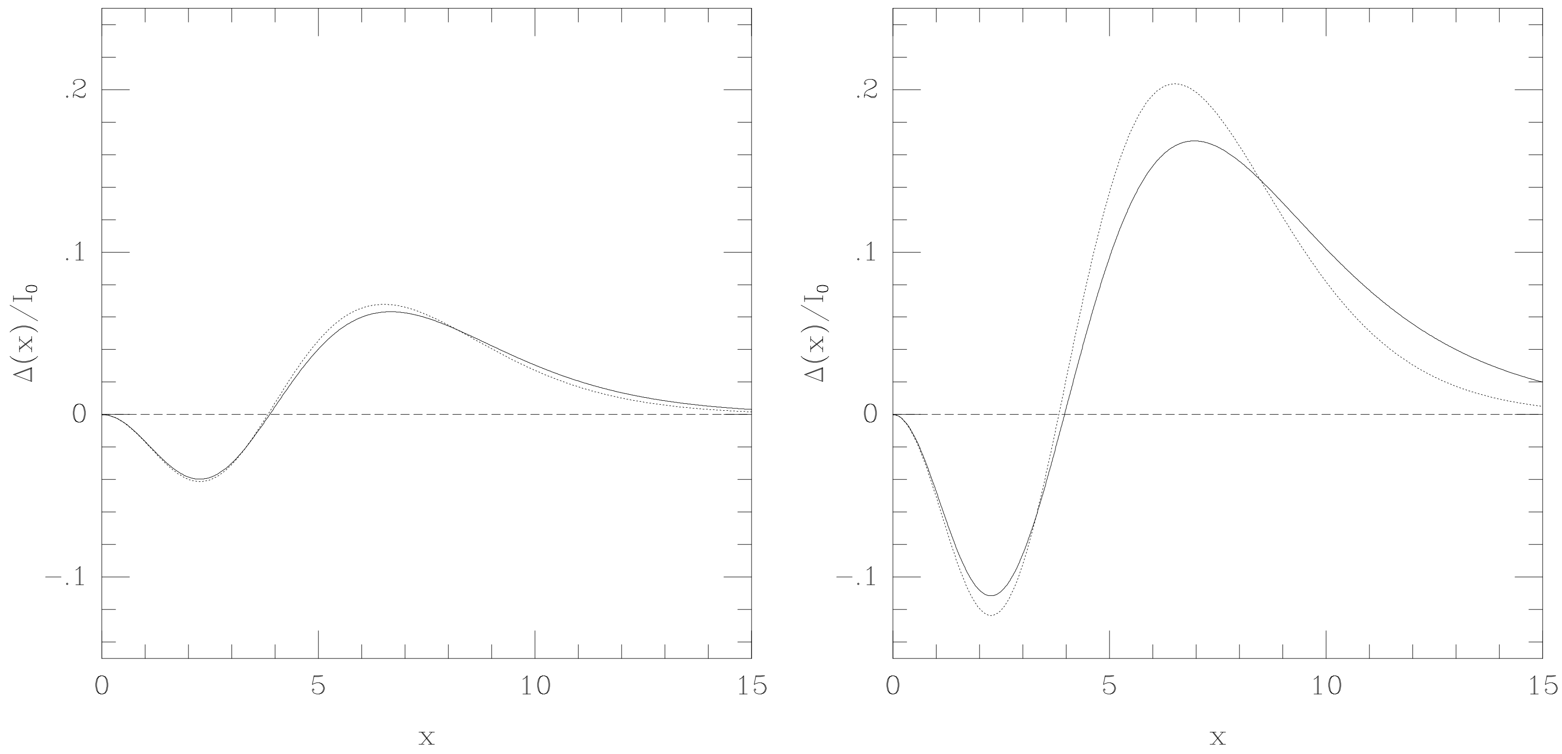
$P_1(s)$: left panel – $k_B T_e = 5.1$keV, right panel – $k_B T_e = 15.3$ keV.

5 Average over incident (blackbody) spectrum $I_0(\nu)$:

$$\Delta I(\nu) = \frac{2h}{c^2}\tau_e \int_{-\infty}^{\infty} P_1(s)\, ds \Big(\frac{\nu_0^3}{e^{h\nu_0/k_B T_{CMB}} - 1} - \frac{\nu^3}{e^{h\nu/k_B T_{CMB}} - 1} \Big);$$

here

$$s = \log(\nu/\nu_0).$$



SZ effect: left panel – $k_B T_e = 5.1$keV, right panel – $k_B T_e = 15.3$ keV.

Kompaneets equation

Nonrelativistic diffusion approximation to the kinetic equation:

$$\frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}x^4\left(\frac{\partial n}{\partial x} + n + n^2\right),$$

- $n(\nu)$ – photon occupation number,
- $x = \frac{h\nu}{k_B T_e}$ – dimensionless frequency,
- $y = \frac{k_B T_e}{m_e c^2}\frac{ct}{\lambda_e}$ – dimensionless time spent among electrons,
- $\lambda_e = \frac{1}{n_e \sigma_T}$ – mean scattering free path,
- $\sigma_T = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2}$ – Thomson scattering cross section.

Derivation in Peebles (1993), pages 603–608.

Comptonization parameter:

$$y = \int n_e \frac{k_B T_e}{m_e c^2} \sigma_T \, dl$$

If $x \ll 1$, Kompaneets reduces to

$$\frac{\partial n}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} x^4 \frac{\partial n}{\partial x}.$$

If $y \ll 1$, write

$$\frac{\partial n}{\partial y} = \frac{\Delta n}{y}$$

Comptonization parameter:

$$y = \int n_e \frac{k_B T_e}{m_e c^2} \sigma_T \, dl$$

If $x \ll 1$, Kompaneets reduces to

$$\frac{\partial n}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} x^4 \frac{\partial n}{\partial x} = \frac{1}{x^2} \frac{\partial}{\partial x} x^4 \frac{\partial n}{\partial x}, \quad x = \frac{h\nu}{k_B T_{CMB}}.$$

If $y \ll 1$, write

$$\frac{\partial n}{\partial y} = \frac{\Delta n}{y}$$

Comptonization parameter:

$$y = \int n_e \frac{k_B T_e}{m_e c^2} \sigma_T \, dl$$

If $x \ll 1$, Kompaneets reduces to

$$\frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x} x^4 \frac{\partial n}{\partial x} = \frac{1}{x^2}\frac{\partial}{\partial x} x^4 \frac{\partial n}{\partial x}, \quad x = \frac{h\nu}{k_B T_{CMB}}.$$

Change colour: substitute x by x.

If $y \ll 1$, write

$$\frac{\partial n}{\partial y} = \frac{\Delta n}{y}$$

The CMB spectrum is thermal:

$$n(\nu) = \left(e^{h\nu/k_B T_{CMB}} - 1\right)^{-1}, \quad n(x) = \left(e^x - 1\right)^{-1}.$$

Substitute and get:

$$\Delta n = \frac{xe^x}{(e^x-1)^2}\left(x\frac{e^x+1}{e^x-1} - 4\right) y I_0,$$

$$I_0 = 2(k_B T_{CMB})^3/(hc^2),$$

Change of n leads to change in specific intensity:

$$I(\nu) = \frac{2h\nu^3}{c^2} n(\nu),$$

$$\Delta I = g(x) y I_0,$$

$$g(x) = \frac{x^4 e^x}{(e^x - 1)^2}\left(x\frac{e^x+1}{e^x-1} - 4\right),$$

Blackbody:

$$I(\nu) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/k_B T_{CMB}} - 1\right)^{-1},$$

Rayleigh-Jeans limit:

$$I(\nu) = 2k_B T_{CMB}\nu^2/c^2,$$

define brightness temperature T_{RJ}:

$$T_{RJ}(\nu) = \frac{c^2 I(\nu)}{2 k_B \nu^2}.$$

SZ change in brightness temperature:

$$\frac{\Delta T_{RJ}}{T_{RJ}} = f(x) y,$$

$$f(x) = \left(x \frac{e^x + 1}{e^x - 1} - 4 \right)$$

Kompaneets approximation:

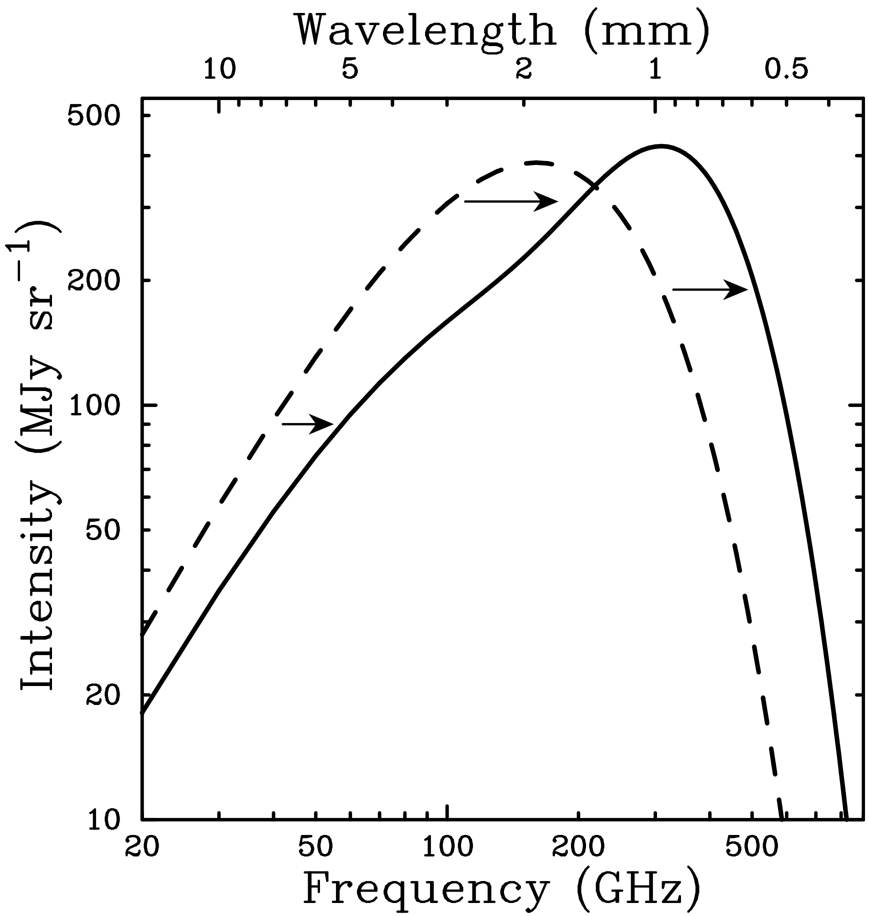
- simple spectrum
- does not depend on T_e
- amplitude depends only on y

Kinetic SZ:

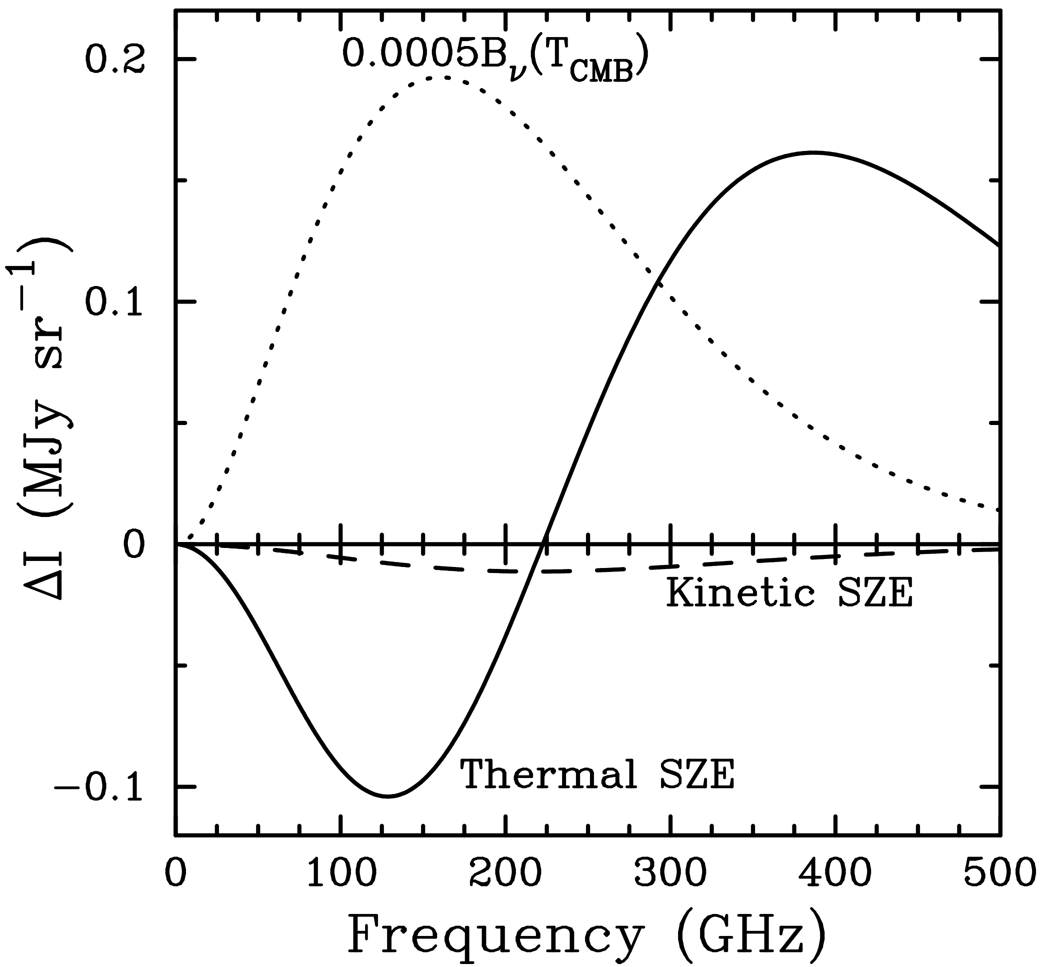
$$\Delta I = -\beta\tau_e I_0 \frac{x^4 e^x}{(e^x-1)^2},$$

$$\Delta T_{RJ} = -\beta\tau_e T_{CMB} \frac{x^2 e^x}{(e^x-1)^2},$$

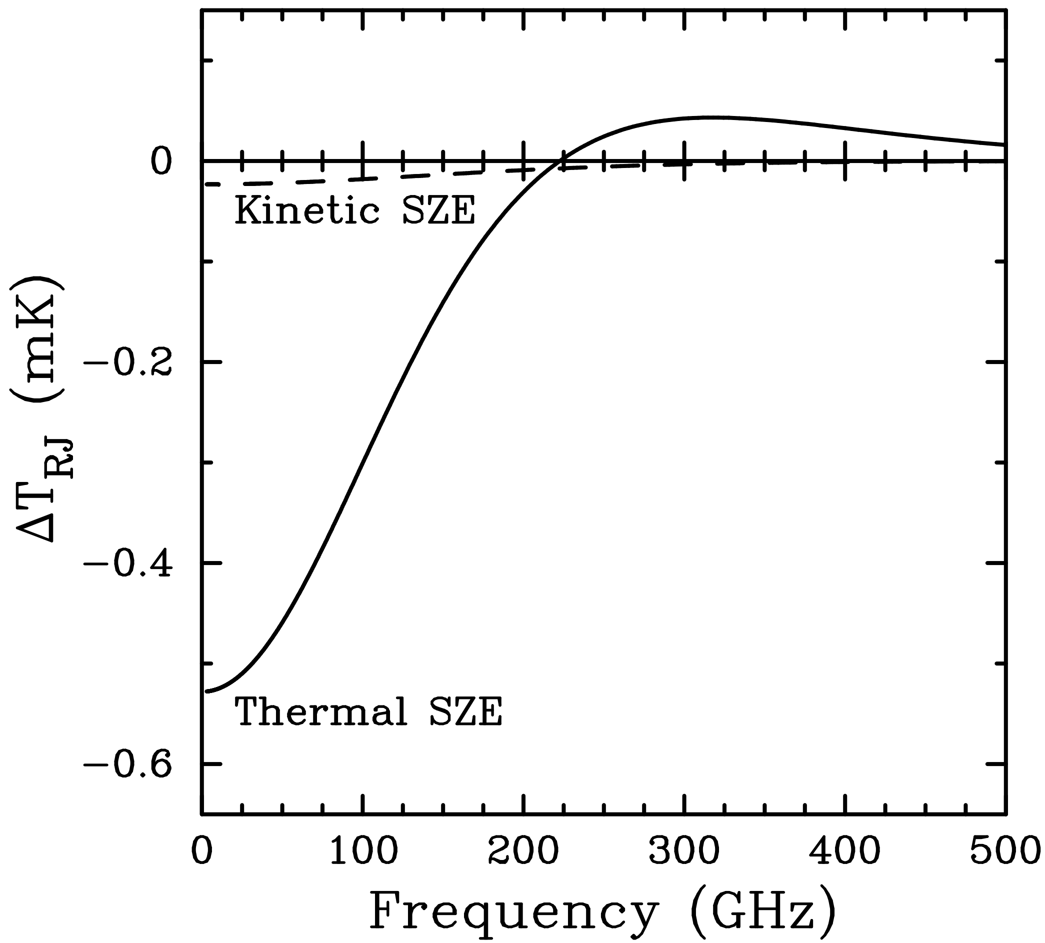
$$\frac{\Delta T_{kin}}{\Delta T_{th}} = \frac{1}{2}\frac{v_r}{c}\left(\frac{k_B T_e}{m_e c^2}\right)^{-1} =$$
$$= 0.085(v_r/1000\text{km/s})(k_b T_e/10keV)^{-1}.$$



SZE for an hypothetical $10^{18} M_\odot$ cluster.



Thermal and kinetic SZ (in specific intensity)

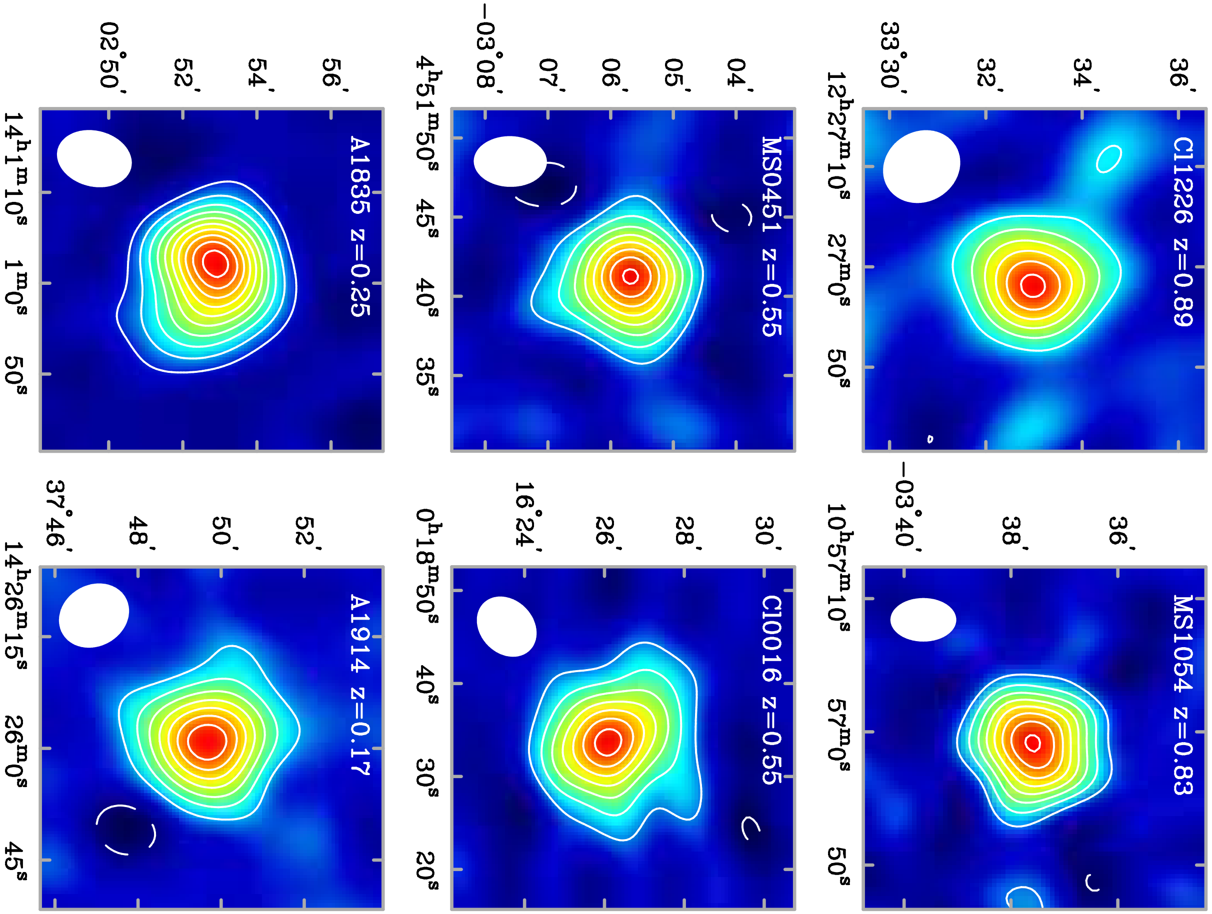


Thermal and kinetic SZ (in RJ temperature)

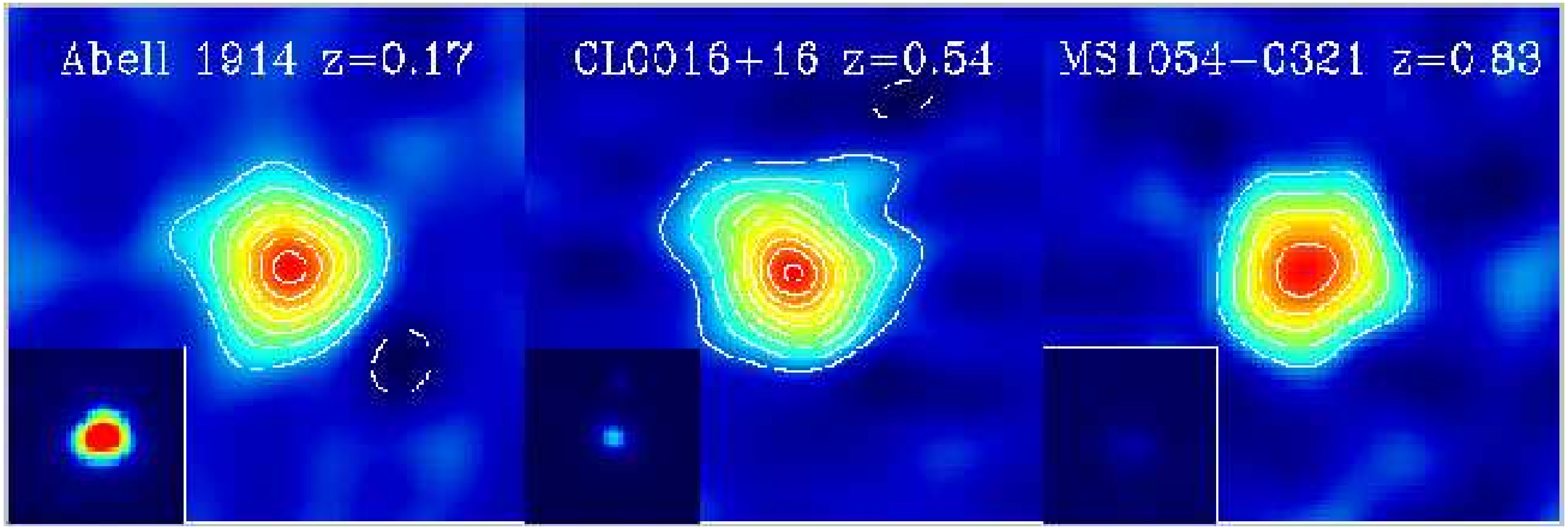
References:

- P.J.E. Peebles, Principles of Physical Cosmology, Princeton University Press, 1993 (pp. 581–608). Derives Kompaneets in 603–608.
- M. Birkinshaw, Phys. Reports 310, 97, 1999 (astro-ph/9808050).

SZ: Observations



Observed SZ clusters



SZ and X from clusters

Applications: H_0

- SZ:

$$\Delta T_{SZ} \sim \int n_e T_e \, d\ell, \qquad d\ell = D_A d\theta;$$

- X:

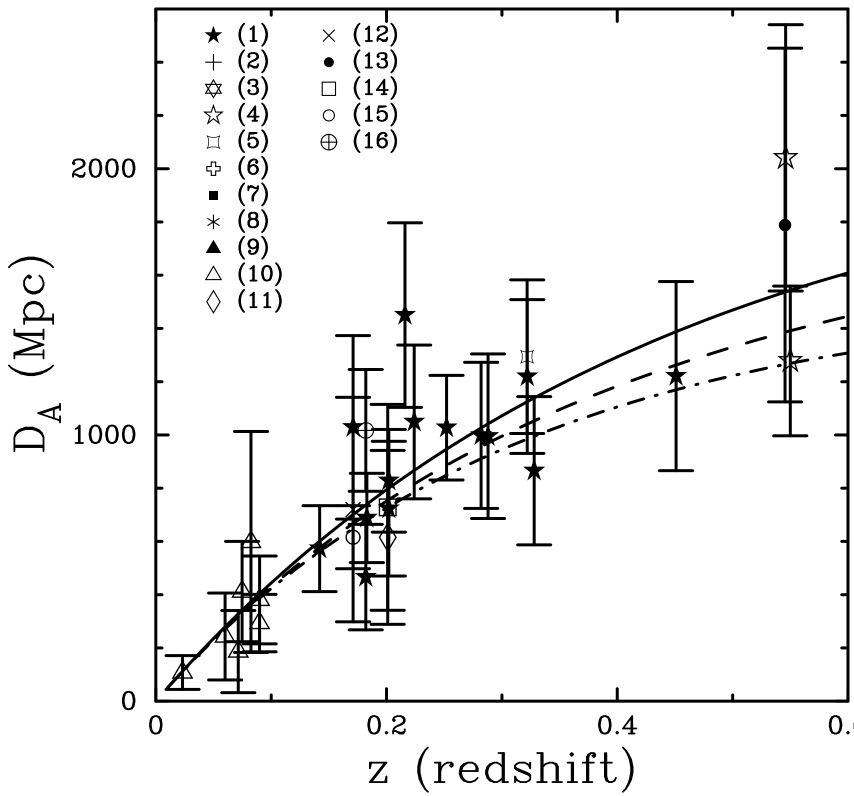
$$S_x \sim \int n_e^2 \Lambda_X .$$

$$D_A \approx \frac{1}{\theta_c} \frac{(\Delta T_c)^2 \Lambda_{Xc}}{S_{x_c} T_{ec}^2}.$$

$D_A \Rightarrow H_0$.

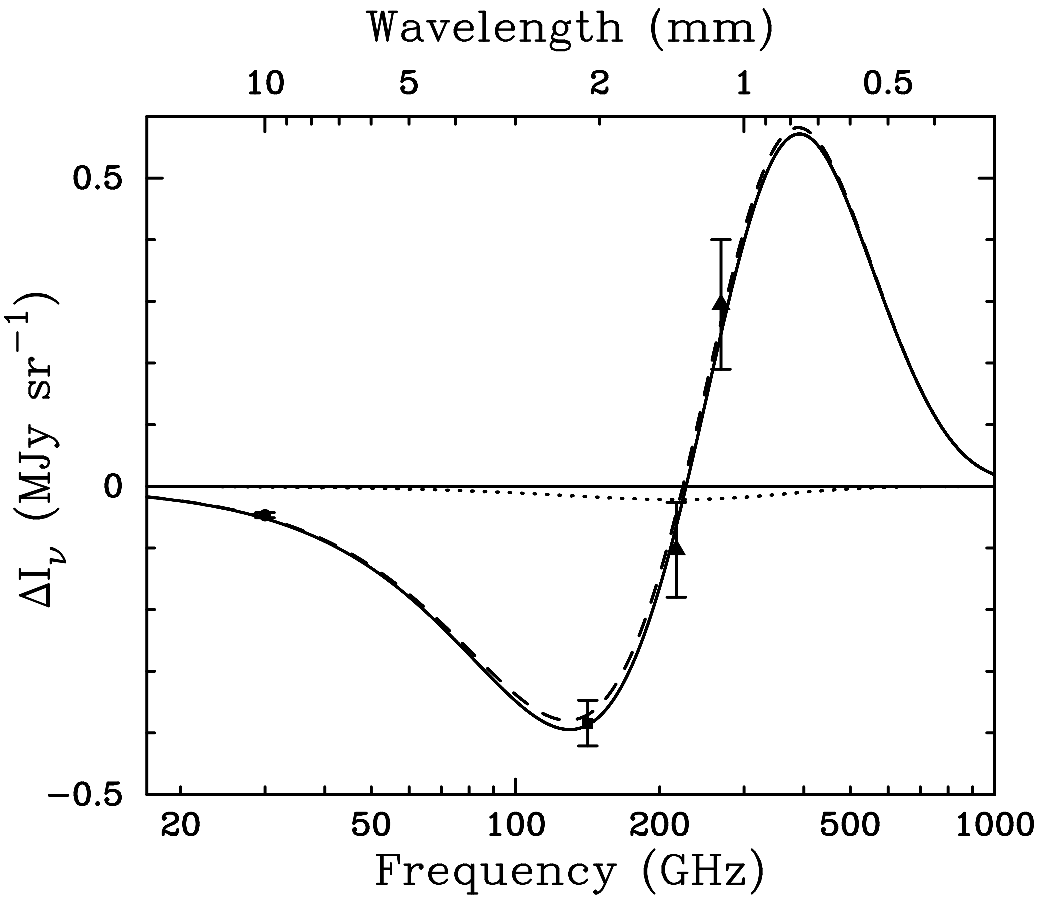
SZ Effect/X-ray H_0 Measurements

Cluster	Redshift	H_0 (km s^{-1} Mpc^{-1})	H_0 Reference
A2256	0.0581	68^{+21}_{-18}	Myers *et al.* 1997
A478	0.0881	30^{+17}_{-13}	Myers *et al.* 1997
A2142	0.0899	46^{+41}_{-28}	Myers *et al.* 1997
A1413	0.143	44^{+20}_{-15}	Saunders 1996
A2218	0.171	59 ± 23	Birkinshaw & Hughes 1994
A2218	0.171	34^{+18}_{-16}	Jones 1995
A665	0.182	46 ± 16	Hughes & Birkinshaw 1998b
A665	0.182	48^{+19}_{-16}	Cooray *et al.* 1998c
A2163	0.201	58^{+39}_{-22}	Holzapfel *et al.* 1997
Cl0016+16	0.5455	47^{+23}_{-15}	Hughes & Birkinshaw 1998a



Angular distances of SZ clusters (β-model, soon MCMC & mapping).

Applications: v_r



Fitting of kinetic and thermal SZ spectra for A2163.

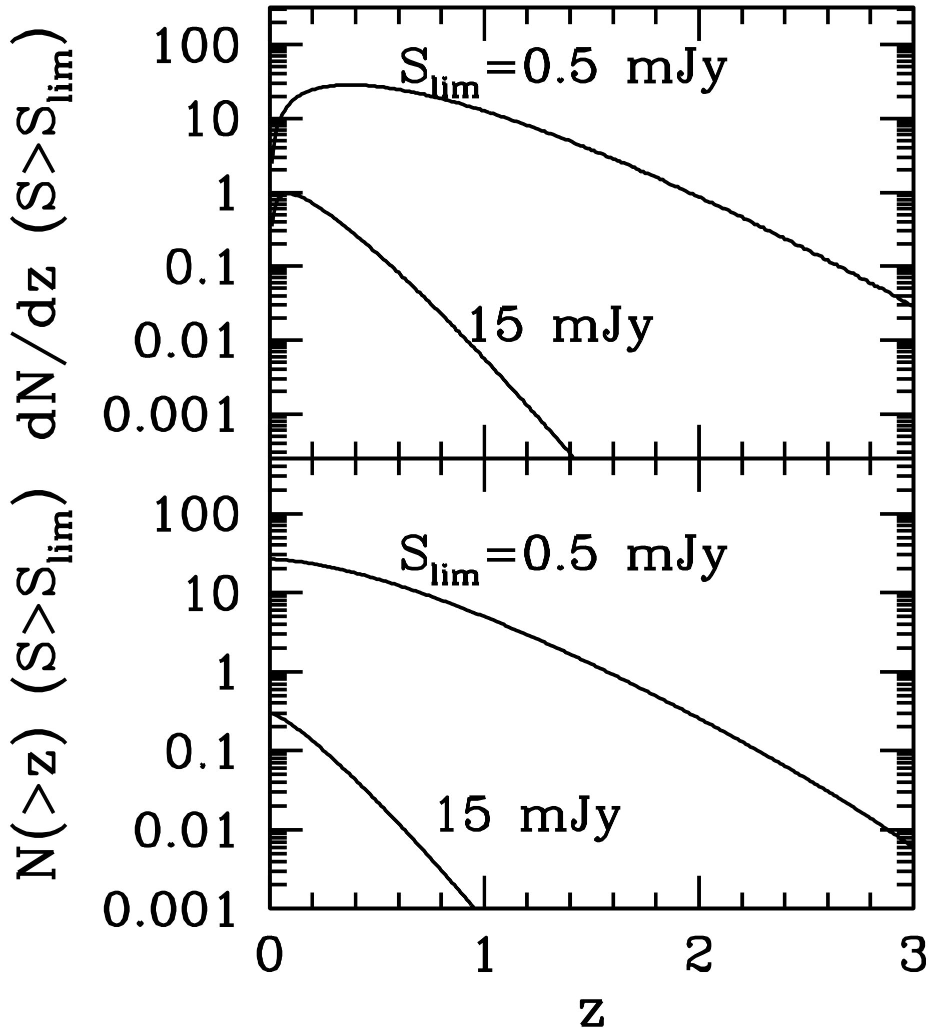
Kinetic SZ effect in clusters

Cluster	Date	Δ RA (arcsec)	$y_0 \times 10^4$	v_{pec} (km s^{-1})
A2261	Mar99	$1.3^{+11.0}_{-11.0}$	$7.58^{+2.24}_{-2.28}$	-1400^{+1725}_{-1050}
A2390	Nov00	$-2.5^{+14.3}_{-14.3}$	$1.67^{+1.03}_{-0.72}$	$+1950^{+6275}_{-2675}$
Zw3146	Nov00	$13.9^{+23.9}_{-22.9}$	$3.60^{+1.79}_{-2.61}$	-650^{+3550}_{-1875}
A1835	Apr96	$-22.5^{+12.0}_{-13.0}$	$7.54^{+1.61}_{-1.61}$	-225^{+1650}_{-1225}
Cl0016	Nov96	$-2.9^{+25.0}_{-26.9}$	$2.98^{+1.32}_{-2.59}$	-4050^{+2900}_{-1775}
MS0451	Nov96	$-1.5^{+16.0}_{-16.0}$	$2.09^{+1.87}_{-0.88}$	$+350^{+6625}_{-2925}$
	Nov97	$22.0^{+9.0}_{-9.0}$	$2.15^{+0.72}_{-0.73}$	$+1650^{+3775}_{-2125}$
	Nov00	$-25.0^{+17.0}_{-17.0}$	$3.06^{+0.83}_{-0.83}$	-300^{+1950}_{-1250}
	Combined Fit		$2.80^{+0.51}_{-0.52}$	$+750^{+1500}_{-1125}$

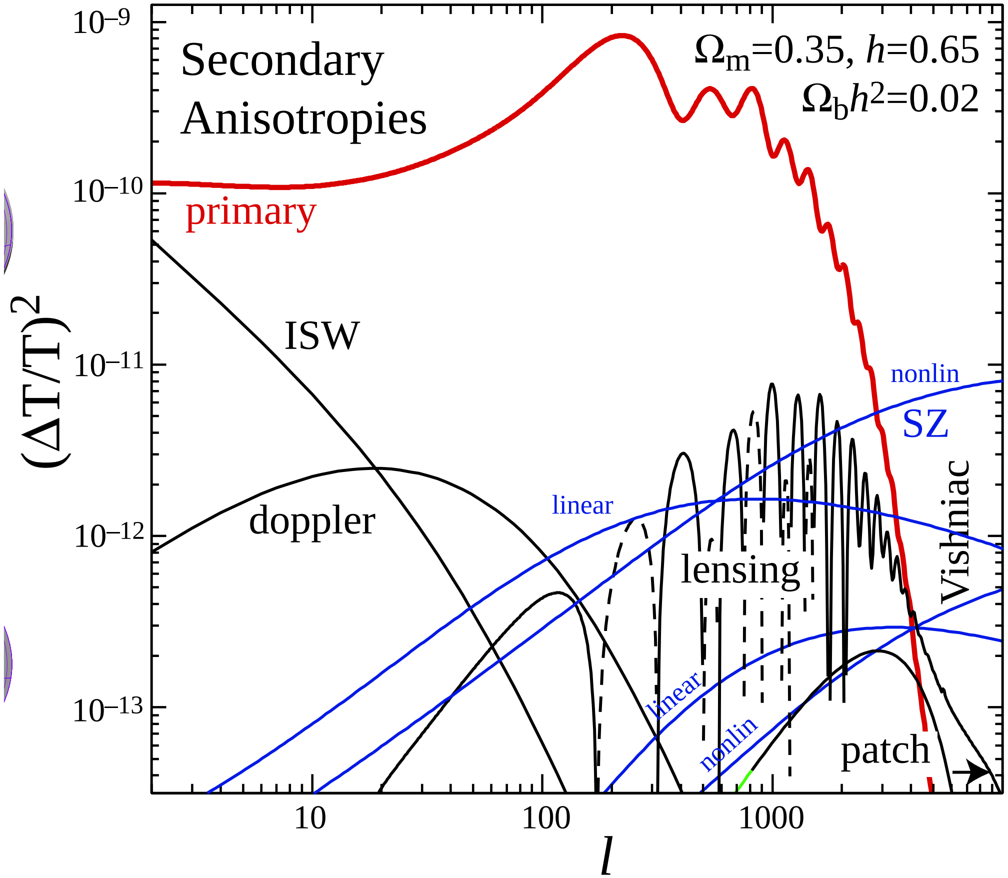
B.A. Benson et al., Ap. J. 592, 674, 2003, astro-ph/0303510, state-of-the-art SZ kinetic effect measurements.

Cluster counts

Predicted SZ cluster counts



Confusion noise



SZ secondary anisotropies

Other applications

- SZ from the Andromeda halo
- SZ from first stars
- SZ and Planck

Look also: J.E. Carlstrom, G.P. Holder, E.D. Reese, ARAA 40, 643, 2002 (astro-ph/0208192).